Phasors & Op-Amp Circuits

ECE 211 Circuits I

Lecture #28

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Circuit Analysis with Op-Amps

Previously, we looked at DC analysis of op-amp circuits

Two methods where used to analyze them

- Voltage Nodes
  - Write N equations for N unknowns
  - Note that Vp = Vm (usually)
- Figure it out
  - Determine what voltages make the currents balance
  - Note that Vp = Vm (usually)

These methods also work with AC signals

- Now using phasors (complex numbers)

This Lecture

This lecture will look at three different op-amp circuits

Each circuit will be analyzed using

- Voltage Nodes
- Figure it out
- CircuitLab

Circuit #1

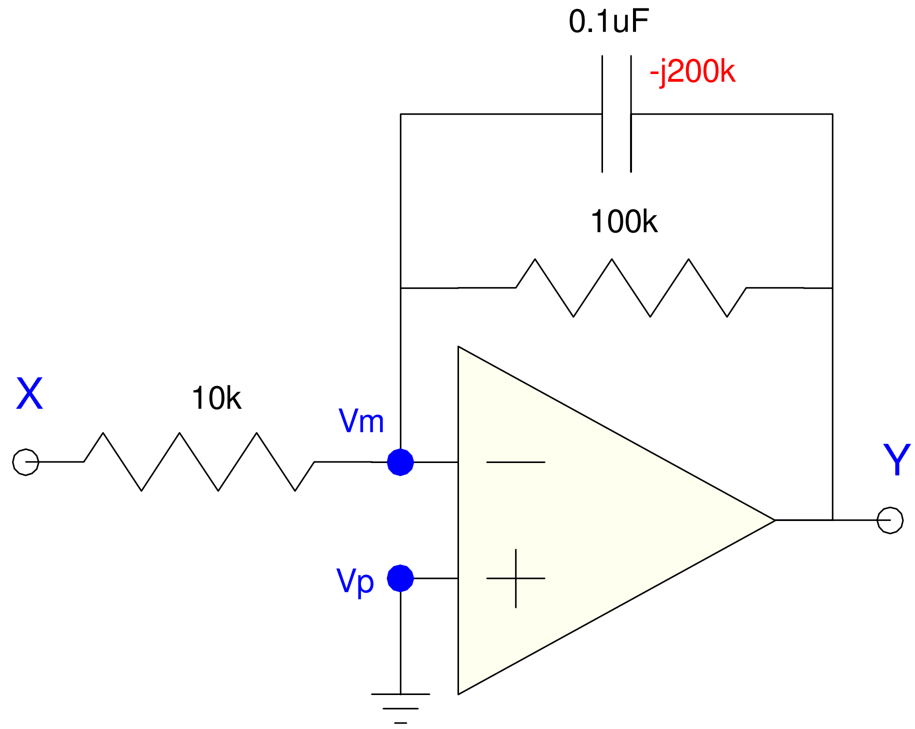
Find y(t) for $x(t) = 3\sin(50t)$

Using phasor notation

$$X = 0 - j3$$

$$\omega = 50\frac{rad}{\sec}$$

$$Z_c = \frac{1}{j\omega C} = -j200k\Omega$$



Circuit #1

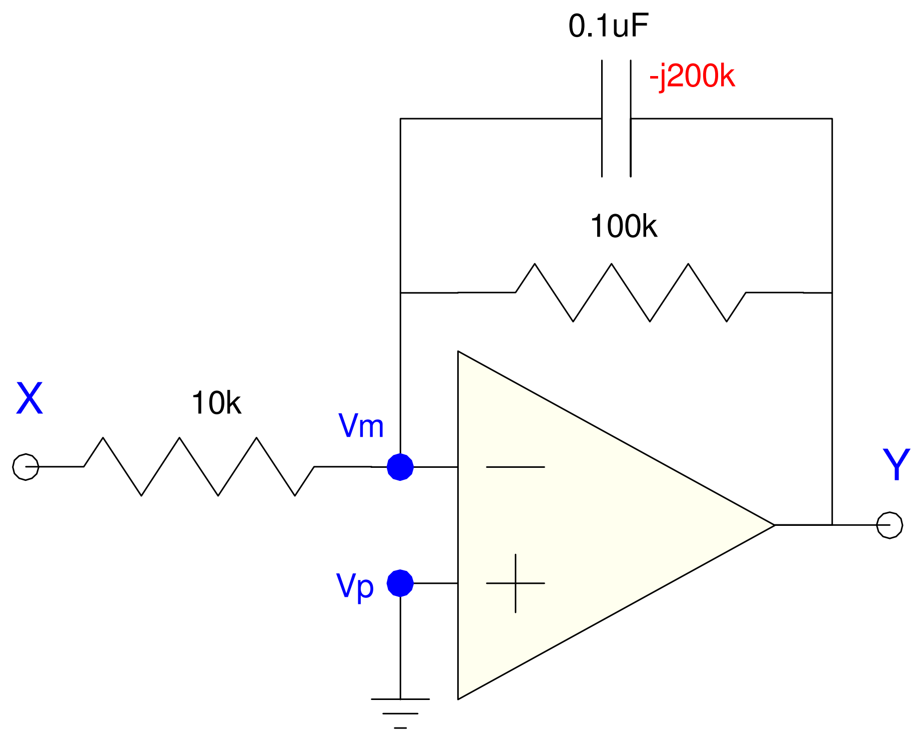
Voltage Nodes Equations

$$X = 0 - j3$$

$$V_p = 0$$

$$V_m = V_p$$

$$\left(\frac{V_m - X}{10k}\right) + \left(\frac{V_m - Y}{100k}\right) + \left(\frac{V_m - Y}{-j200k}\right) = 0$$



Solve

$$\left(\frac{-(-j3)}{10k}\right) + \left(\frac{-Y}{100k}\right) + \left(\frac{-Y}{-j200k}\right) = 0$$

$$Y = 12 + j24$$

$$\boxed{y(t) = 12\cos(50t) - 24\sin(50t)}$$

Circuit #1

Figure it out...

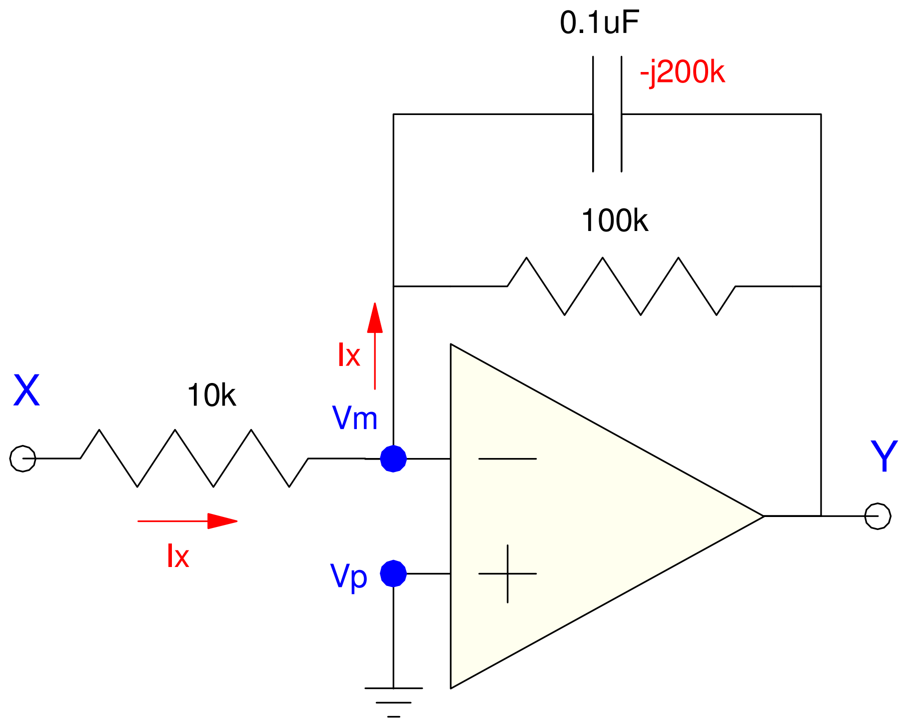
- Vp = Vm = 0

$$I_x = \left(\frac{0 - j3}{10k}\right)$$

$$Y = -(-j200k || 100k) I_x$$

$$Y = -(80k - j40k) I_x$$

$$Y = 12 + j24$$



$$y(t) = 12\cos(50t) - 24\sin(50t)$$

Circuit #1

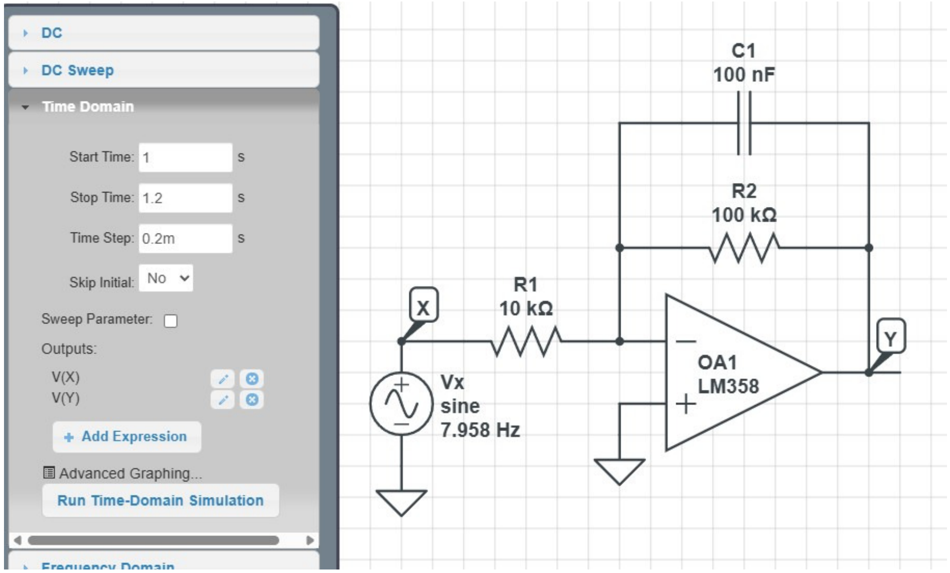
- CircuitLab

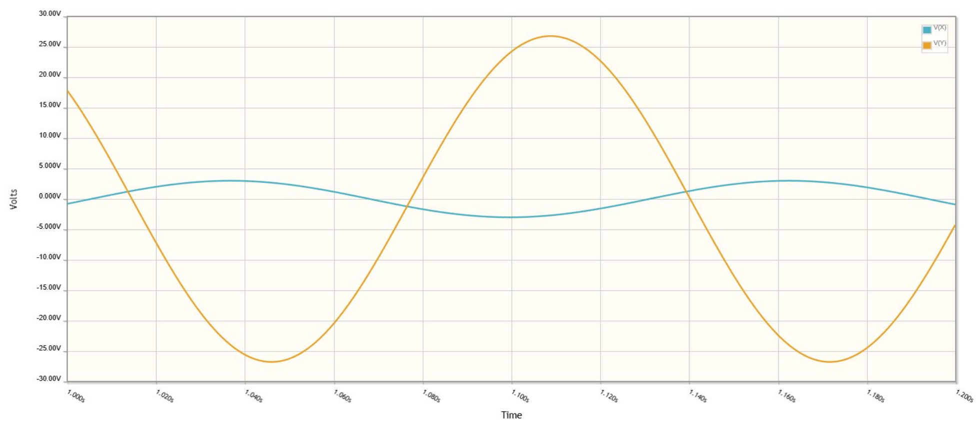
Polar form is easier to see

$$Y = 12 + j24$$

$$Y = 26.8328\angle 63.4349^{0}$$

	CircuitLab	Matlab
Vx	3.000V	3.000V
Vy	26.78V	26.83V





Circuit #2

Find the voltages

Assume

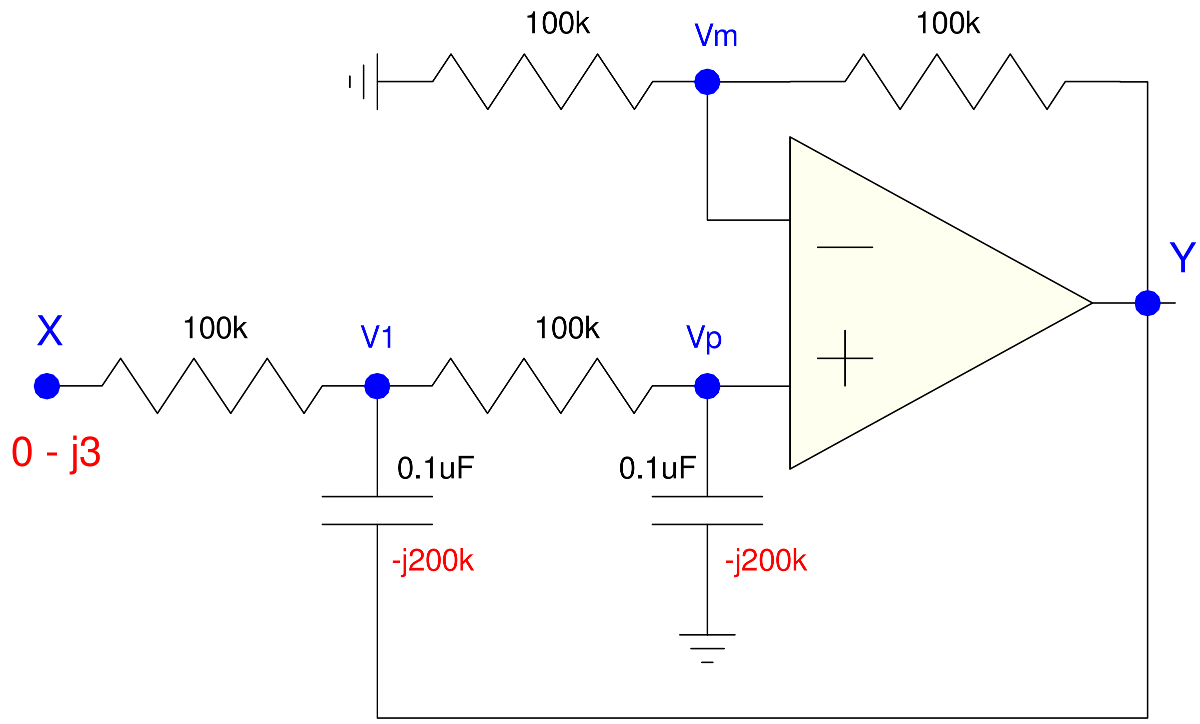
$$x(t) = 3\sin(50t)$$

Using phasors

$$\omega = 50$$

$$X = 0 - j3$$

$$\frac{1}{j\omega C} = -j200k\Omega$$



Circuit #2

Voltage Nodes

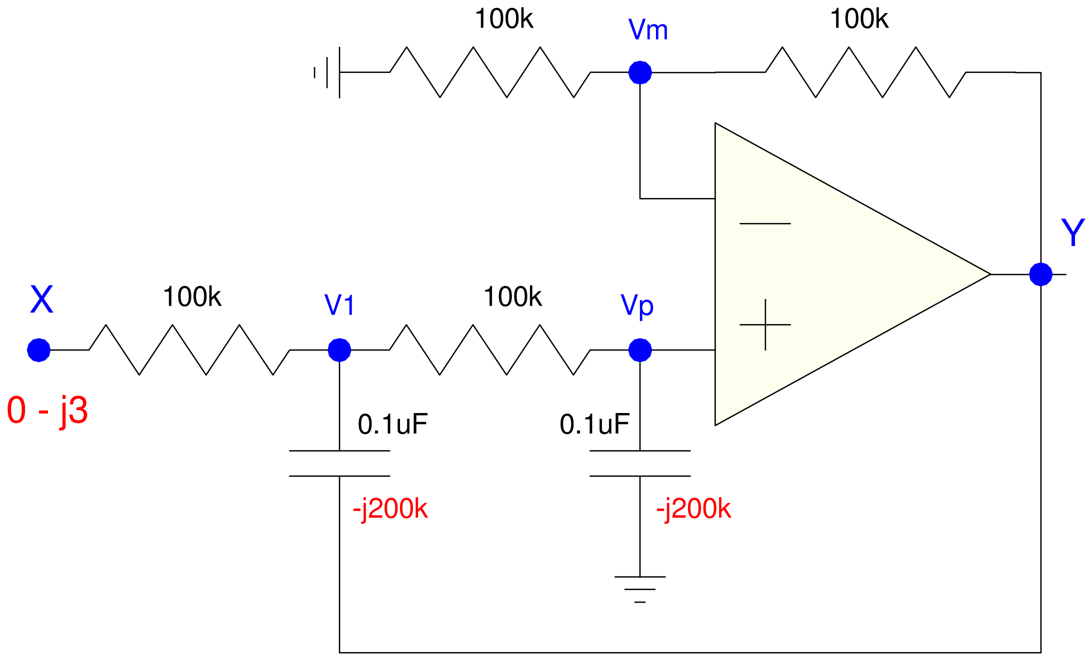
$$X = 0 - j3$$

$$V_p = V_m$$

$$\left(\frac{V_1 - X}{100k}\right) + \left(\frac{V_1 - V_p}{100k}\right) + \left(\frac{V_1 - Y}{-j200k}\right) = 0$$

$$\left(\frac{V_p - V_1}{100k}\right) + \left(\frac{V_p}{-j200k}\right) = 0$$

$$\left(\frac{V_m}{100k}\right) + \left(\frac{V_m - Y}{100k}\right) = 0$$

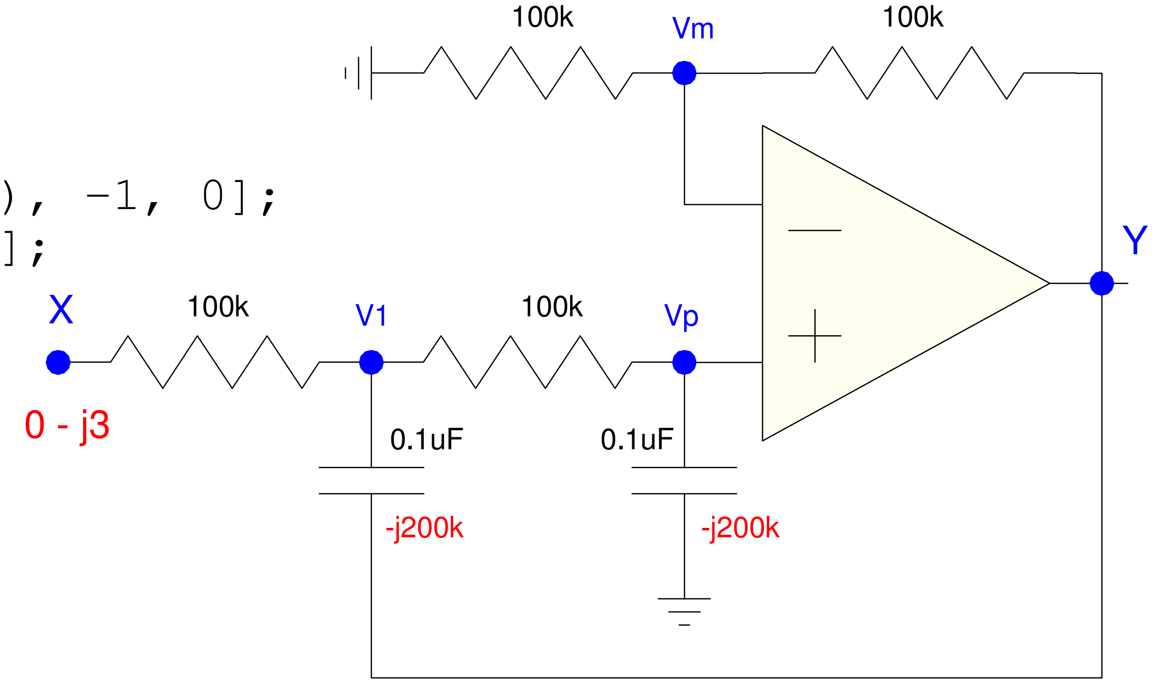


Circuit 2

Solve using Matlab

```
% X   Y   V1   Vp   Vm
a1 = [1 0 0 0 0];
a2 = [0 0 0 1 -1];
a3 = [-1, 1/(2j), 1+1-1/(2j), -1, 0];
a4 = [0, 0, -1, 1-1/(2j), 0];
a5 = [0, -1, 0, 0, 2];
A = [a1;a2;a3;a4;a5];
B = [-3j;0;0;0;0];
V = inv(A)*B


Vx        0 - 3.0000i
Vy  -3.6923 - 5.5385i
V1  -0.4615 - 3.6923i
Vp  -1.8462 - 2.7692i
Vm  -1.8462 - 2.7692i
```



Circuit #2

- Figure it out...

$$V_m = V_p = \frac{1}{2}Y$$

$$V_p = \frac{1}{2}Y = \left(\frac{-j200k}{100k - j200k}\right)V_1$$

meaning

$$V_1 = (0.5 + j0.25)Y$$

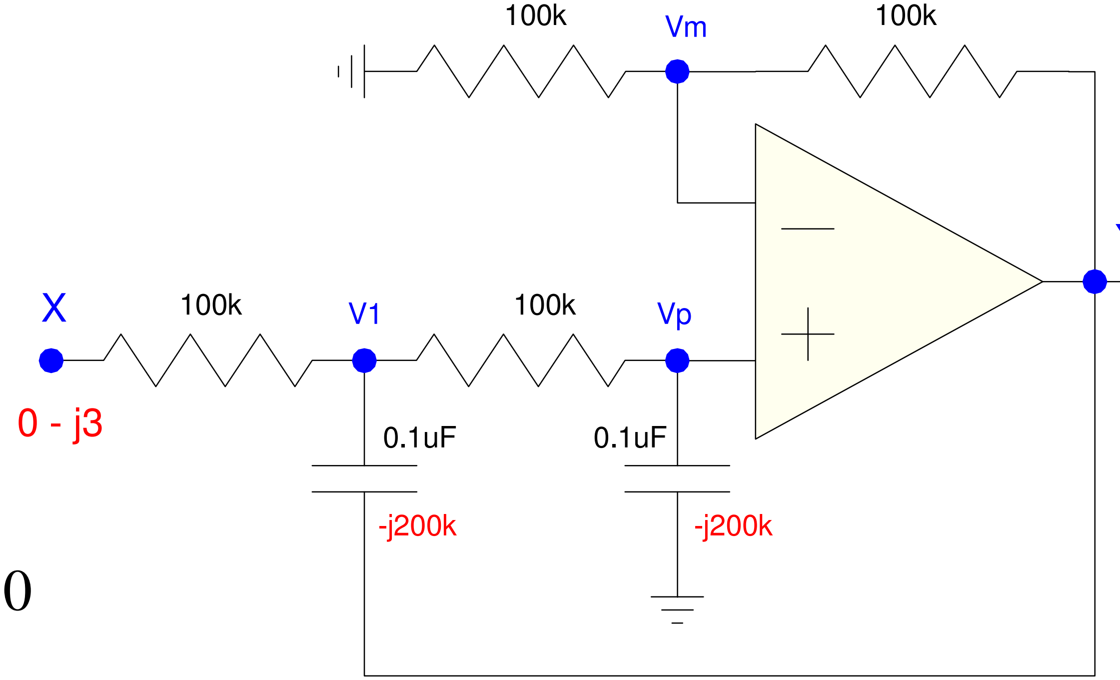
Voltage node at V1

$$\left(\frac{V_1 - X}{100k}\right) + \left(\frac{V_1}{100k - j200k}\right) + \left(\frac{V_1 - Y}{-j200k}\right) = 0$$

Substitute

$$\left(\frac{0.5 + j0.25}{100k}\right)Y + \left(\frac{0.5 + j.25}{100k - j200k}\right)Y + \left(\frac{(0.5 + j0.25) - 1}{-j200k}\right)Y = \left(\frac{1}{100k}\right)X$$

$$Y = -3.6923 - j5.5385$$

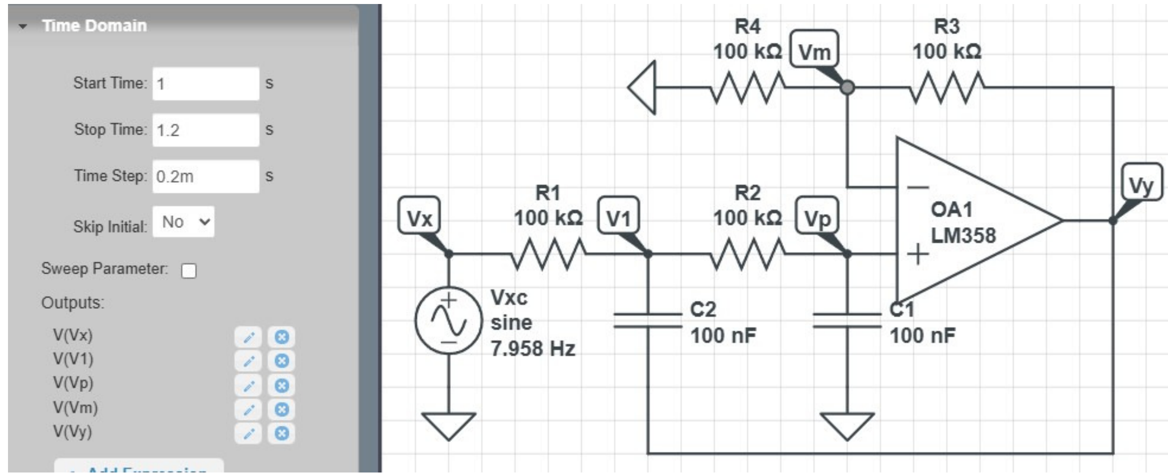


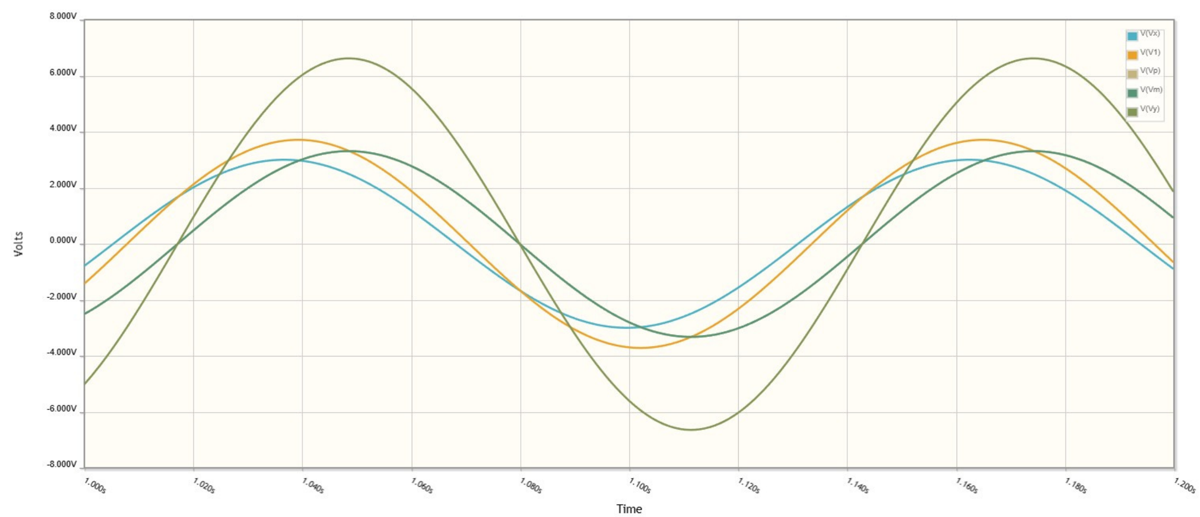
Circuit 2

CircuitLab

- Voltage magnitudes match

	CircuitLab	Matlab
Vx	3.000V	3.000V
V1	3.690V	3.721V
Vp	3.298V	3.328V
Vy	6.616V	6.656V

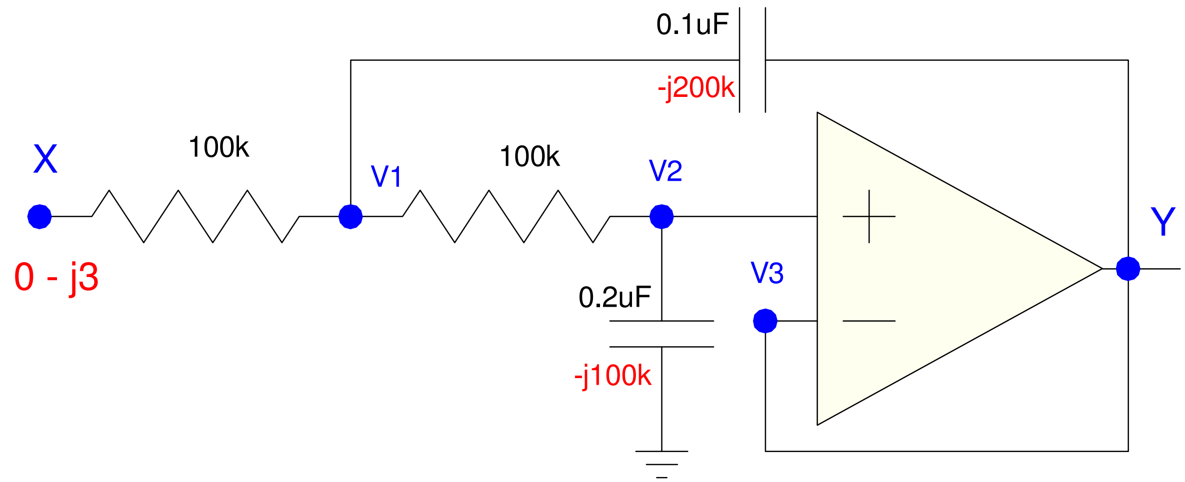




Circuit #3

Find the node votlages

$$X = 3\sin(50t)$$



Using phasors

$$\omega = 50$$

$$X = 0 - j3$$

$$0.2\mu F \rightarrow \frac{1}{j\omega C} = -j100k\Omega$$

$$0.1\mu F \rightarrow \frac{1}{j\omega C} = -j200k\Omega$$

Circuit #3

Voltage Nodes

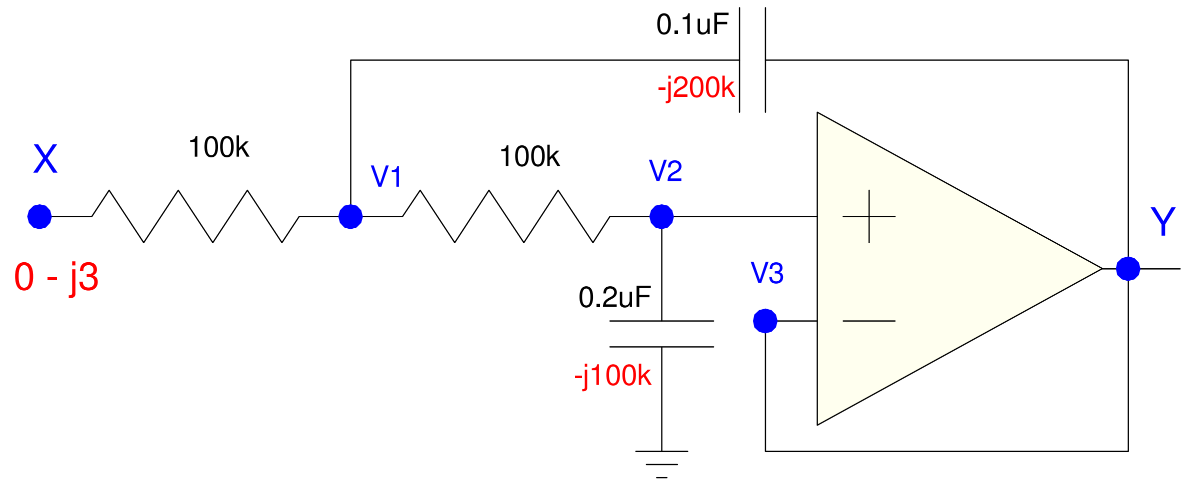
$$X = 0 - j3$$

$$V_3 = V_2$$

$$V_3 = Y$$

$$\left(\frac{V_1 - X}{100k}\right) + \left(\frac{V_1 - Y}{-j200k}\right) + \left(\frac{V_1 - V_2}{100k}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100k}\right) + \left(\frac{V_2}{-j100k}\right) = 0$$

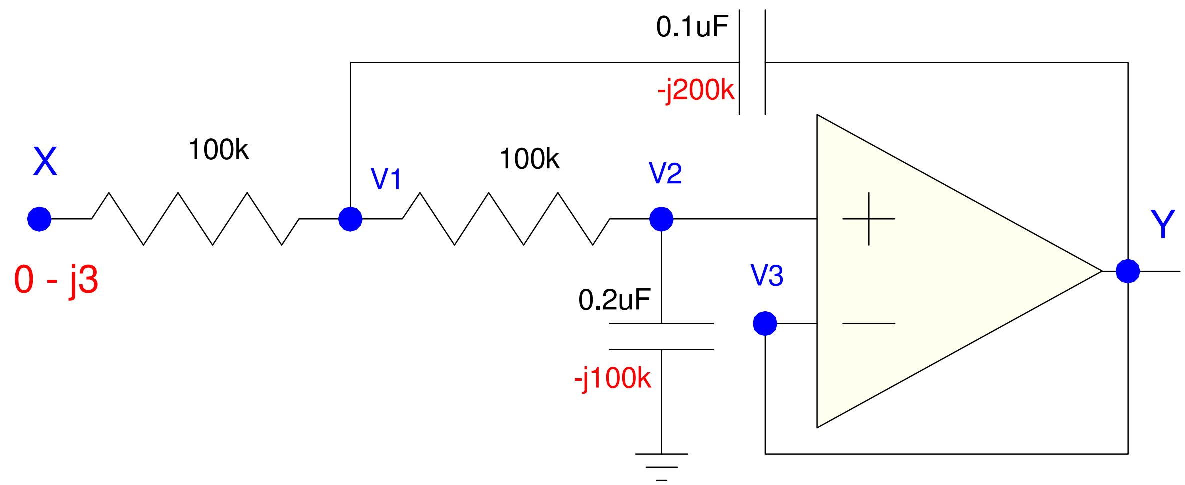


Circuit 3

Solve in Matlab

```
% X   Y   V1   V2   V3
a1 = [1 0 0 0 0];
a2 = [0 0 0 1 -1];
a3 = [0, 1, 0, 0, -1]
a4 = [-1, 1/(2j), 1-1/(2j)+1,-1,0];
a5 = [0, 0, -1, 1-1/j, 0];
A = [a1;a2;a3;a4;a5];
B = [-3j;0;0;0;0];
V = inv(A)*B

Vx        0 - 3.0000i
Vy  -1.4118 - 0.3529i
V1  -1.0588 - 1.7647i
V2  -1.4118 - 0.3529i
V3  -1.4118 - 0.3529i
```



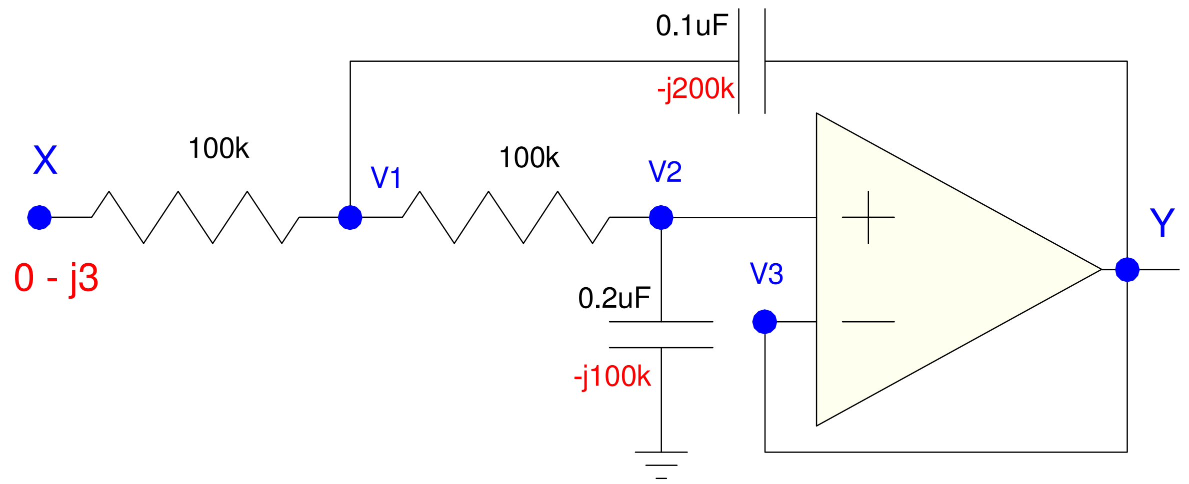
Circuit3

Find the votlages that balance

$$V_2 = V_3 = Y$$

$$V_1 = \left(\frac{100k - j100k}{-j100k}\right) V_2$$

$$V_1 = (1 + j)Y$$



Node equation at V1

$$\left(\frac{V_1 - X}{100k}\right) + \left(\frac{V_1 - Y}{-j200k}\right) + \left(\frac{V_1}{100k - j100k}\right) = 0$$

Substitute and scale by 100k

$$\left(\frac{(1+j)Y - X}{1}\right) + \left(\frac{(1+j)Y - Y}{-j2}\right) + \left(\frac{(1+j)Y}{1-j}\right) = 0$$

Solve

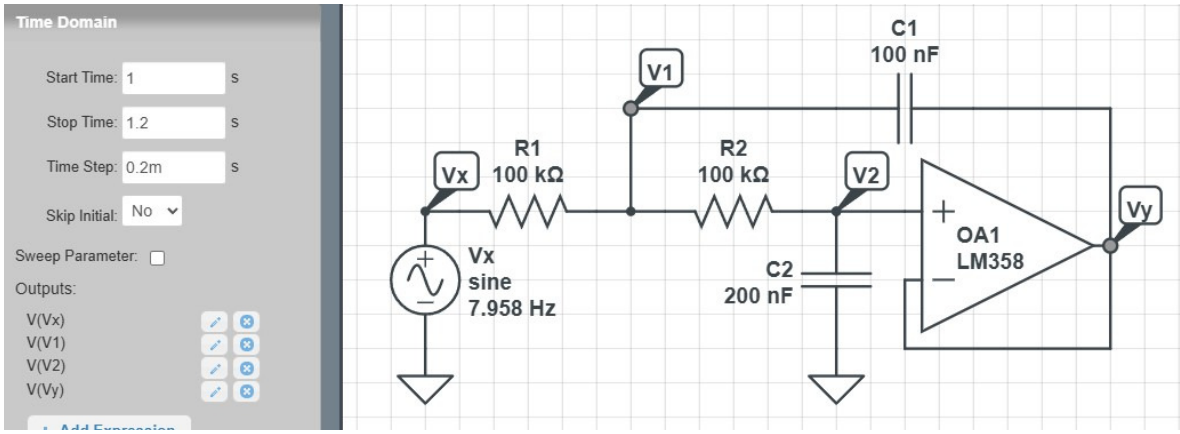
$$Y = -1.4118 - j0.3529$$

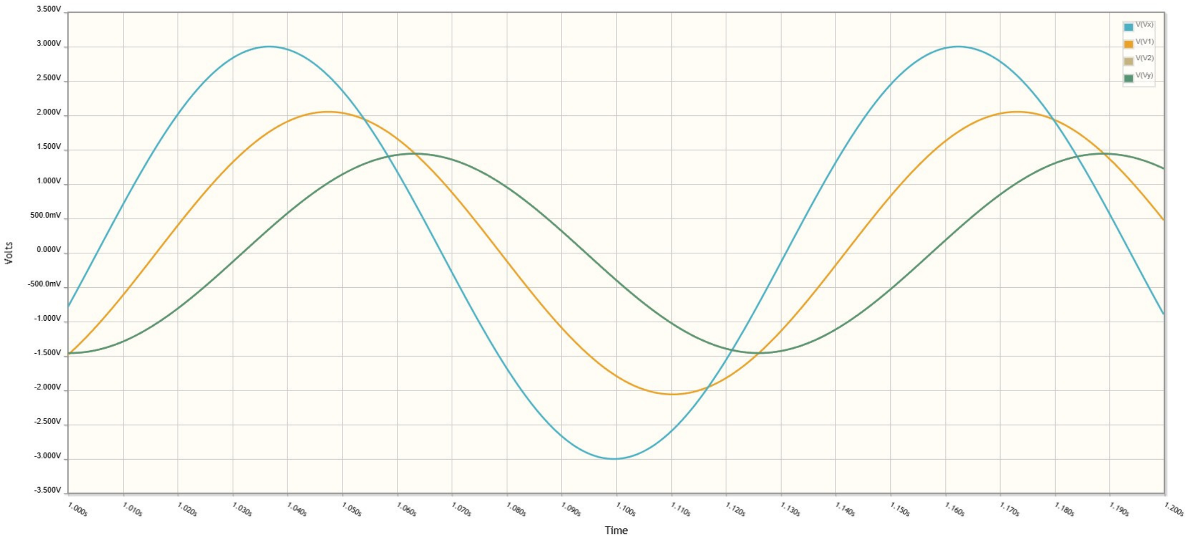
Circuit 3

Compare magnitude

- polar form

	CircuitLab	Matlab
Vx	3.000V	3.000V
V1	2.051V	2.058V
Vy	1.440V	1.455V





Summary

Phasors work with op-amp circuits

- Use the same procedure we used with DC signals
- The only complication is you're now using complex numbers

CircuitLab is a good way to check your answers

- Polar form is easier to see
- The peak voltage in CircuitLab should match magnitude of the voltages you computed