Superposition with Phasors

ECE 211 Circuits I

Lecture #31

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

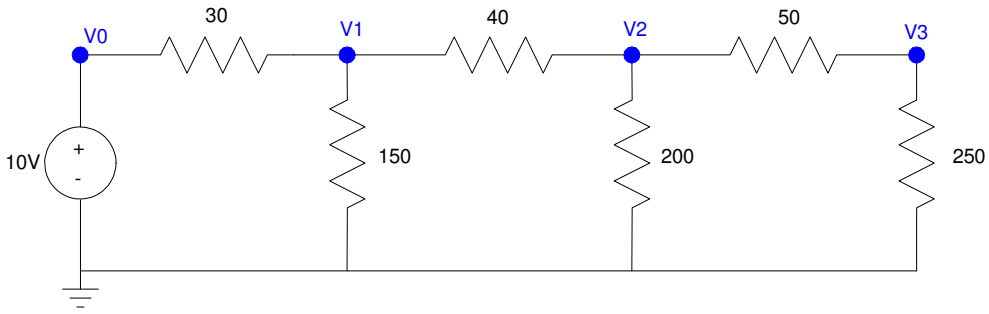
Recap

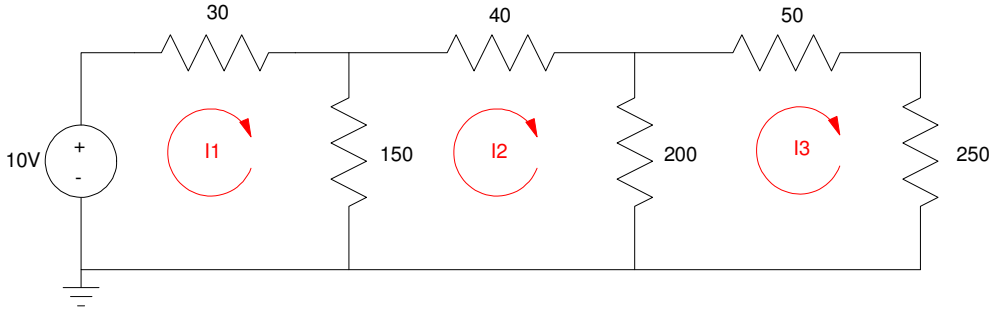
At the start of this course, we looked at circuits with constant inputs

- DC voltage and/or current sources

The resulting votlages and currents can be found using

- Real numbers,
- Voltage Nodes,
- Current Loops, and
- Thevenin equivalents





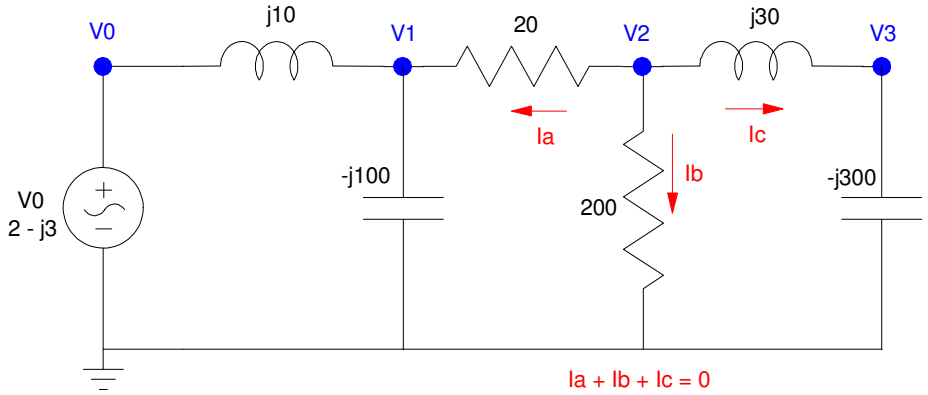
Recap (cont'd)

We then looked at circuits with sinusoidal inputs

- Votlage sources and/or current sources

The resulting votlages and currents can be found using

- Complex numbers (phasors),
- Voltage Nodes,
- Current Loops, and
- Thevenin equivalents



Problem:

How do you analyze a circuit with both?

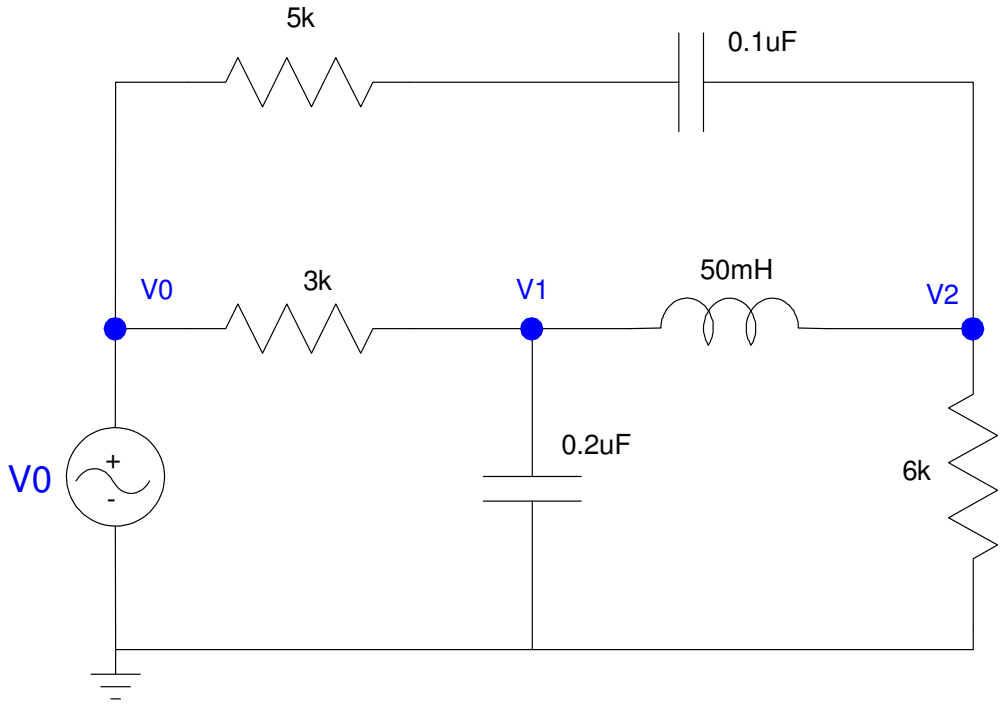
- DC source and
- AC source?

How do you analyze a circuit with

- Multiple AC sources,
- Each at a different frequency?

The answer is superposition

- Topic for this lecture



a) V0 = 2 + 3 cos(200t)

b) V0 = 3 cos(200t) + 2 sin(300t)

Linear Systems

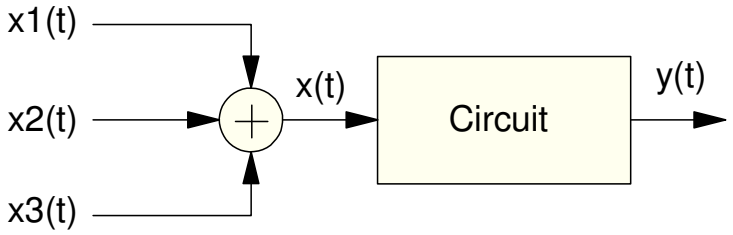
Op-Amp circuits with RLC components are linear circuits.

Linear means

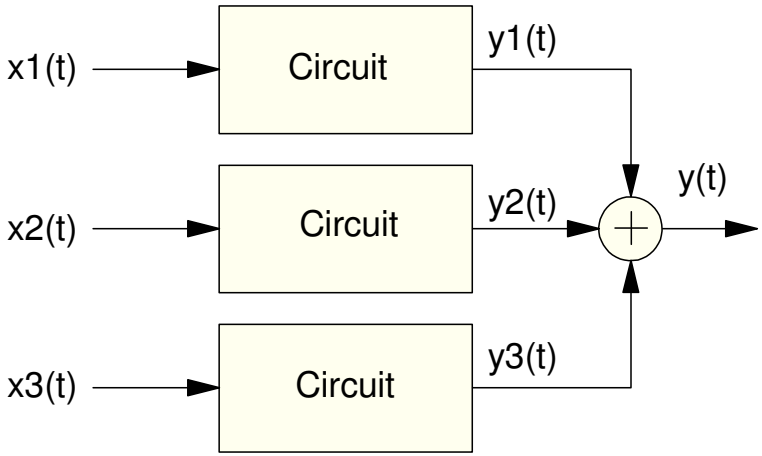
$$f(x_1 + x_2 + x_3) = f(x_1) + f(x_2) + f(x_3)$$

In English, this means that

- If the input has N terms,
- You can treat this as N separate problems
- Solve each problem separately
- Add them together to get the total output



Actual Problem



Analysis Using Superposition

DC + AC

Example: RLC Filter

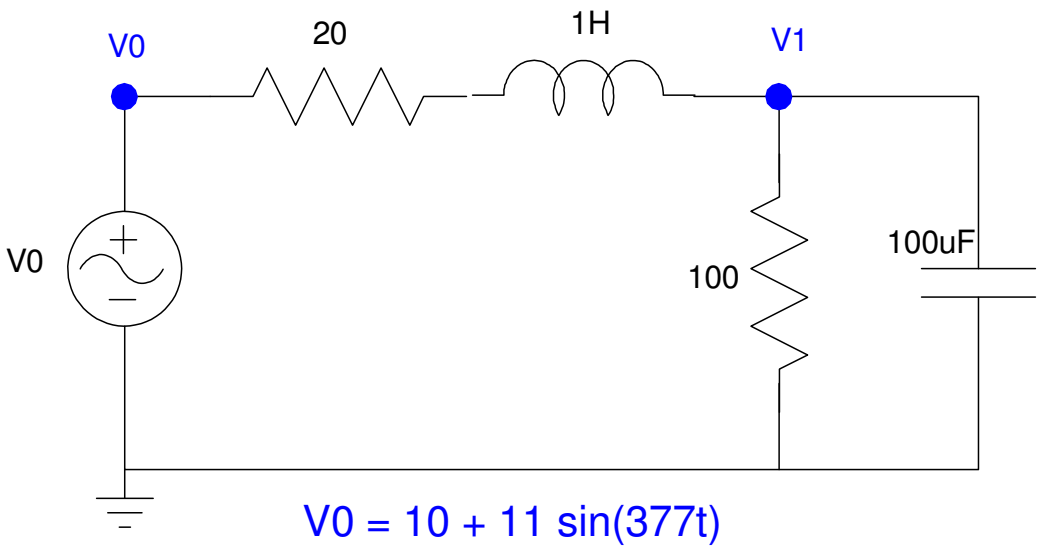
- Determing V1(t)

Why?

- Similar to a AC to DC converter
- Covered in Electronics

V1 represents

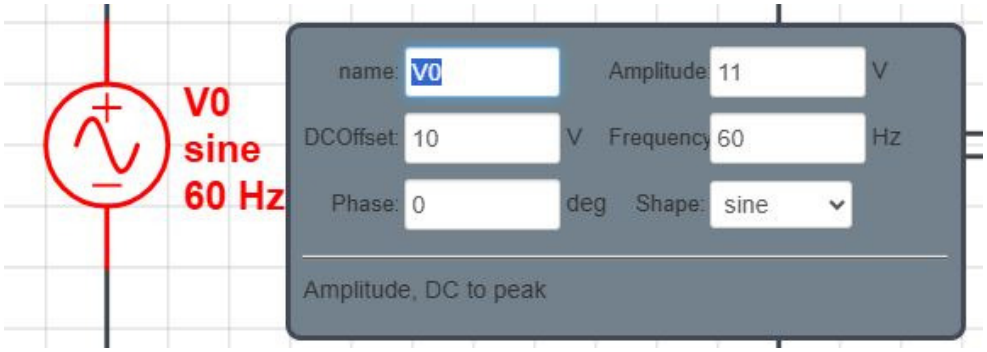
- The DC voltage generated
- With some ripple (AC)



CircuitLab

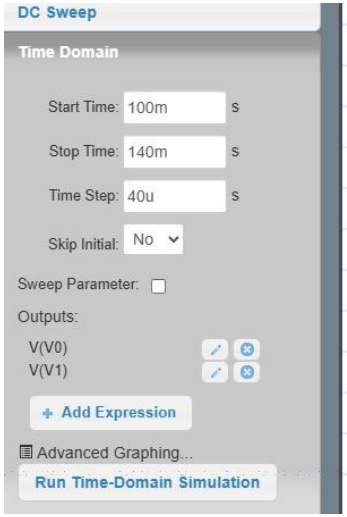
Set V0 to

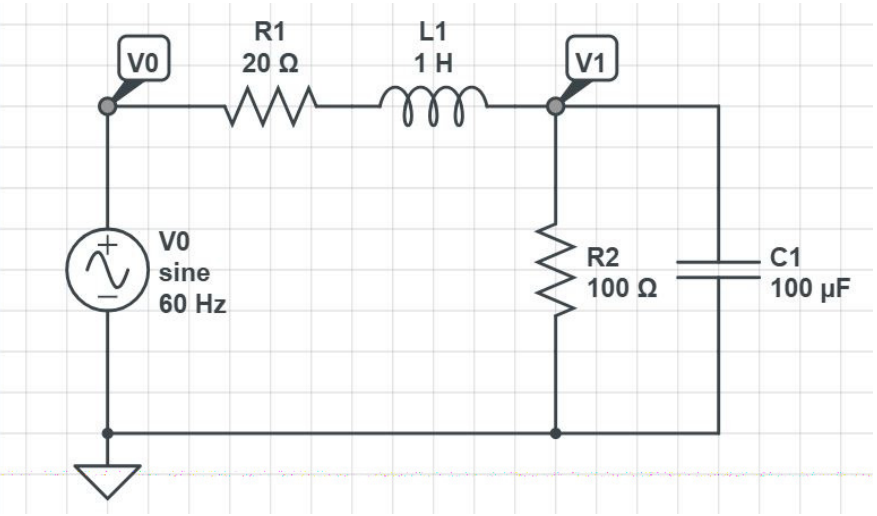
- 10V Offset (DC)
- 11V Amplitude (AC)
- 60Hz (377 rad/sec)



Time-Domain Simulation

- Start at 100ms
  - Let the transient die out
- Run for 40ms
  - 3 cycles @ 60Hz
- Step size 40us
  - 1000x smaller than run time
  - 1000 points in the plot





CircuitLab Result

V1(t) (orange line)

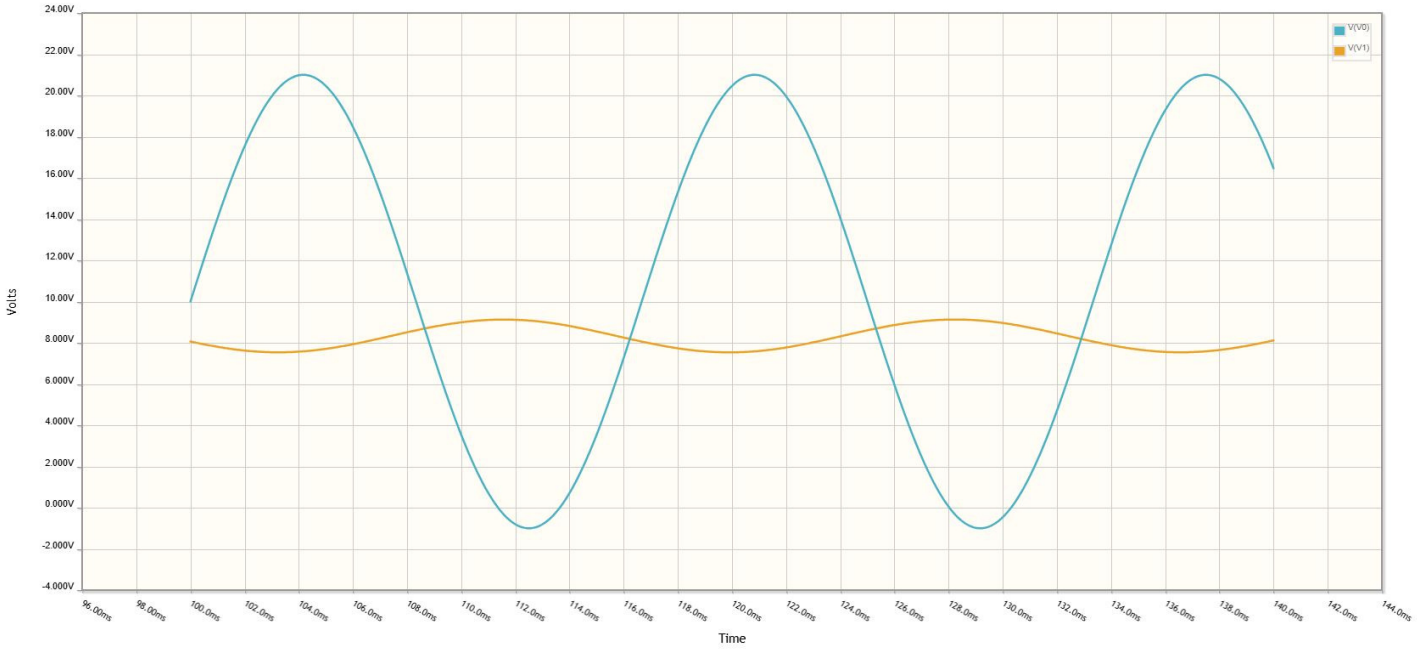
- max = 9.127V
- min = 7.536V

DC term

- 8.332V
  - (max + min) / 2

AC term

- 1.591Vpp
  - (max - min)
- 795.5mVp
  - Vpp / 2
- 562.5mVrms
  - Vp / $\sqrt{2}$



Analyze at w = 0 (DC)

$$V_0(t) = 10$$

Phasor analysis at $\omega = 0$

- $L \rightarrow j\omega L = 0$
- $C \rightarrow \frac{1}{j\omega C} = \infty$

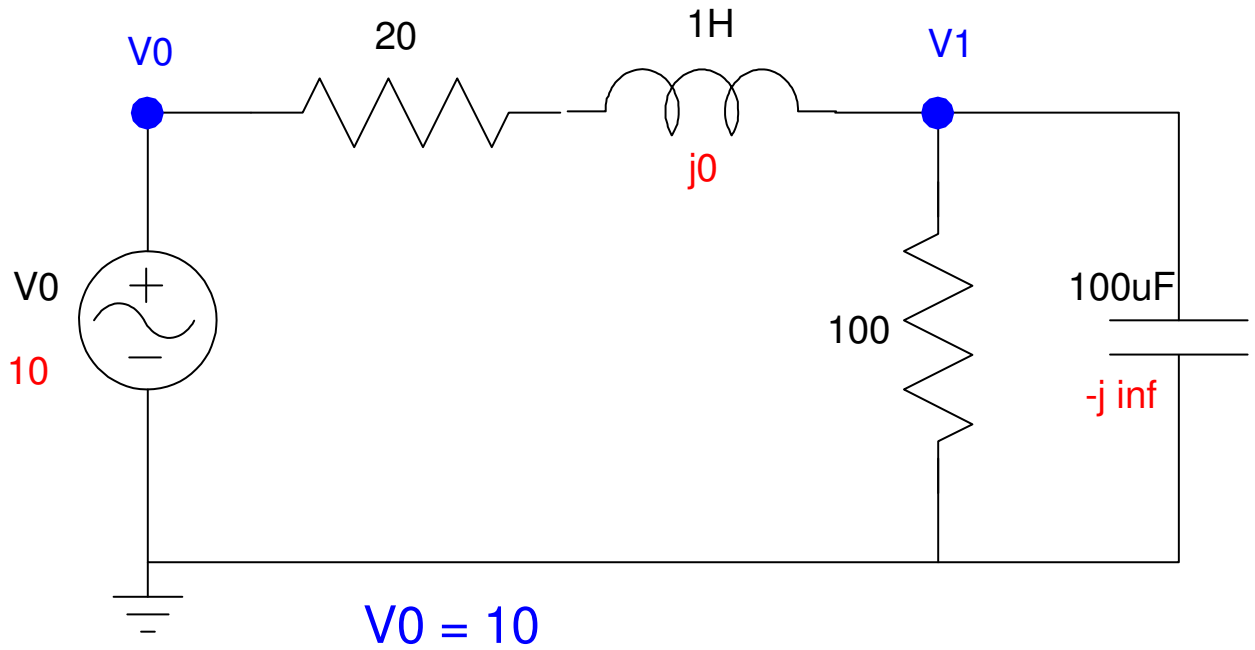
By voltage division

$$V_1 = \left(\frac{100}{100+20}\right) 10$$

$$V_1 = 8.333V$$

Same as CircuitLab

- DC term for V1

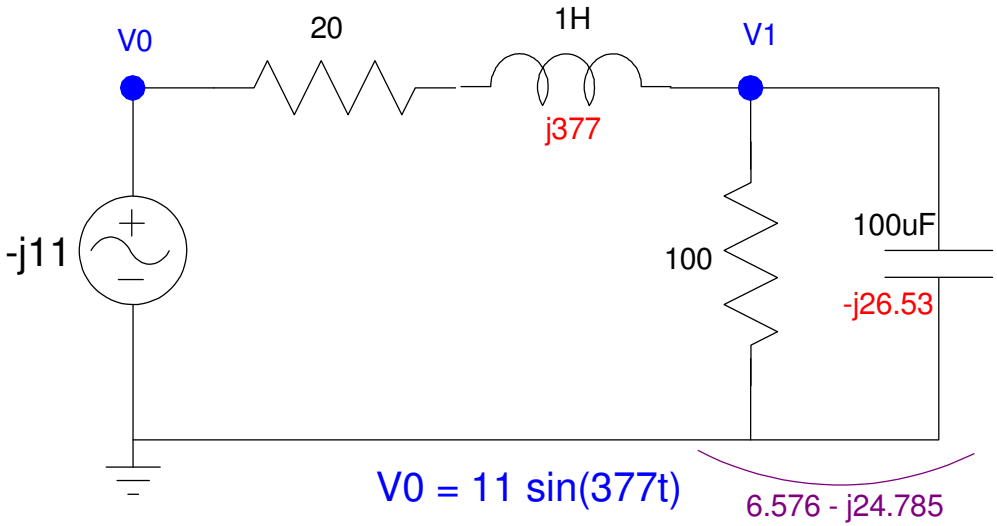


Analyze at w = 377 rad/sec

$$V_0(t) = 11\sin(377t)$$

At this frequency

- $\omega = 377$
- $V_0 = 0 - j11$
- $L \rightarrow j\omega L = j377$
- $C \rightarrow \frac{1}{j\omega C} = -j26.5$



Find V1

$$(100)||(-j26.54) = 6.576 - j24.785$$

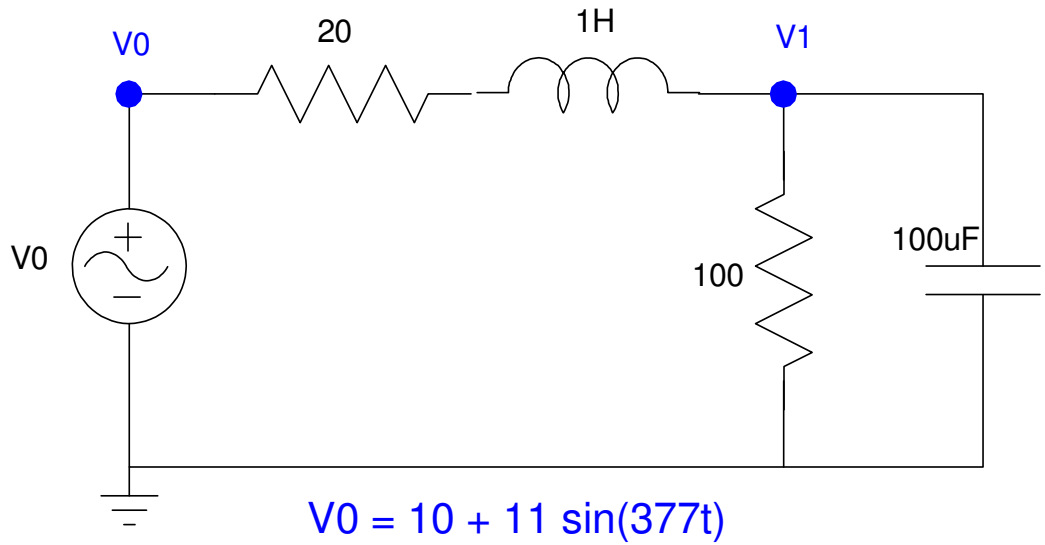
$$V_1 = \left(\frac{(6.576 - j24.785)}{(6.576 - j24.785) + (20 + j377)}\right) \cdot (0 - j11)$$

$$V_1 = -0.241 + j0.763$$

$$|V_1| = 0.800V$$

- Matches CircuitLab
- V1p = 795.5mV

Total Answer



DC:

$$V_1 = 8.333$$

$$v_1(t) = 8.333$$

AC:

$$V_1 = -0.241 + j0.763$$

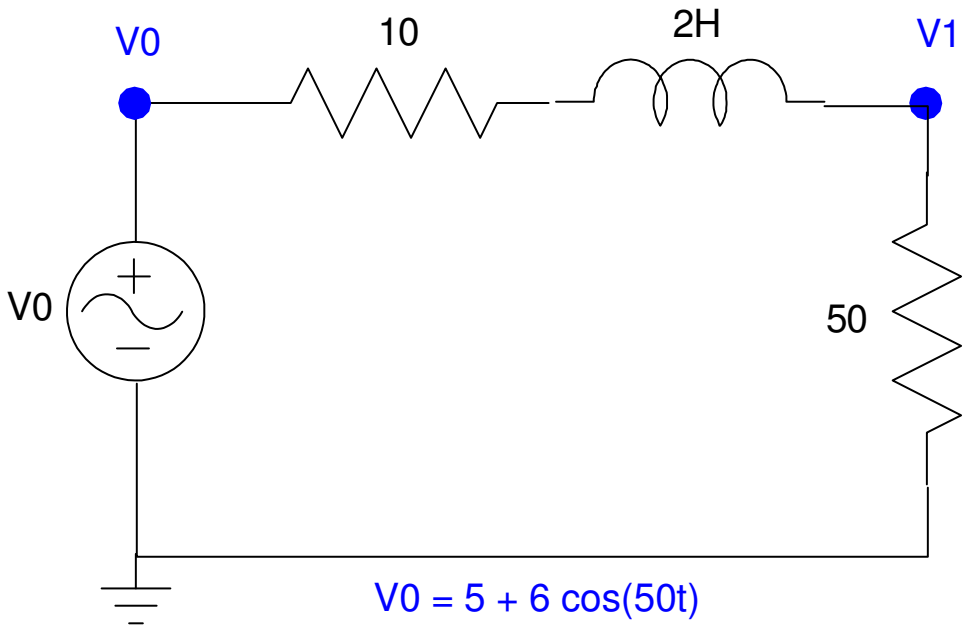
$$v_1(t) = -0.241\cos(377t) - 0.763\sin(377t)$$

Total = DC + AC

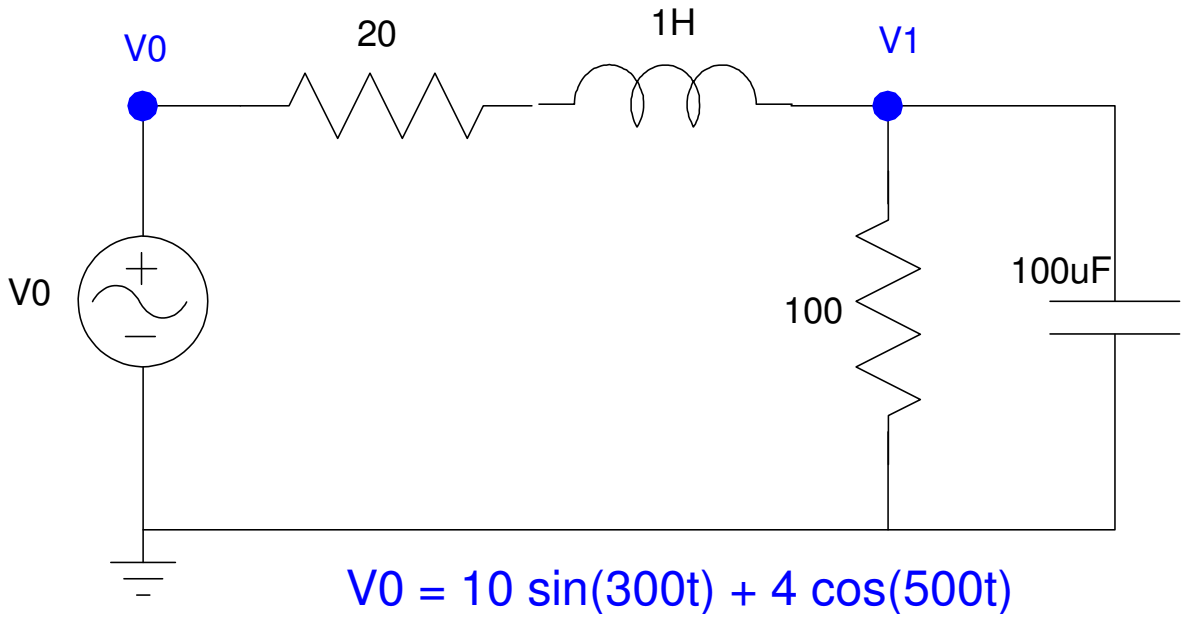
$$v_1(t) = 8.333 - 0.241\cos(377t) - 0.763\sin(377t)$$

Practice Problem

Find v1(t)



What if V1(t) has two AC terms?



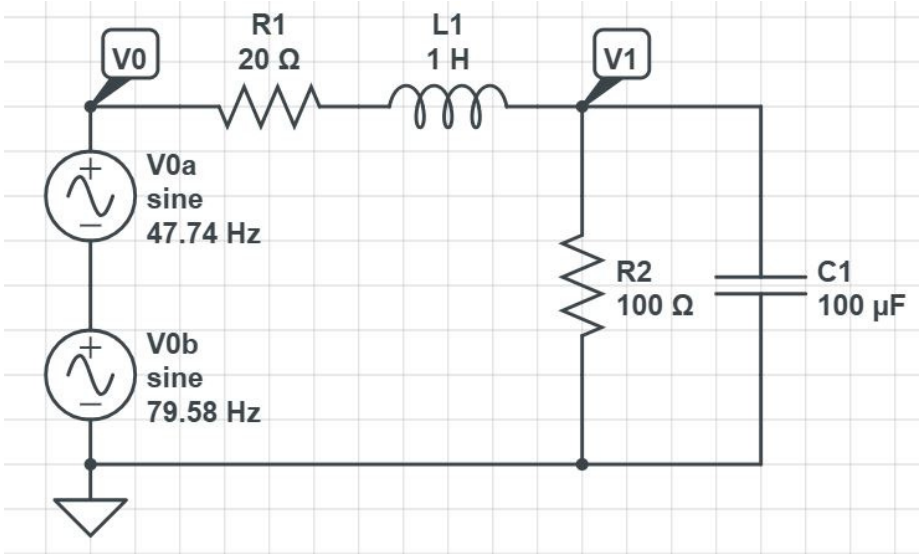
$V0 = 10 \sin(300t) + 4 \cos(500t)$

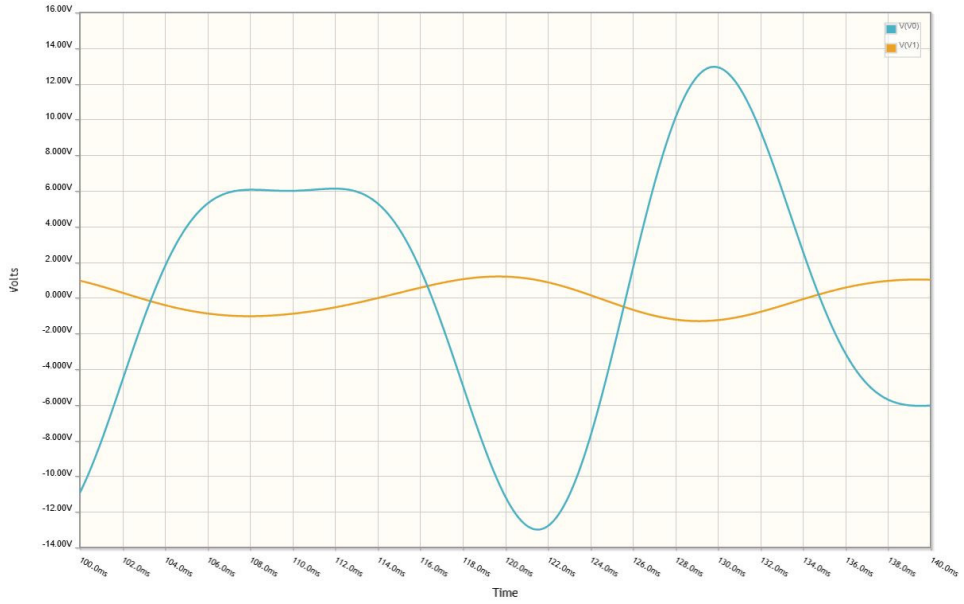
CircuitLab

Use two inputs in series

Run a transient simulation

- Somewhat erratic signal





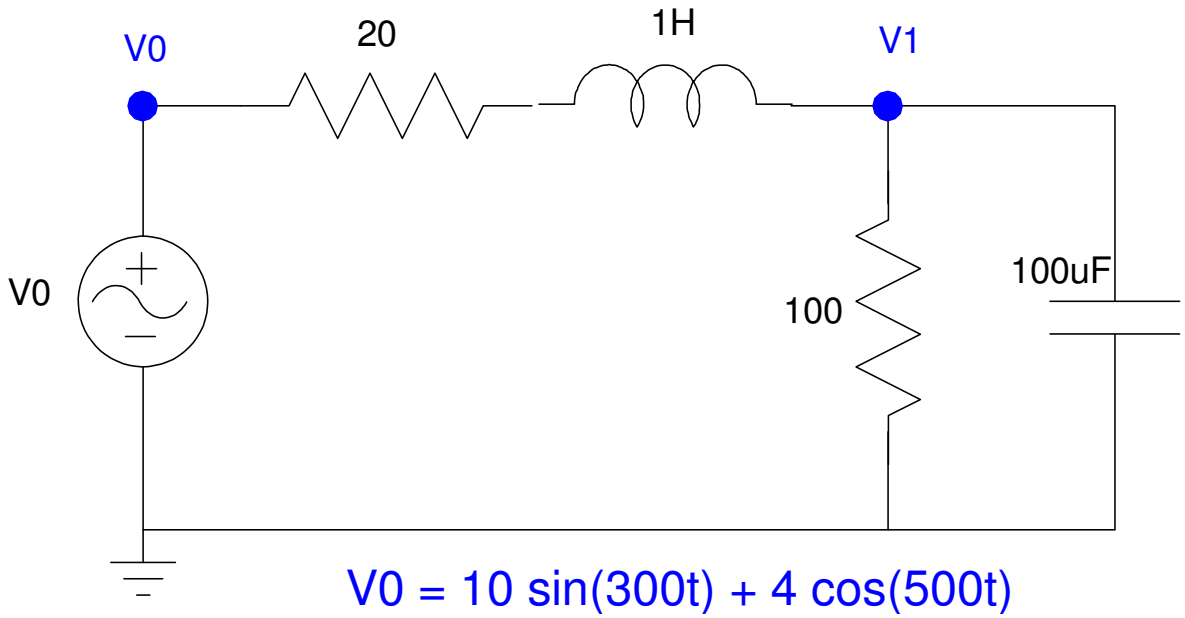
Analysis

Treat as two separate problems

- V0 = 10 sin(300t)
- V0 = 4 cos(500t)

Find V1 for each case

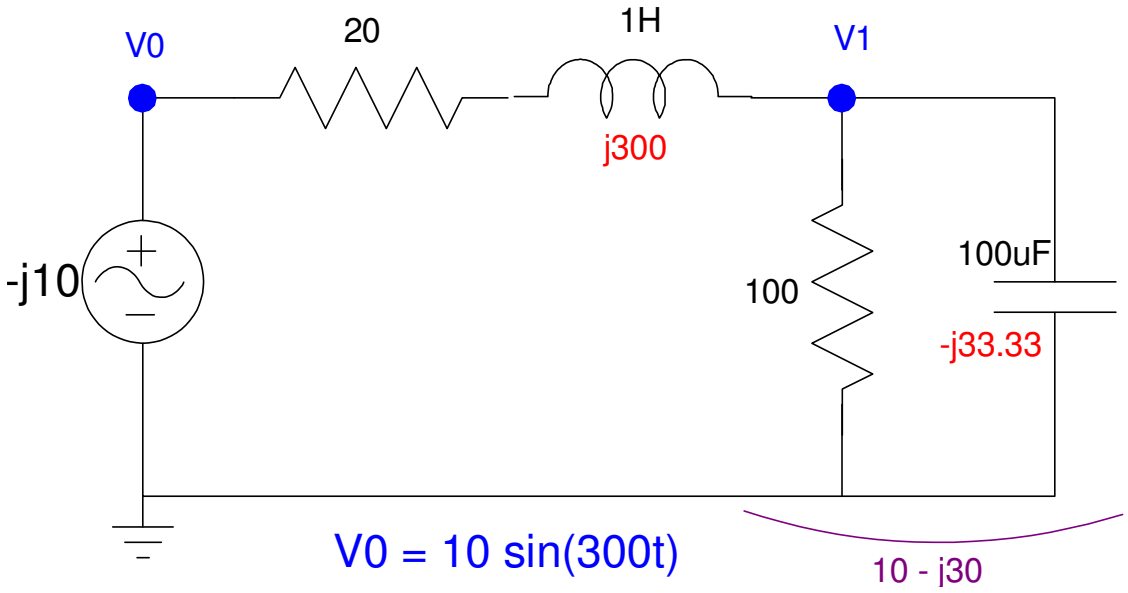
The total answer is the sum of the two



First freqency

V0 = 10 sin(300t)

- $\omega = 300$
- $V_0 = 0 - j10$
- $L \rightarrow j\omega L = j300$
- $C \rightarrow \frac{1}{j\omega C} = -j33.33$



By voltage division

$$V_1 = \left(\frac{(10-j30)}{(10-j30)+(20+j300)}\right)(-j10)$$

$$V_1 = -0.4878 + j1.0568$$

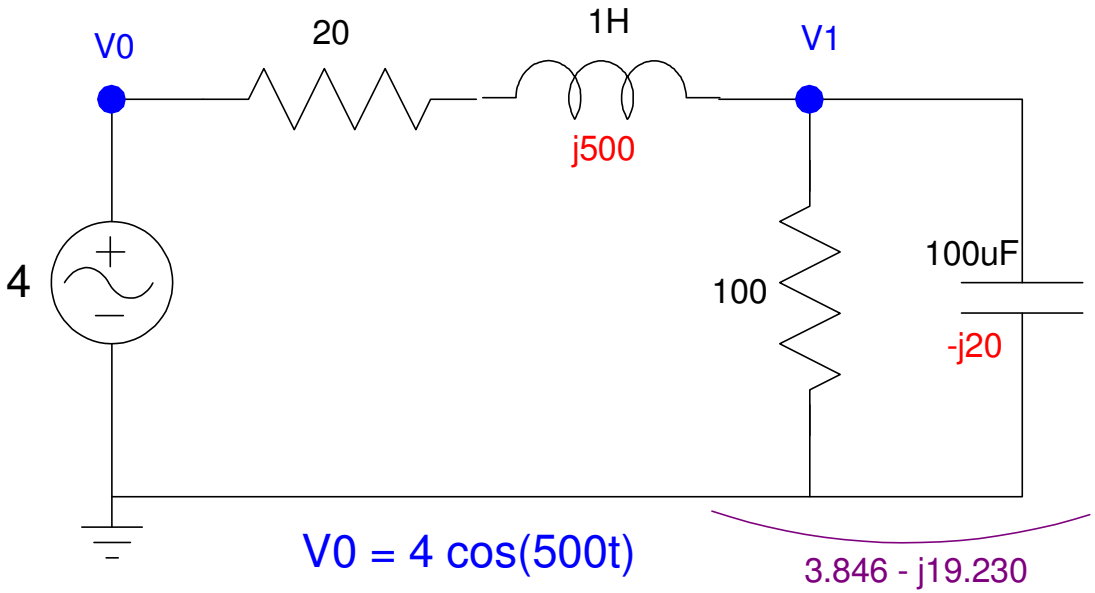
meaning

$$v_1(t) = -0.4878\cos(300t) - 1.0569\sin(300t)$$

Second Frequency

$V0 = 4\cos(500t)$

- $\omega = 500$
- $V_0 = 4 - j0$
- $L \rightarrow j\omega L = j500$
- $C \rightarrow \frac{1}{j\omega C} = -j20$



Find V1

$$V_1 = \left(\frac{(3.846 - j19.23)}{(3.846 - j19.23) + (20 + j300)}\right)(4)$$

$$V_1 = -0.1580 - j0.0398$$

meaning

$$v_1(t) = -0.1580\cos(500t) + 0.0398\sin(500t)$$

Total Answer

From superposition, v1(t) is the sum of these two answers

300 rad/sec

$$v_1(t) = -0.4878\cos(300t) - 1.0569\sin(300t)$$

500 rad/sec

$$v_1(t) = -0.1580\cos(500t) + 0.0398\sin(500t)$$

Total Answer

$$v_1(t) = -0.4878\cos(300t) - 1.0569\sin(300t) - 0.1580\cos(500t) + 0.0398\sin(500t)$$

This is the function that CircuitLab plotted for V1(t)

- Orange line

Common Mistake:

A common mistake is to try to simplify

At 300 rad/sec

$$V_1 = -0.4878 + j1.0568$$

At 500 rad/sec

$$V_1 = -0.1580 - j0.0398$$

Add the two

$$V_1 = -0.6458 + j1.0170$$

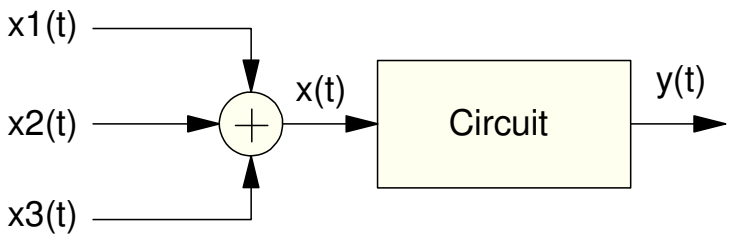
You can't do that

- Each phasor represents a different frequency
- cos(300t) + cos(500t) doesn't simplify

Summary

Superposition uses the property of linarity

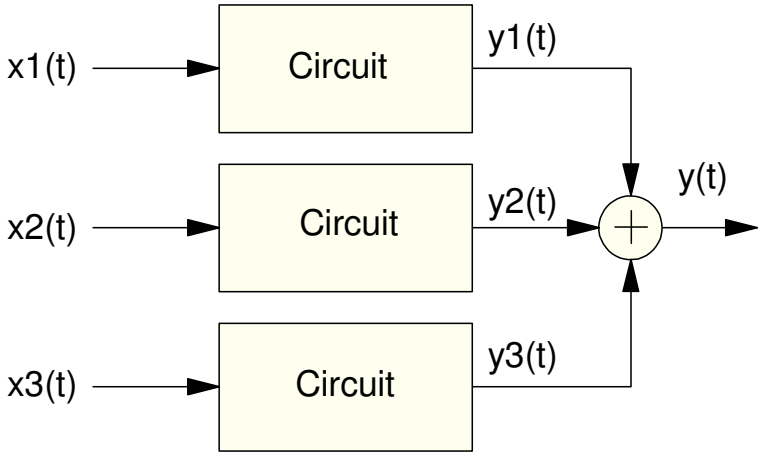
$$f(x_1 + x_2 + x_3) = f(x_1) + f(x_2) + f(x_3)$$



Actual Problem

If a circuit has an input containing two different frequencies, treat it as two separate problems

- Find the votlages and currents for each frequency separately
- Add the two together to get the total output



Analysis Using Superposition