Active Filters

ECE 211: Circuits I

Lecture #29

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Background:

Up to now, we've been analyzing RLC circuits

- With constant inputs, and
- With a sinusoidal input

If you have an RLC circuit with a specific input this works

- Phasor analysis

What if the frequency of the input varies?

Active Filters

A filter is any circuit whose behaviour changes with frequency

- Any circuit with capacitors and/or inductors
- An active filter is an RLC circuit which uses an op-amp

Op-amp circuits are usually used for filters

- They allow the gain to be > 1
- They allow you to avoid using inductors

Element	Phasor Impedance
R	R
L	jwL
C	1 / jwC

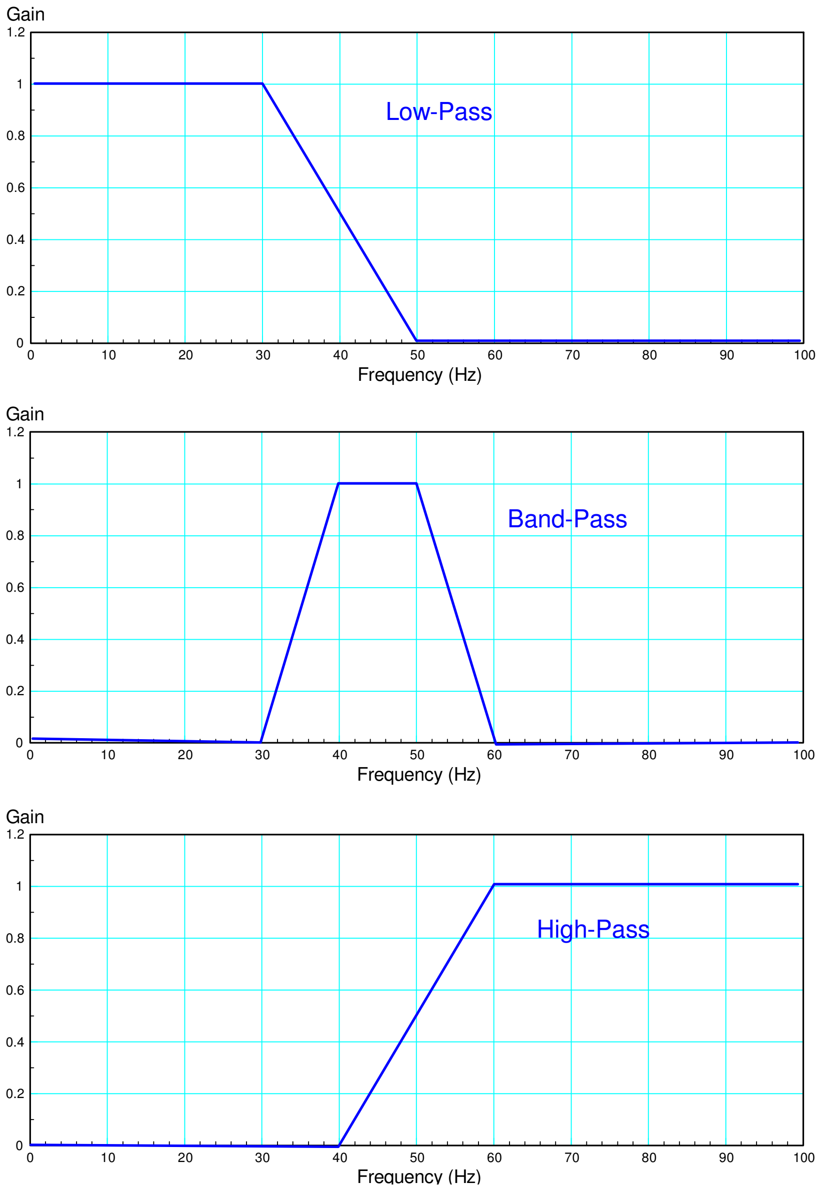
Bode Plots

If the frequency of the input is unknown or varies, the gain can be plotted vs. frequency

- The result is called a *Bode Plot*

Bode plots describe how a filter behaves

- Low-Pass:
  - Pass low frequencies
  - Reject high frequencies
- Band-Pass:
  - Pass freq over a range
  - Reject others
- High-Pass:
  - Pass high frequencies
  - Reject low frequencies



This Lecture

In this lecture, we'll look at five active filters

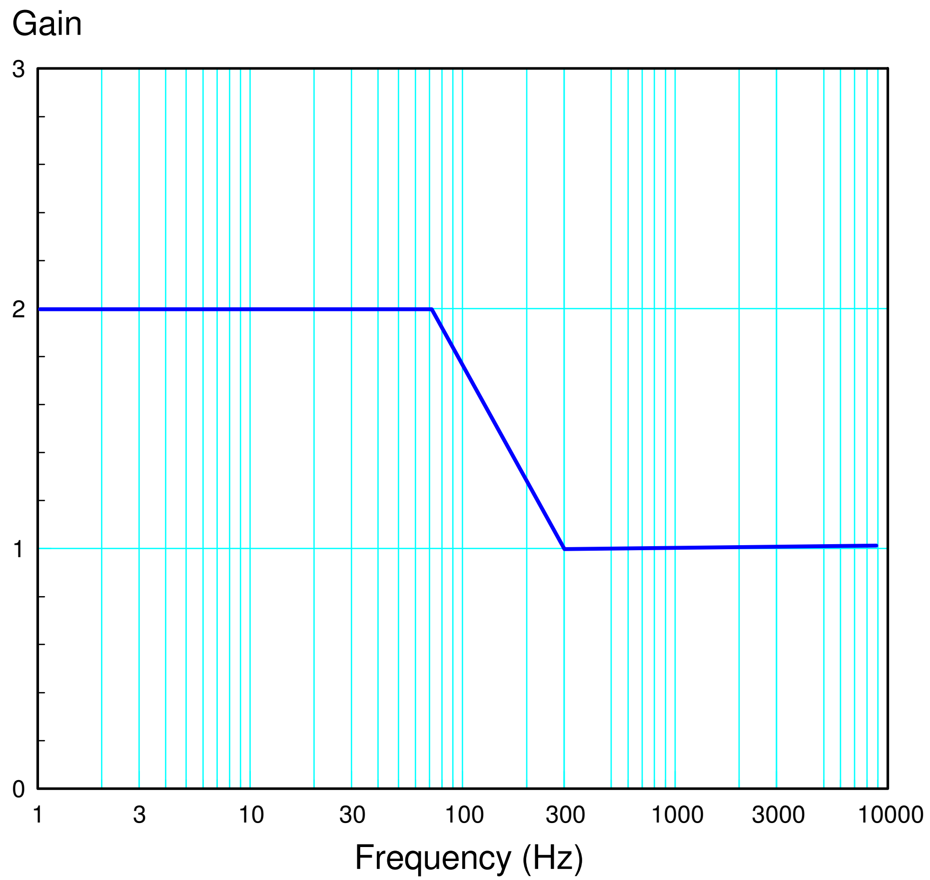
- Bass-boost
- 1st-order low-pass filter
- 2nd-order low-pass filter
- Bandpass filter

Circuits II and Electronics covers filter design in more detail.

Bass-Boost Filter

Design a filter where

- Gain = 2
  - f << 100Hz
- Gain = 1
  - f >> 100Hz



Purpose:

- Amplify the bass of a song

Bass-Boost

- Calculations

The gain will be

$$gain = -\left(\frac{Z_1}{Z_2}\right)$$

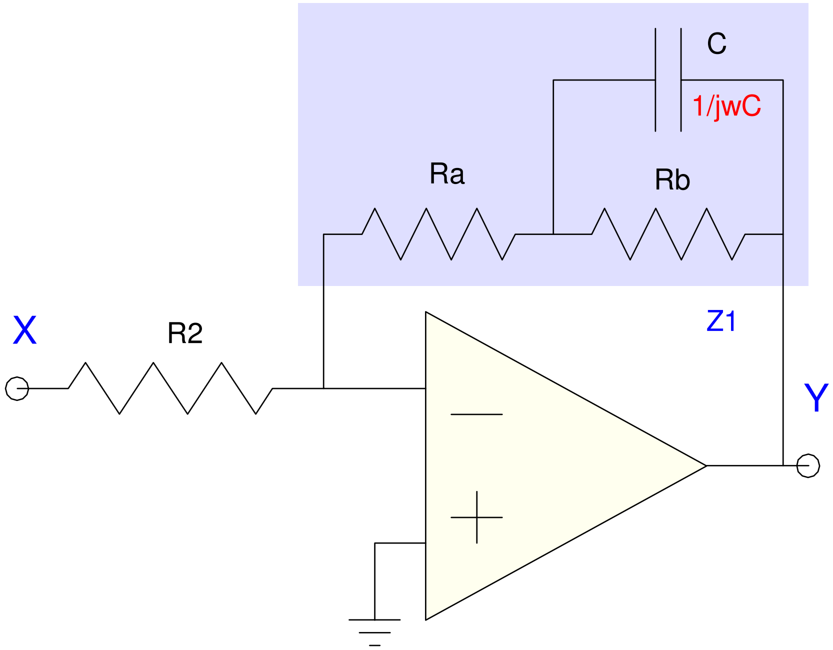
Z1 is the impedance of Ra, Rb, and C

$$Z_1 = R_a + R_b || \frac{1}{j\omega C}$$

$$Z_1 = R_a + \left(\frac{R_b}{1+j\omega R_b C}\right)$$

The overall gain is then

$$G = -\left(\frac{R_a + R_b + j\omega C R_a R_b}{R_2(1+j\omega R_b C)}\right)$$

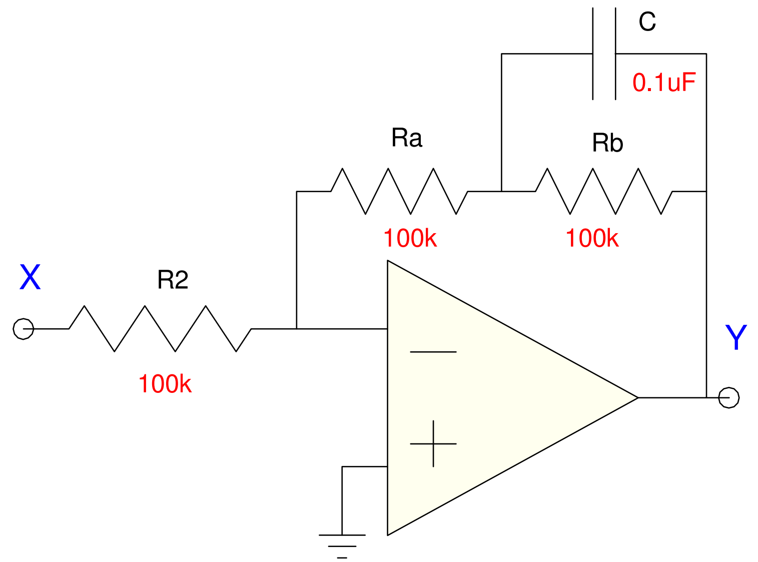


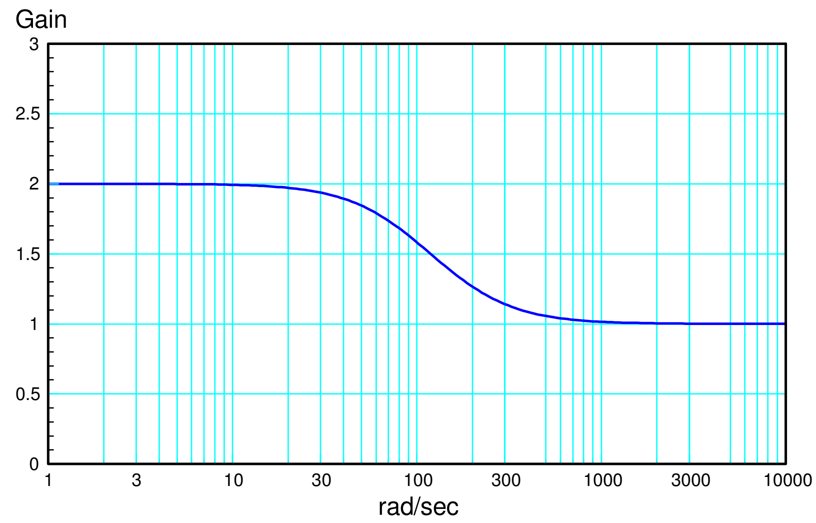
Bass-Boost: Analysis

$$G = -\left(\frac{R_a + R_b + j\omega C R_a R_b}{R_2(1 + j\omega R_b C)}\right)$$

Plot the gain vs. frequency

```
w = logspace(0,4,200)';
Ra = 1e5;
Rb = 1e5;
R2 = 1e5;
C = 0.1e-6;
num = Ra+Rb+j*w*C*Ra*Rb;
den = R2*(1+j*w*C*Rb);
G = -num ./ den;
semilogx(w,abs(G))
xlabel('rad/sec')
ylabel('Gain')
```



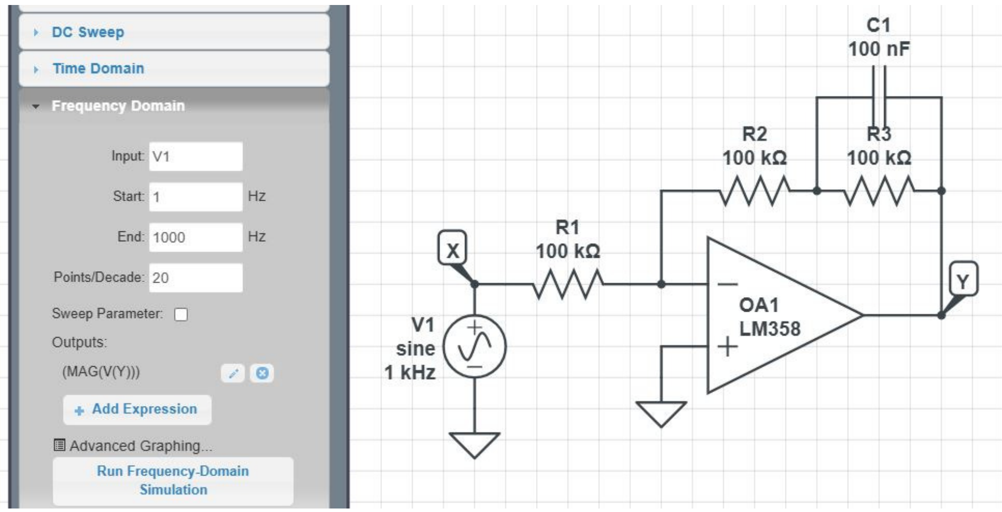


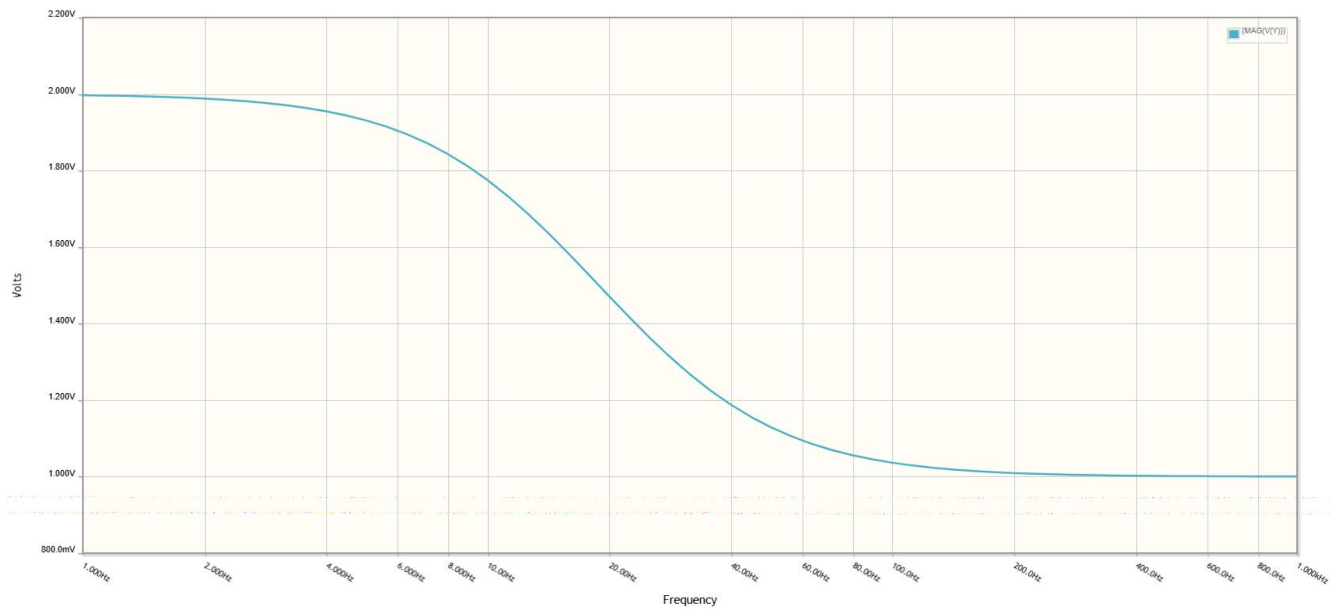
CircuitLab Check

Frequency Domain

- Set the input to V1
- Sweep from 1Hz to 1000Hz
- 20 points per decade

Matches calculations





Bass-Boost: Design

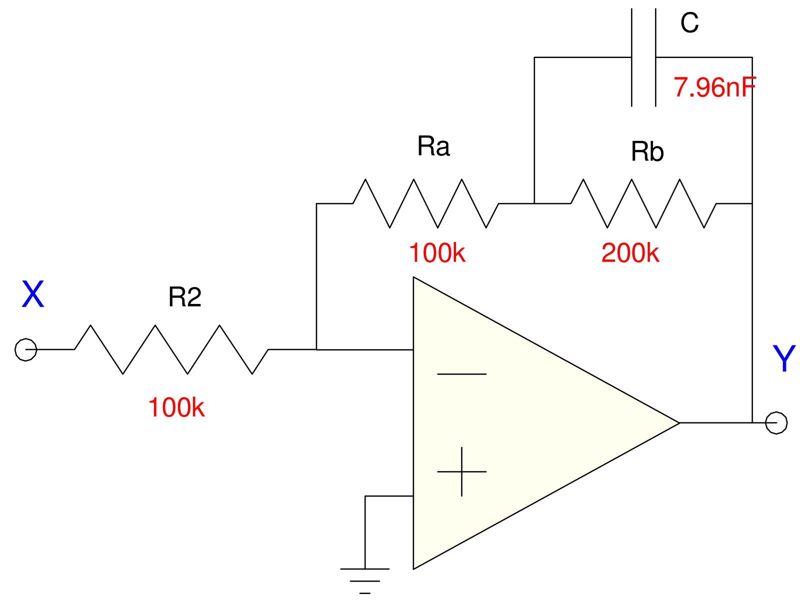
Design a circuit with

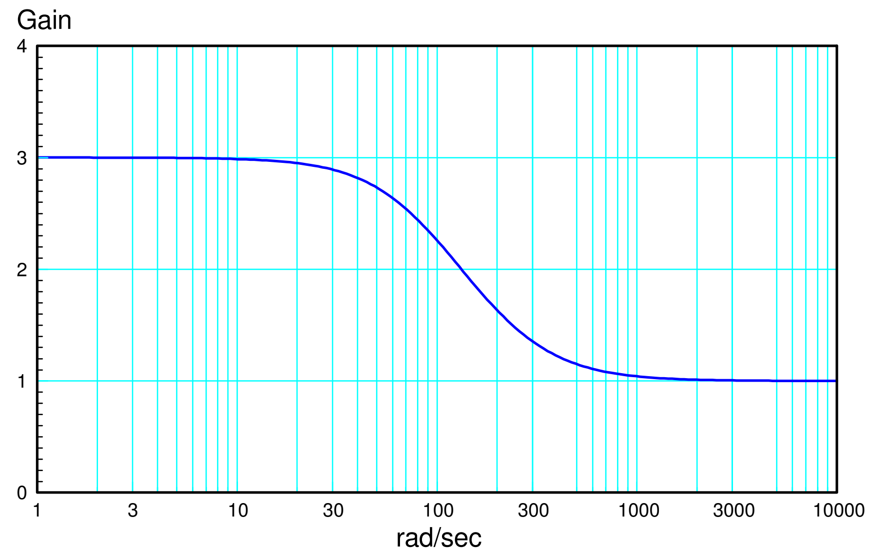
- DC gain = 3
- High-freq gain = 1
- Corner = 100Hz

Solution:

- R2 = 100k
- Ra = 100k (hf gain = 1)
- Rb = 200k (DC gain = 3)
- C = 7.76nF

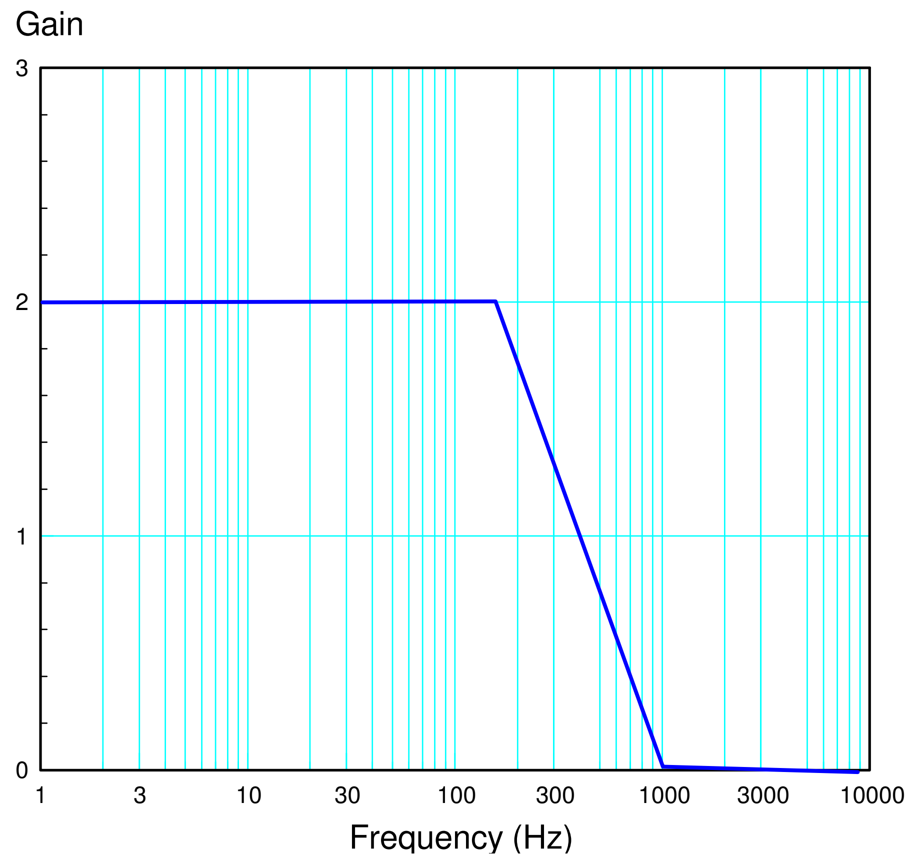
$$\omega R_b C = 1 \quad @ \ 100\text{Hz}$$





1st-Order Low-Pass Filter

- Pass low frequencies
- Reject high frequencies
- Corner = 250Hz
- DC gain = 2.00



Purpose

- Subwoofer crossover
- Remove noise from a signal

1st-Order LPF: Calculations

V1 = Vp = Vm = Y

Non-inverting amplifier

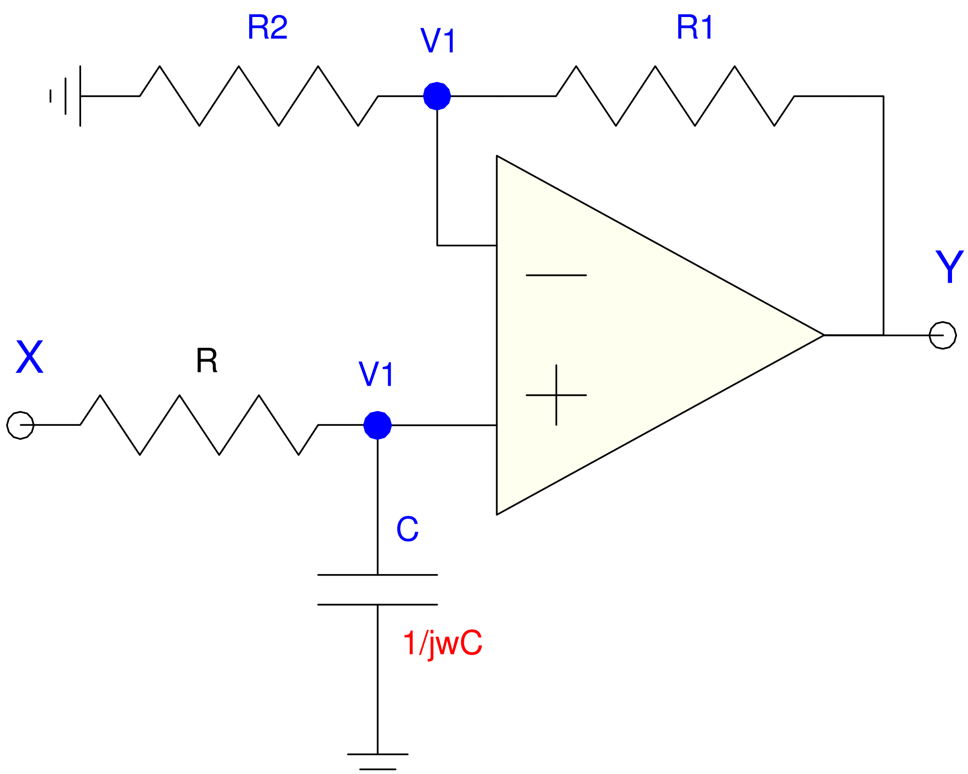
$$Y = \left(1 + \frac{R_1}{R_2}\right) V_1 = k V_1$$

Voltage Nodes

$$\left(\frac{V_1 - X}{R}\right) + \left(\frac{V_1}{1/j\omega C}\right) = 0$$

$$V_1 = \left(\frac{1}{j\omega RC + 1}\right) X$$

$$\boxed{Y = \left(\frac{k}{j\omega RC + 1}\right) X}$$



Analysis

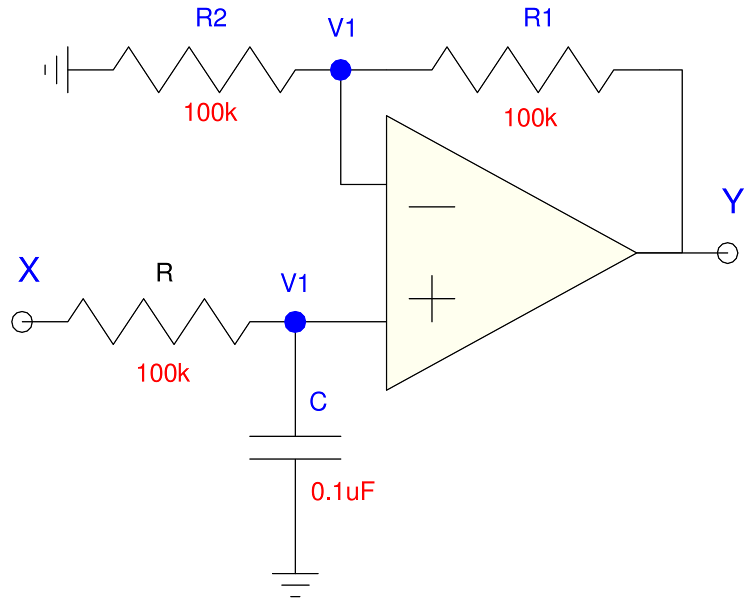
Plot the gain vs. frequency

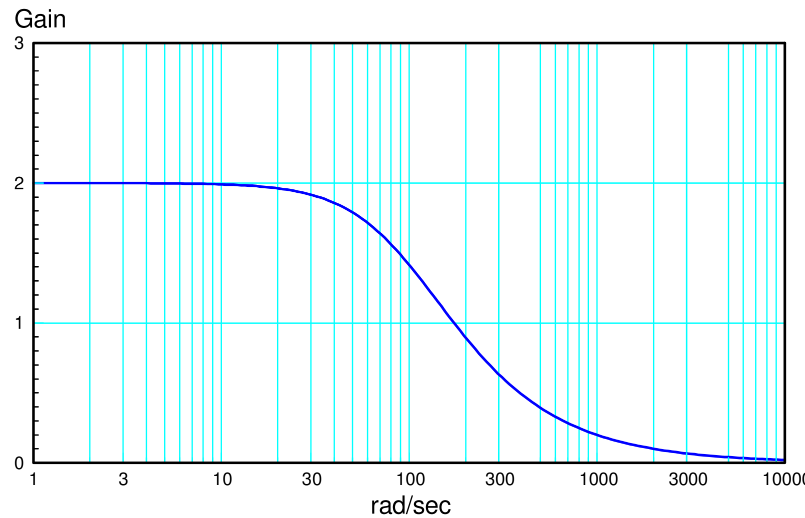
$$G = \left(\frac{k}{j\omega RC+1}\right)$$

$$k = 1 + \frac{R_1}{R_2}$$

In Matlab

```
w = logspace(0,4,200)';
R1 = 1e5;
R2 = 1e5;
R = 1e5;
C = 0.1e-6;
k = 1 + R1/R2;
G = k ./ (1 + j*w*R*C);
semilogx(w,abs(G))
xlabel('rad/sec')
ylabel('Gain')
```



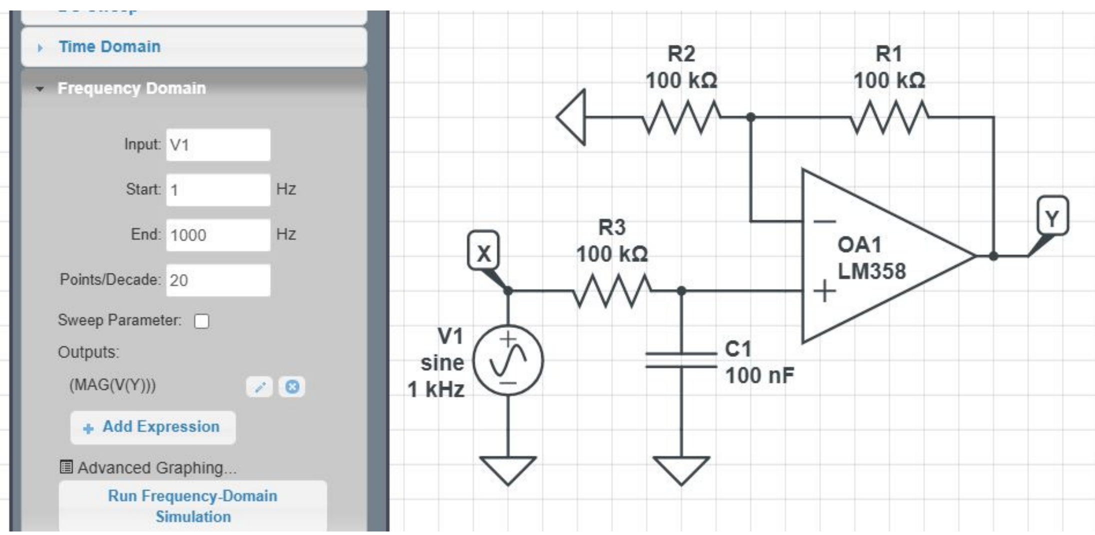


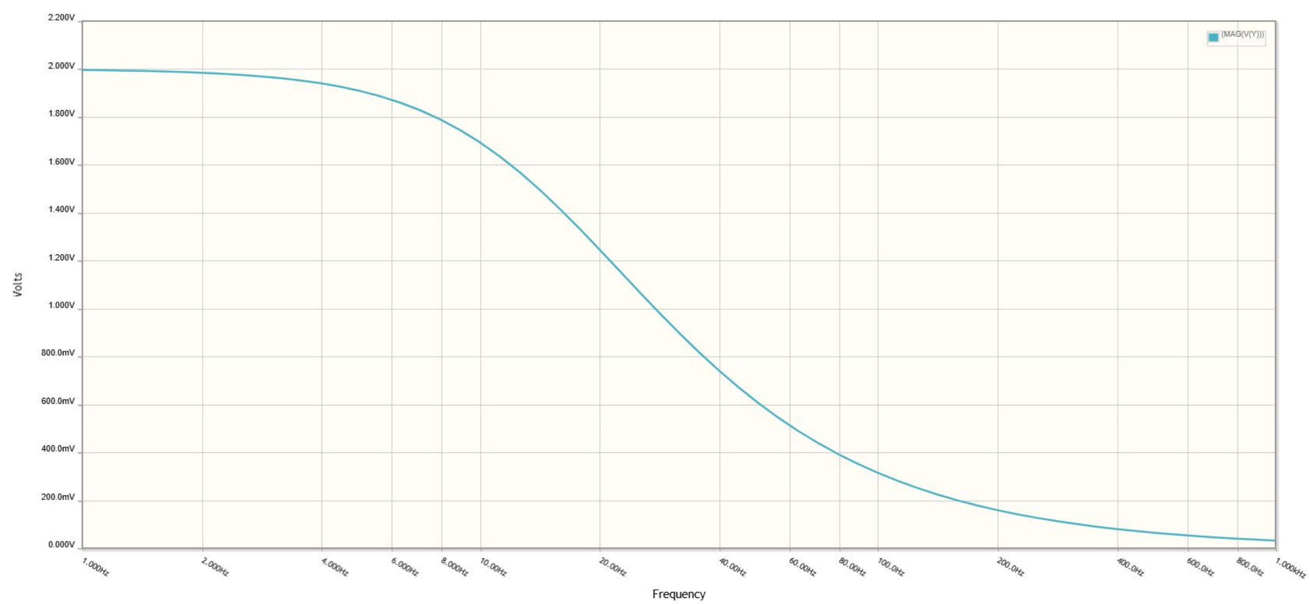
CircuitLab Check

Frequency Domain

- Set the input to V1
- Sweep from 1Hz to 1000Hz
- 20 points per decade

Matches calculations



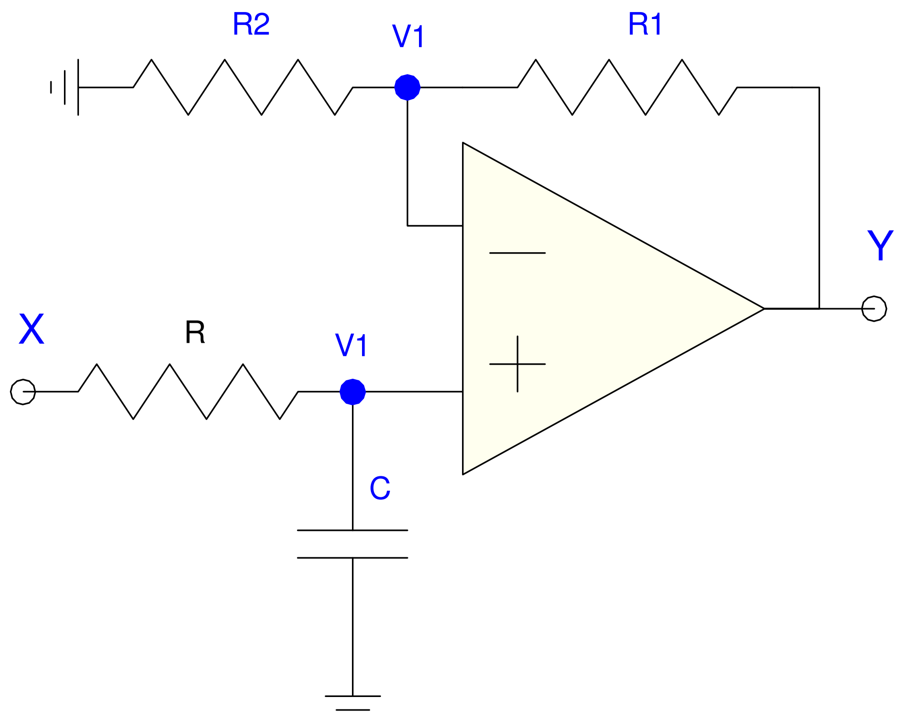


Design

$$G = \left(\frac{k}{j\omega RC+1}\right)$$

Design a filter

- DC gain = 3
- Corner = 100Hz (628 rad/sec)

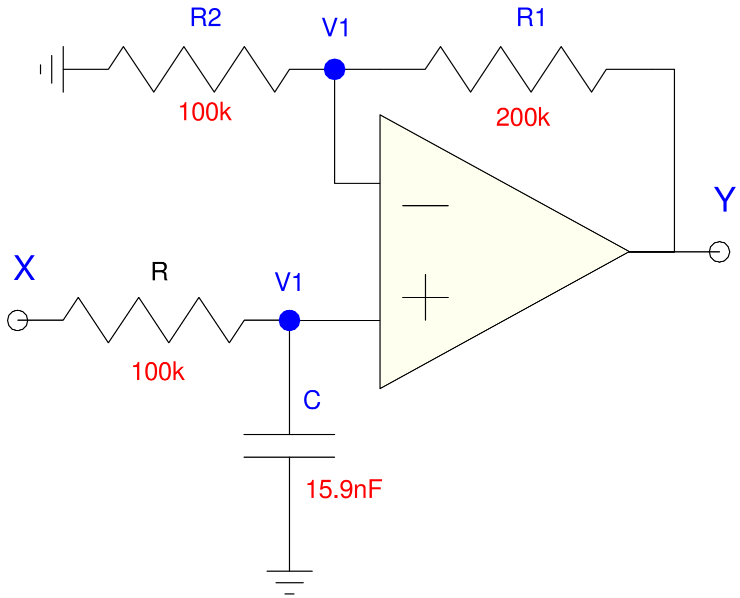


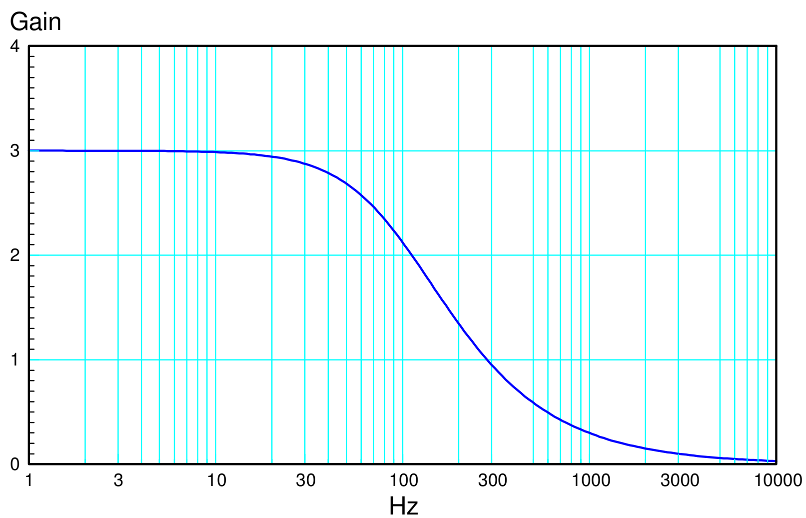
Design

Solution:

- R2 = 100k (arbitrary)
- R1 = 200k (gain = 3)
- R = 100k (arbitrary)
- C = 15.9nF

$$\omega RC = 1 \text{ @ } 100\text{Hz}$$

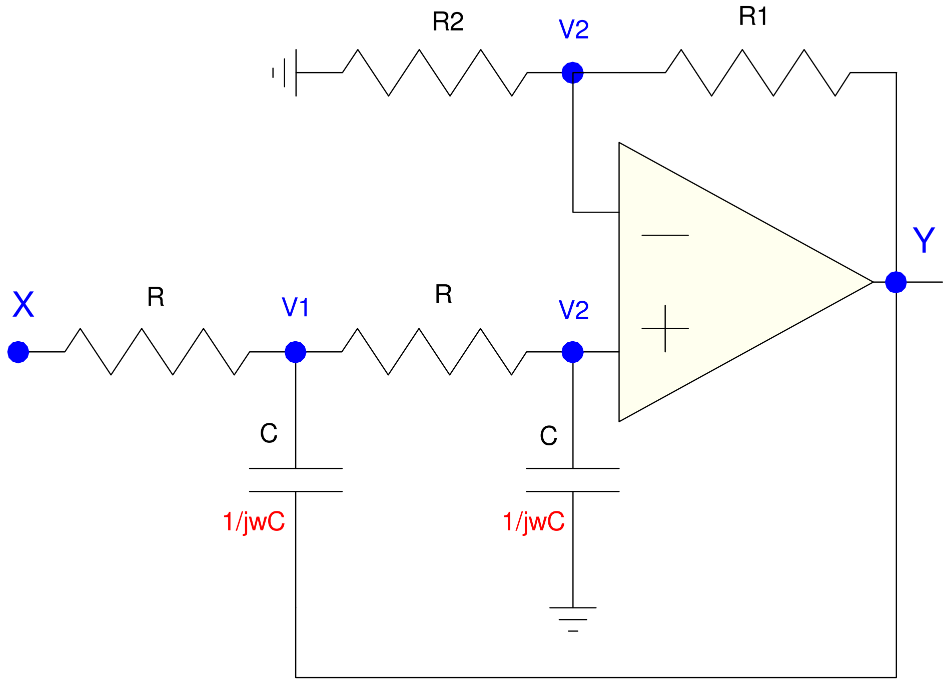




2nd-Order LPF

Not an obvious circuit

- Write the node equations



Voltage Node Equations

Non-Inverting Amplifier

$$Y = \left(1 + \frac{R_1}{R_2}\right) V_2 = kV_2$$

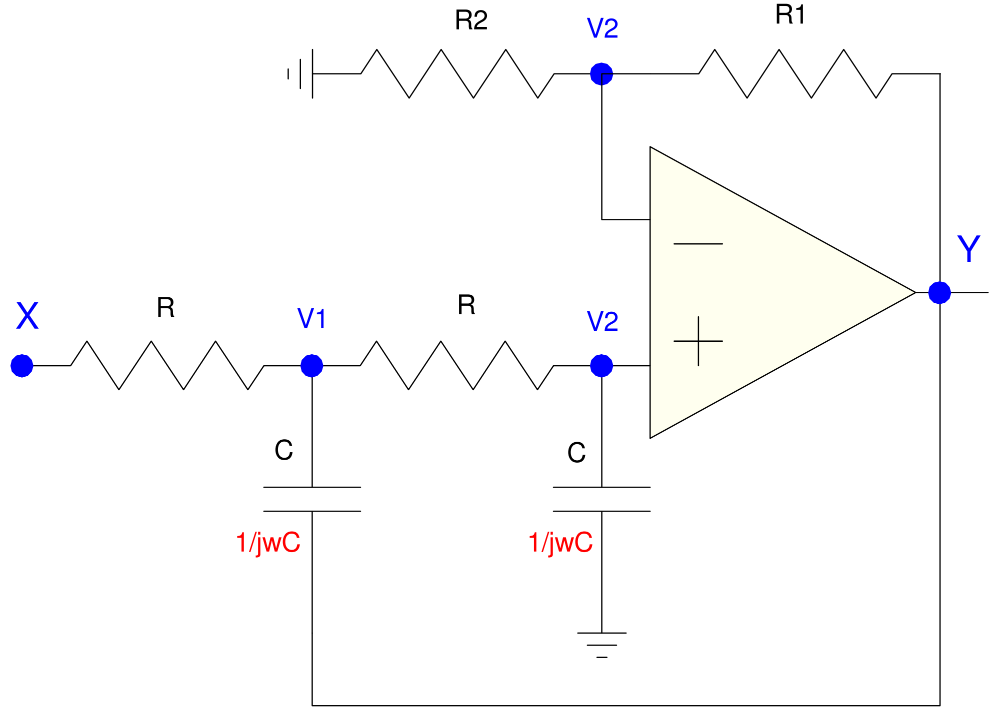
Voltage nodes

$$\left(\frac{V_1 - X}{R}\right) + \left(\frac{V_1 - V_2}{R}\right) + \left(\frac{V_1 - Y}{1/j\omega C}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R}\right) + \left(\frac{V_2}{1/j\omega C}\right) = 0$$

Simplify

$$Y = \left(\frac{k}{(j\omega RC)^2 + (3-k)(j\omega RC) + 1}\right) X$$

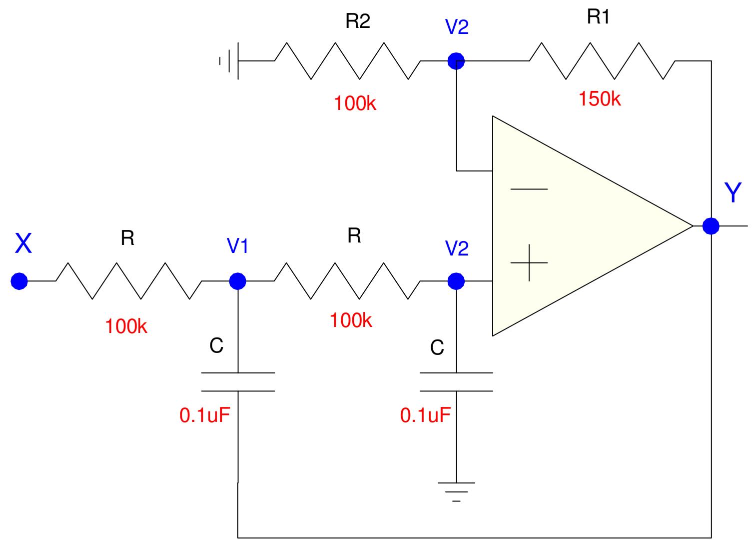


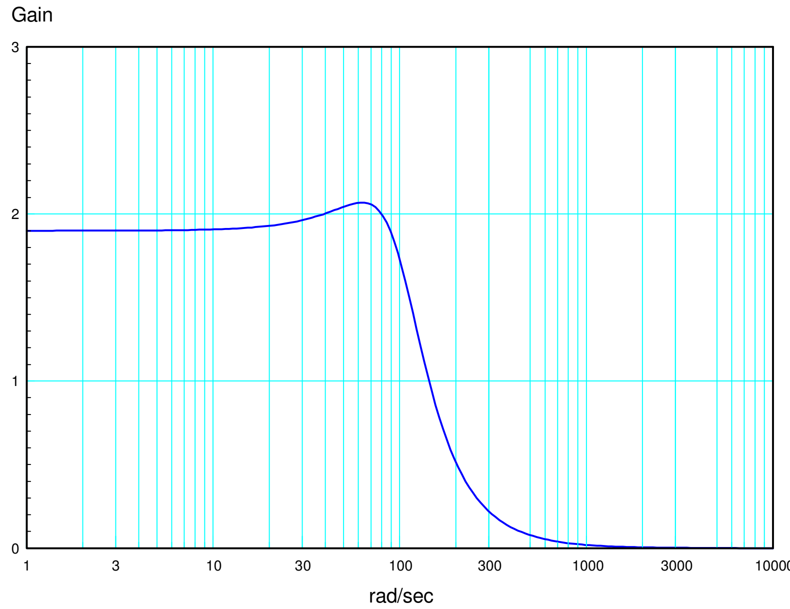
Analysis

$$Y = \left(\frac{k}{(j\omega RC)^2 + (3-k)(j\omega RC) + 1}\right) X$$

In Matlab

```
w = logspace(0,4,200)';
R1 = 90e3;
R2 = 100e3;
R = 100e3;
C = 0.1e-6;
k = 1 + R1/R2;
jwRC = j*w*R*C;
G = k./(jwRC.^2+(3-k)*jwRC+1);
semilogx(w,abs(G))
xlabel('rad/sec')
ylabel('Gain')
```



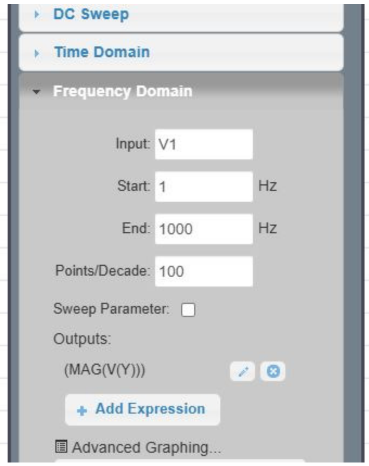


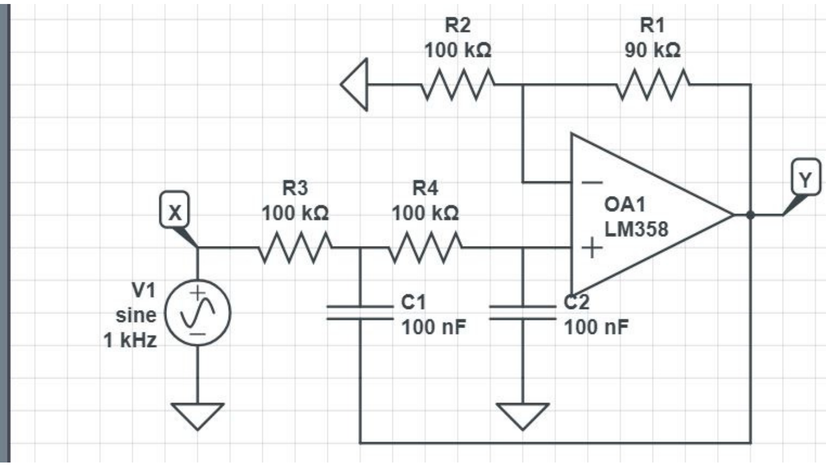
CircuitLab

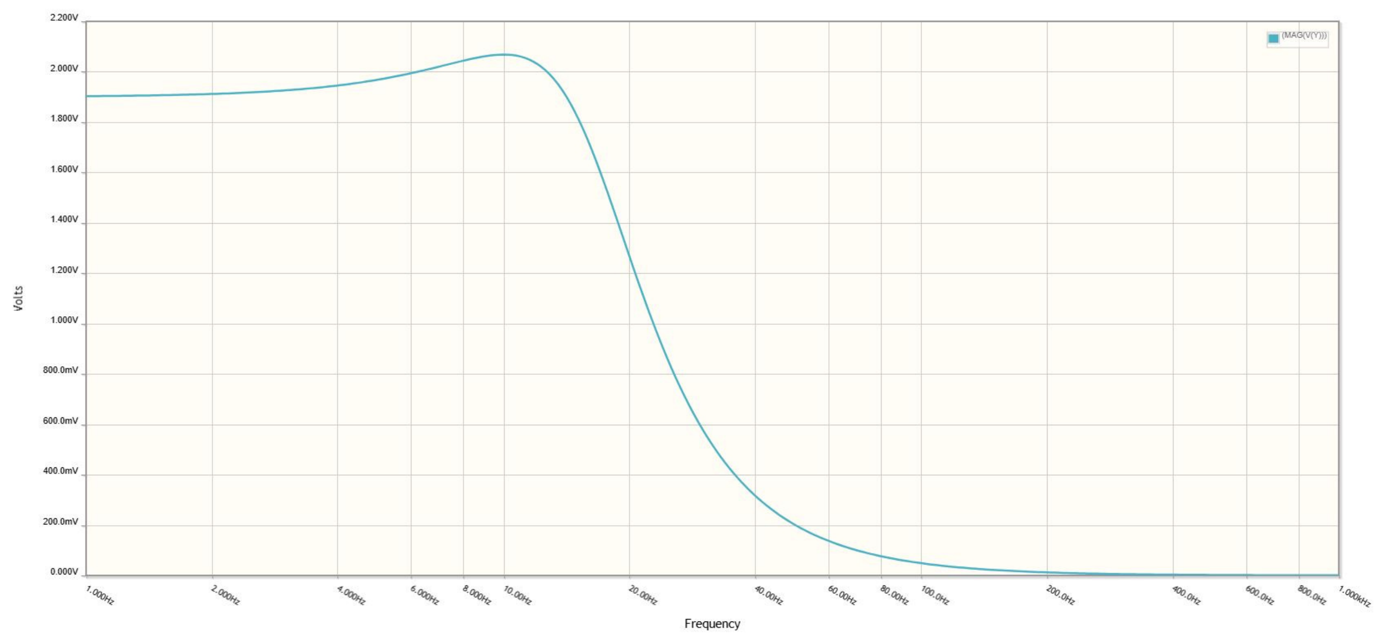
Frequency Domain

- Set the input to V1
- Sweep from 1Hz to 1000Hz
- 20 points per decade

Matches calculations







Design

- Pass frequencies below 250Hz
- Subwoofer crossover

The corner frequency is 1/RC

$$\frac{1}{RC} = 500\pi \qquad \mathit{250Hz}$$

R = 100k

C = 6.37nF

k sets the angle of the pole

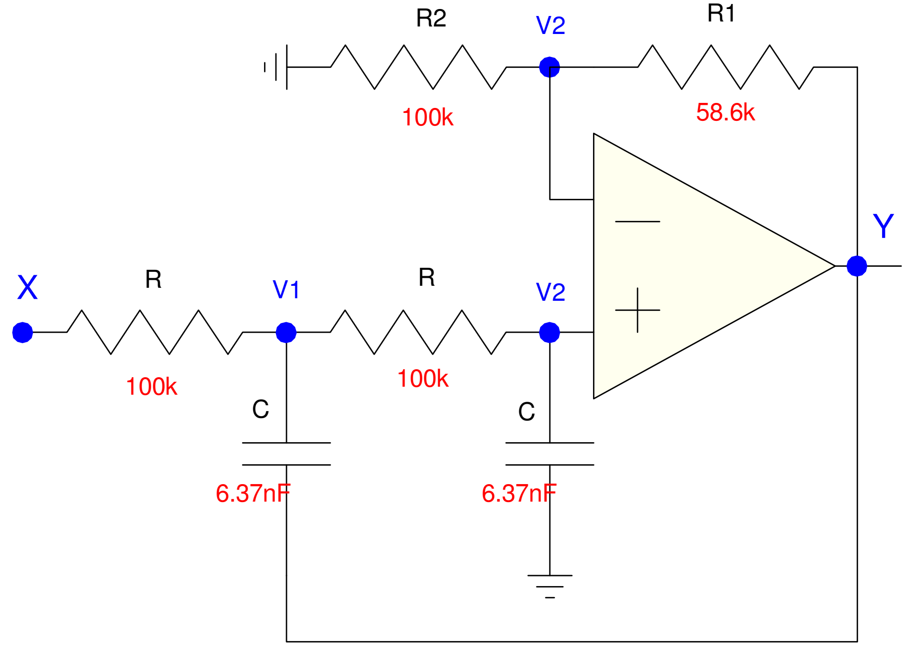
$$3 - k = 2\cos\theta$$

$$\theta = 45^0$$

$$k = 1.586 = 1 + \frac{R_1}{R_2}$$

R2 = 100k

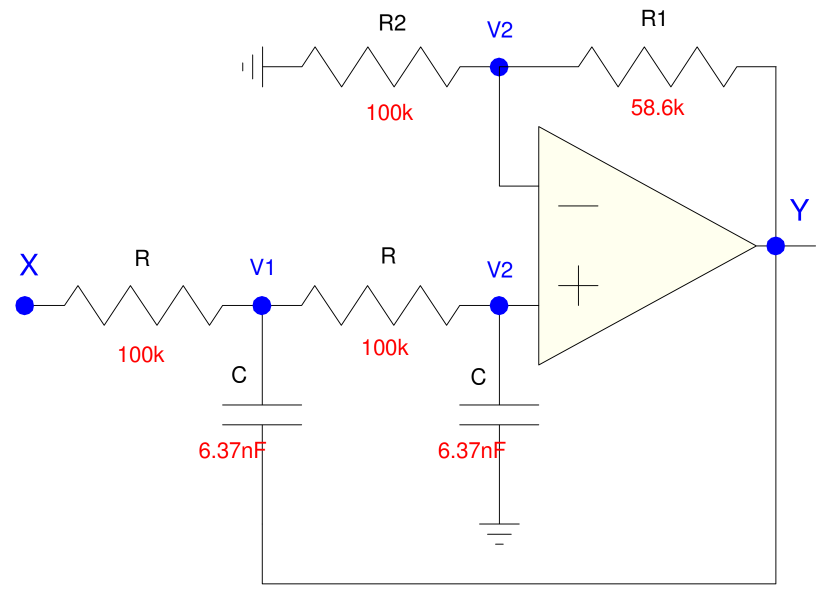
R1 = 58.6k

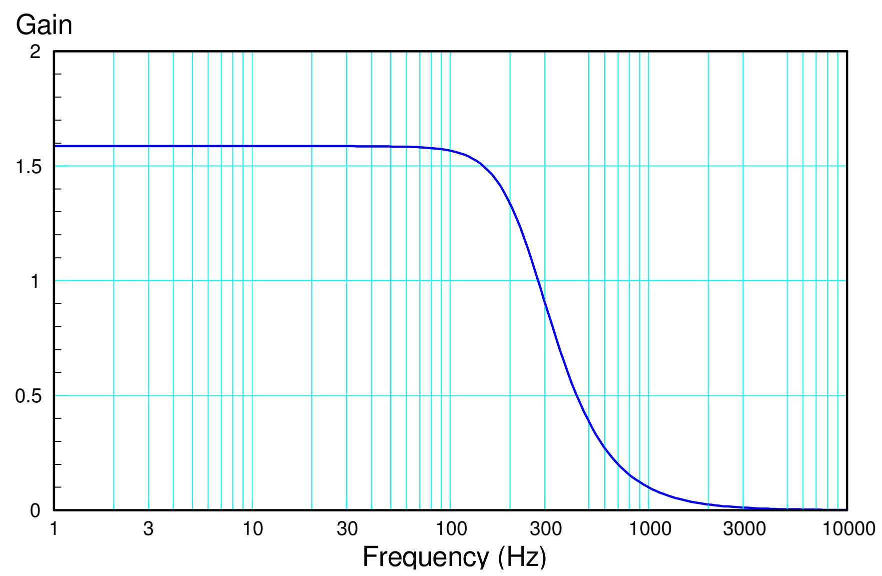


Design

Check the design in Matlab

```
f = logspace(0,4,200)';
w = 2*pi*f;
k = 1.586;
R = 100e3;
C = 6.37e-9;
jwRC = j*w*R*C;
G = k./(jwRC.^2+(3-k)*jwRC+1);
semilogx(w,abs(G))
xlabel('Hz')
ylabel('Gain')
```



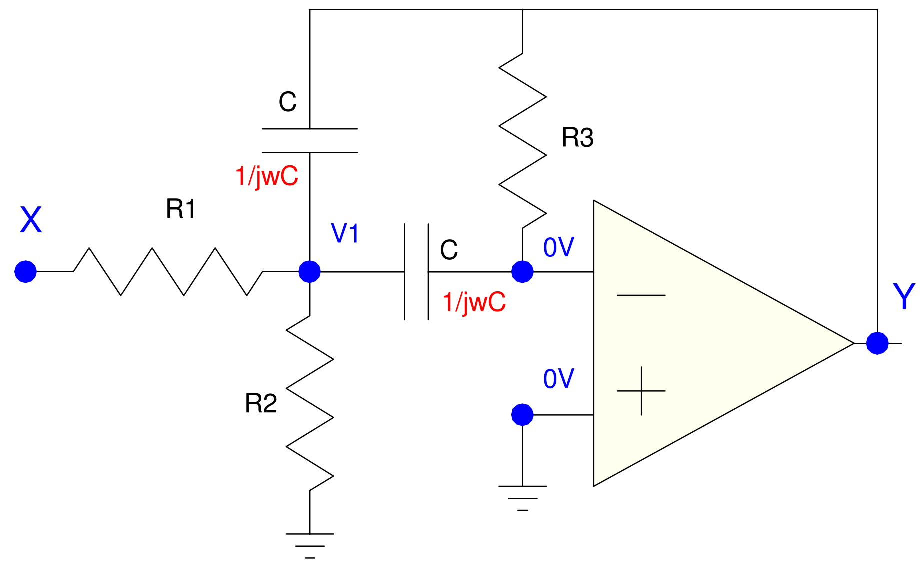


Band-Pass Filter

Also not an obvious topopogy

- DC gain = 0
- High-frequency gain = 0

Write the node equations

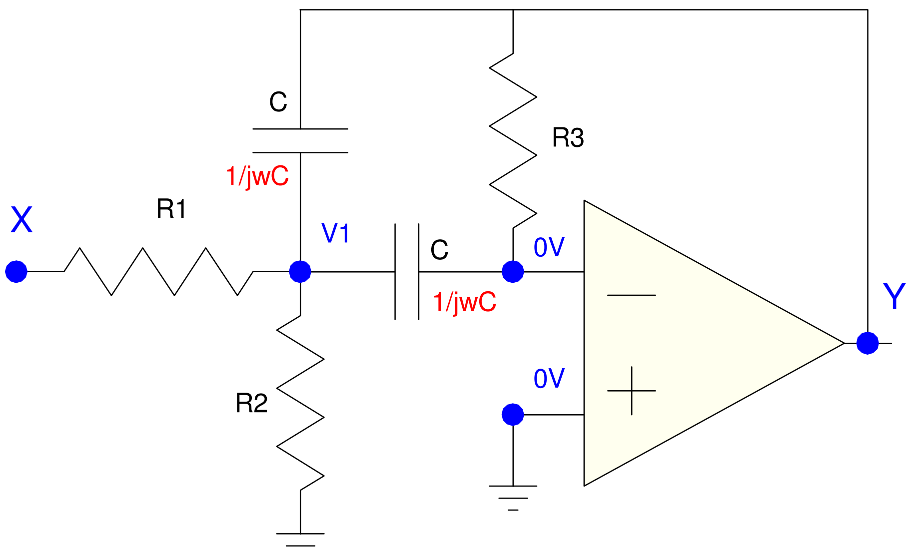


Voltage Nodes

Node equtation

$$\left(\frac{V_1 - X}{R_1}\right) + \left(\frac{V_1 - 0}{1/j\omega C}\right) + \left(\frac{V_1 - Y}{1/j\omega C}\right) + \left(\frac{V_1}{R_2}\right) = 0$$

$$\left(\frac{0 - V_1}{1/j\omega C}\right) + \left(\frac{0 - Y}{R_3}\right) = 0$$



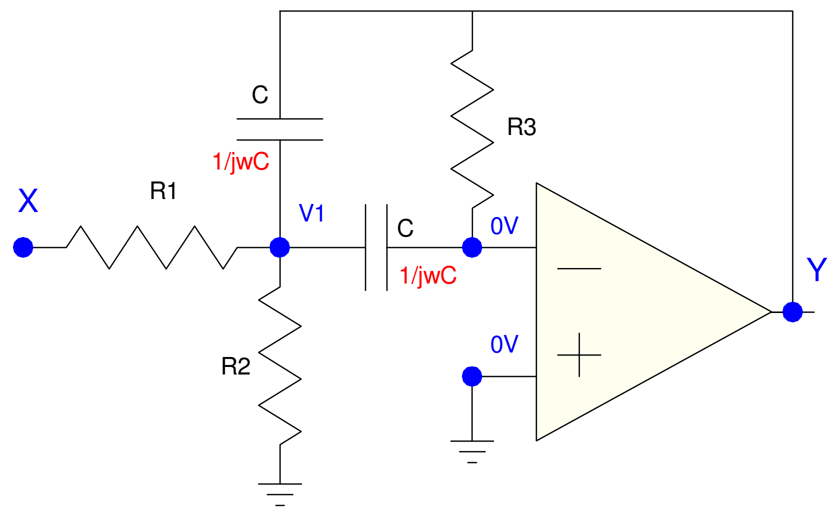
Simplify

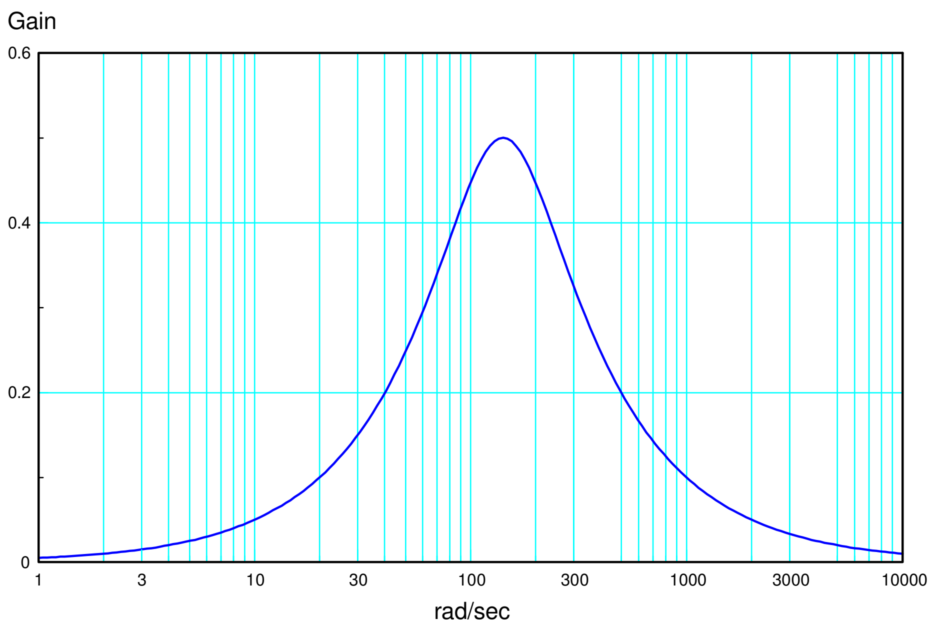
$$Y = \left(\frac{-j\omega\left(\frac{1}{CR_1}\right)}{(j\omega)^2 + \left(\frac{2}{CR_3}\right)(j\omega) + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)}\right) X$$

Analysis

$$G = \left(\frac{-j\omega\left(\frac{1}{CR_1}\right)}{(j\omega)^2 + \left(\frac{2}{CR_3}\right)(j\omega) + \left(\frac{R_1+R_2}{R_1R_2R_3C^2}\right)} \right)$$

```
w = logspace(0,4,200)';
R1 = 100e3;
R2 = 100e3;
R3 = 100e3;
C = 0.1e-6;
num = -(j*w)/(C*R1);
d2 = (j*w).^2;
d1 = (j*w)*2/(C*R3);
d0 = (R1+R2)/(R1*R2*R3*C*C);
den = d2 + d1 + d0;
G = -num ./ den;
semilogx(w,abs(G))
xlabel('rad/sec')
ylabel('Gain')
```

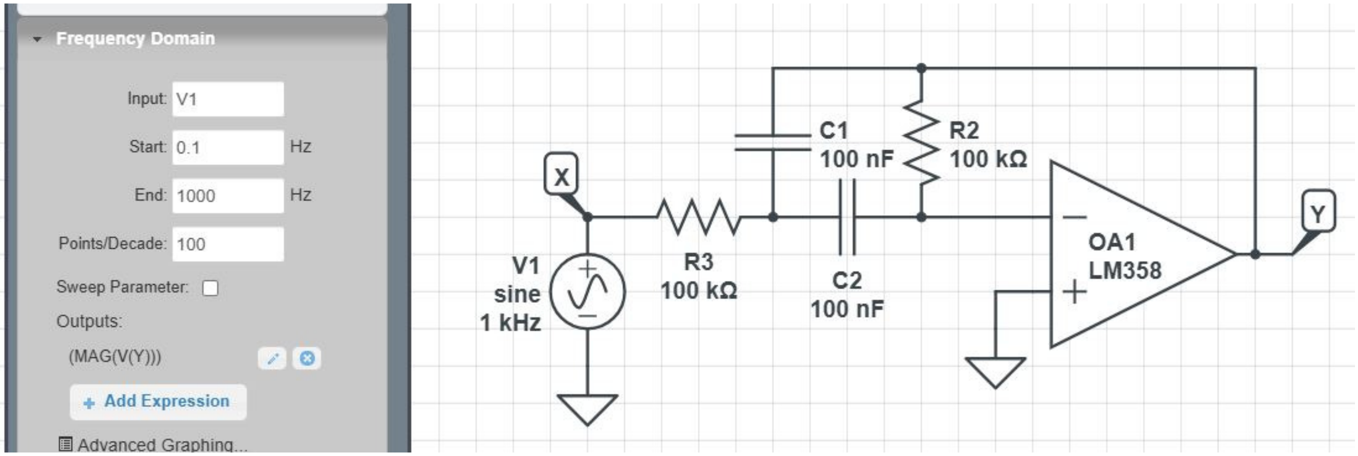




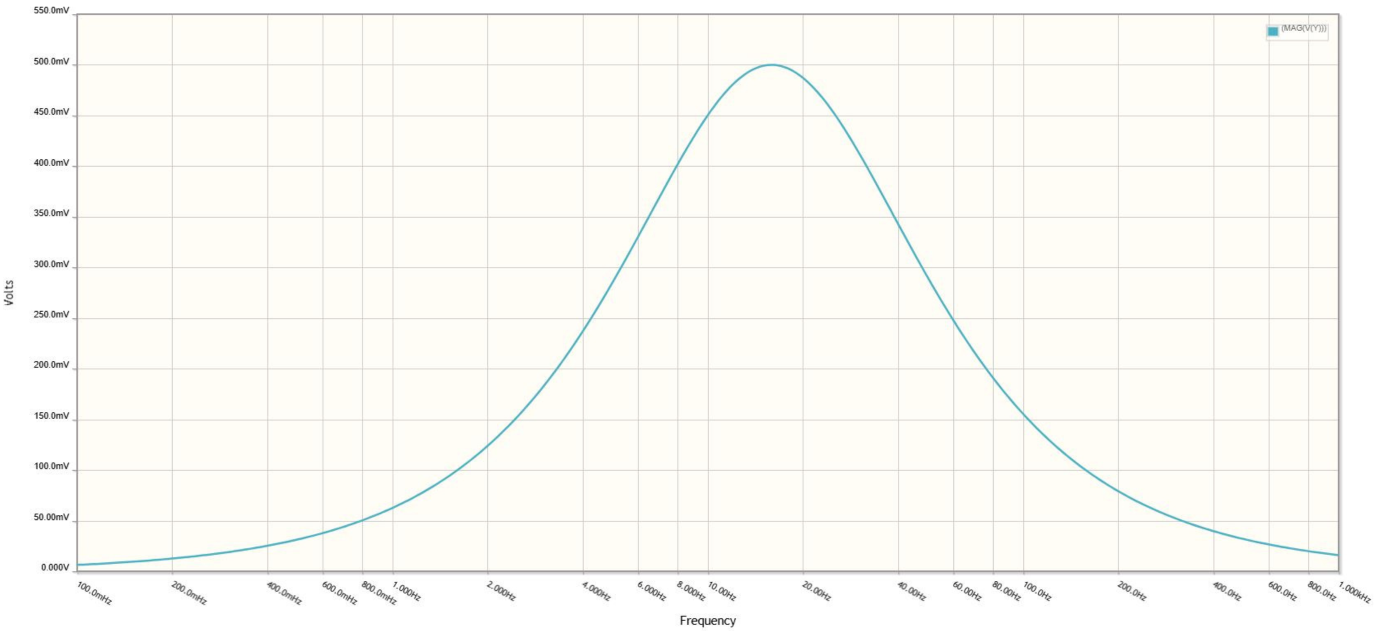
CircuitLab

Frequency Domain

- Set the input to V1
- 0.1Hz to 1000Hz
- 100 points per decade



Matches calculations



Design

Find R and C to implement

$$G = \left(\frac{-20(j\omega)}{(j\omega)^2 + 10(j\omega) + 1000} \right)$$

From before

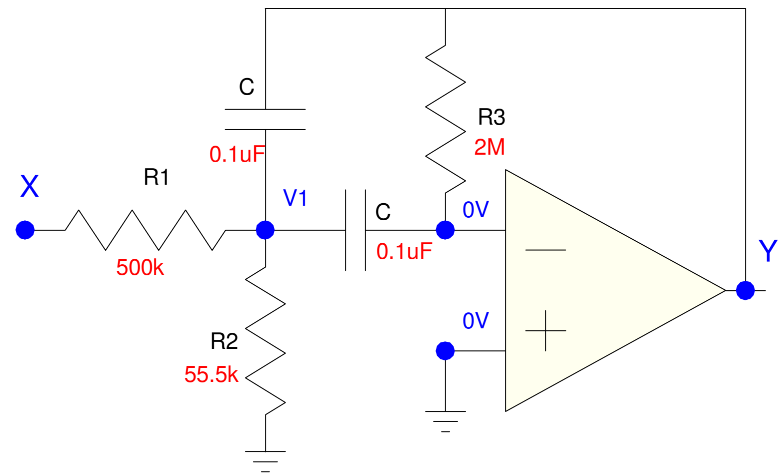
$$G = \left(\frac{-j\omega\left(\frac{1}{CR_1}\right)}{(j\omega)^2 + \left(\frac{2}{CR_3}\right)(j\omega) + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2}\right)} \right)$$

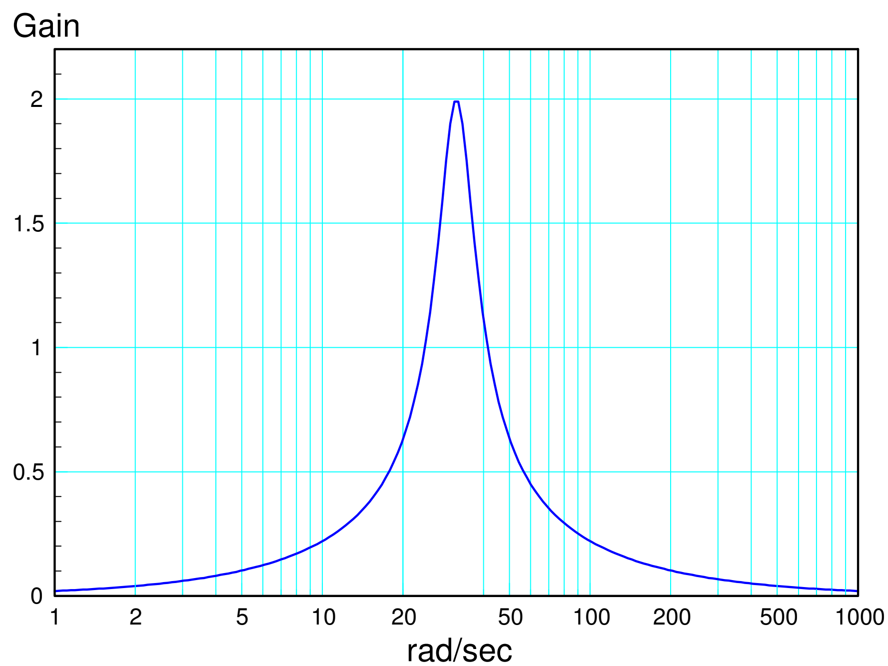
Matching terms

$$\frac{1}{R_1 C} = 20$$

$$\frac{2}{R_3 C} = 10$$

$$\frac{1}{C^2 R_1 R_3} = 1000$$





Summary

Filters are circuits where the gain varies with frequency

- Any circuit with inductors and/or capacitors

Active filters are op-amp circuits where the gain varies with frequency

- Op-amps allow gains > 1
- They also allow you to avoid using inductors

If the operating frequency is unknown, leave it as ω

- You can then use voltage nodes to determine the output as a function of frequency