
Phasors & Current Loops

ECE 211 Circuits I Lecture #26

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

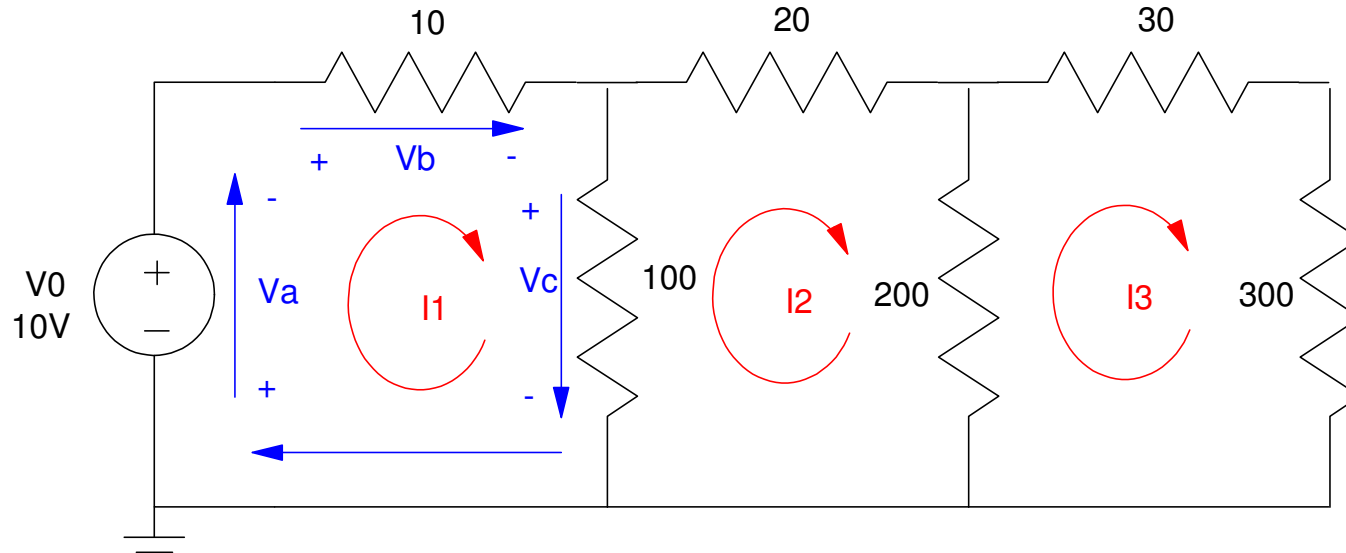
Current Loops (review)

Another way to solve for voltages and currents in a circuit

- Write N equations for N unknowns

Based on Conservation of Voltage

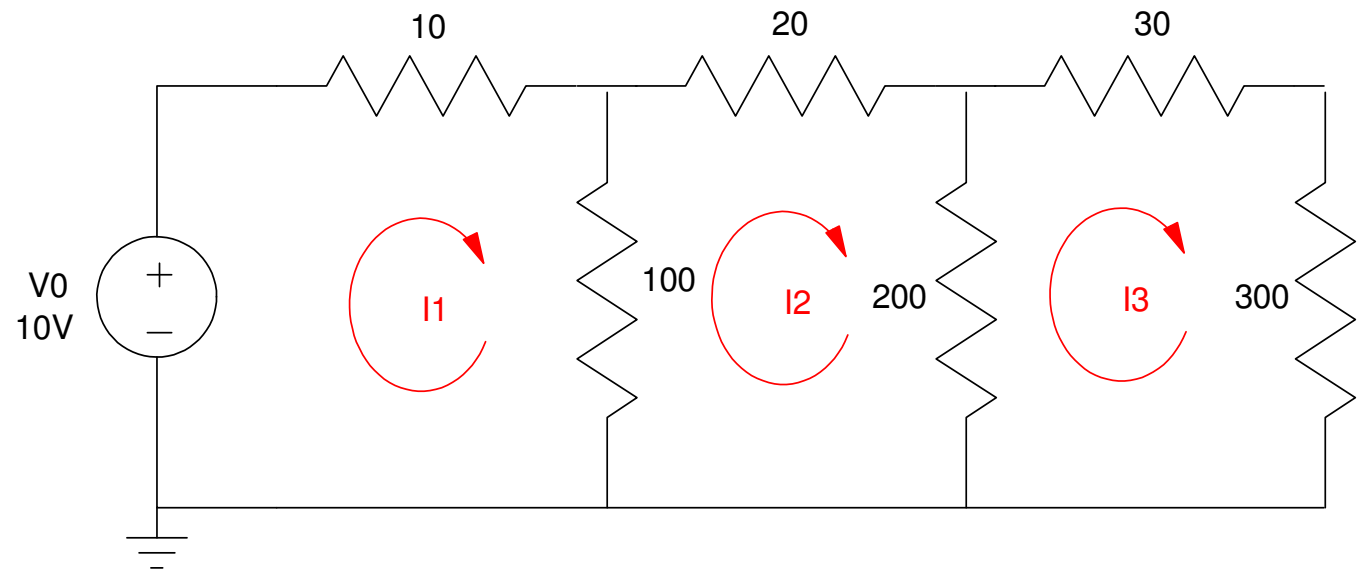
- The sum of voltages around a closed-path must be zero
- $V_a + V_b + V_c = 0$



Current Loops with Resistor Circuits

Current-Loops works with resistor circuits

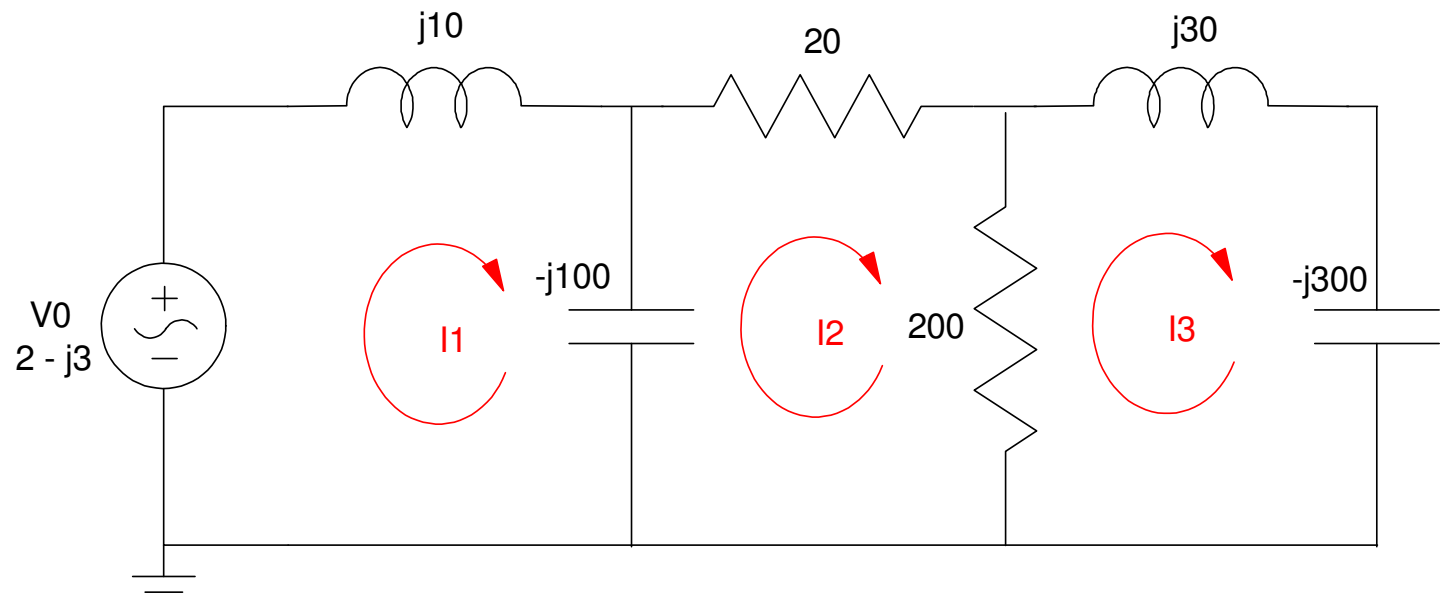
- Write N equations for N unknowns
- Voltages and currents are real numbers
- All numbers are real



Current Loops with RLC Circuits

Current-Loops also works for RLC circuits

- Use phasors
- Write N equations for N unknowns
- Using complex numbers



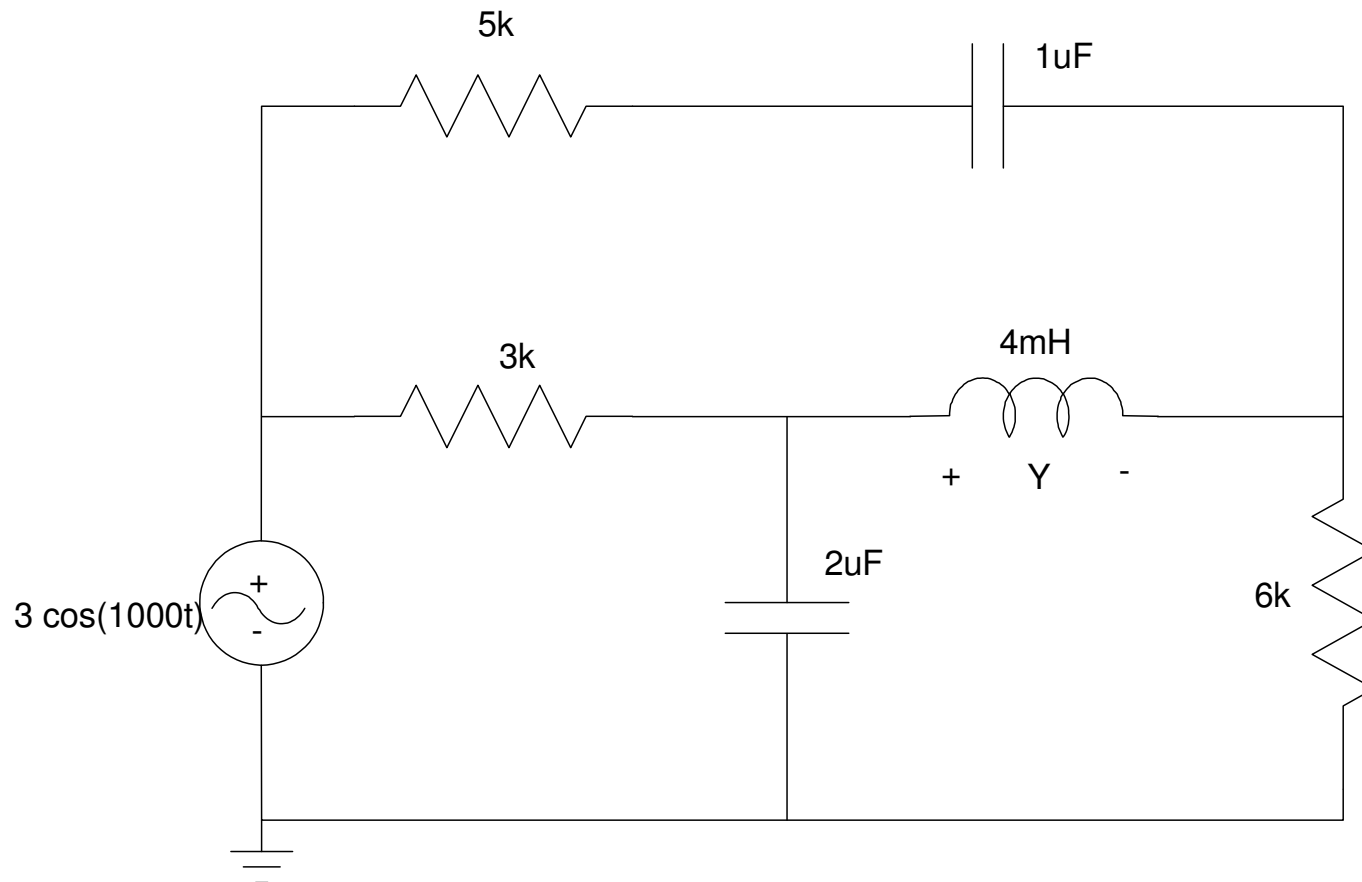
Phasors (recap)

When signals are sinusoids, use phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a \cos(\omega t) + b \sin(\omega t)$	$V = a - jb$
Current	$i(t) = a \cos(\omega t) + b \sin(\omega t)$	$I = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L \frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C \frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example 1:

Determine the currents in the following circuit



Step 1:

- Define the currents
- Convert to Phasors

$$V = 3 \cos(1000t)$$

$$\omega = 1000$$

$$V = 3 + j0$$

$$C = 1\mu\text{F}$$

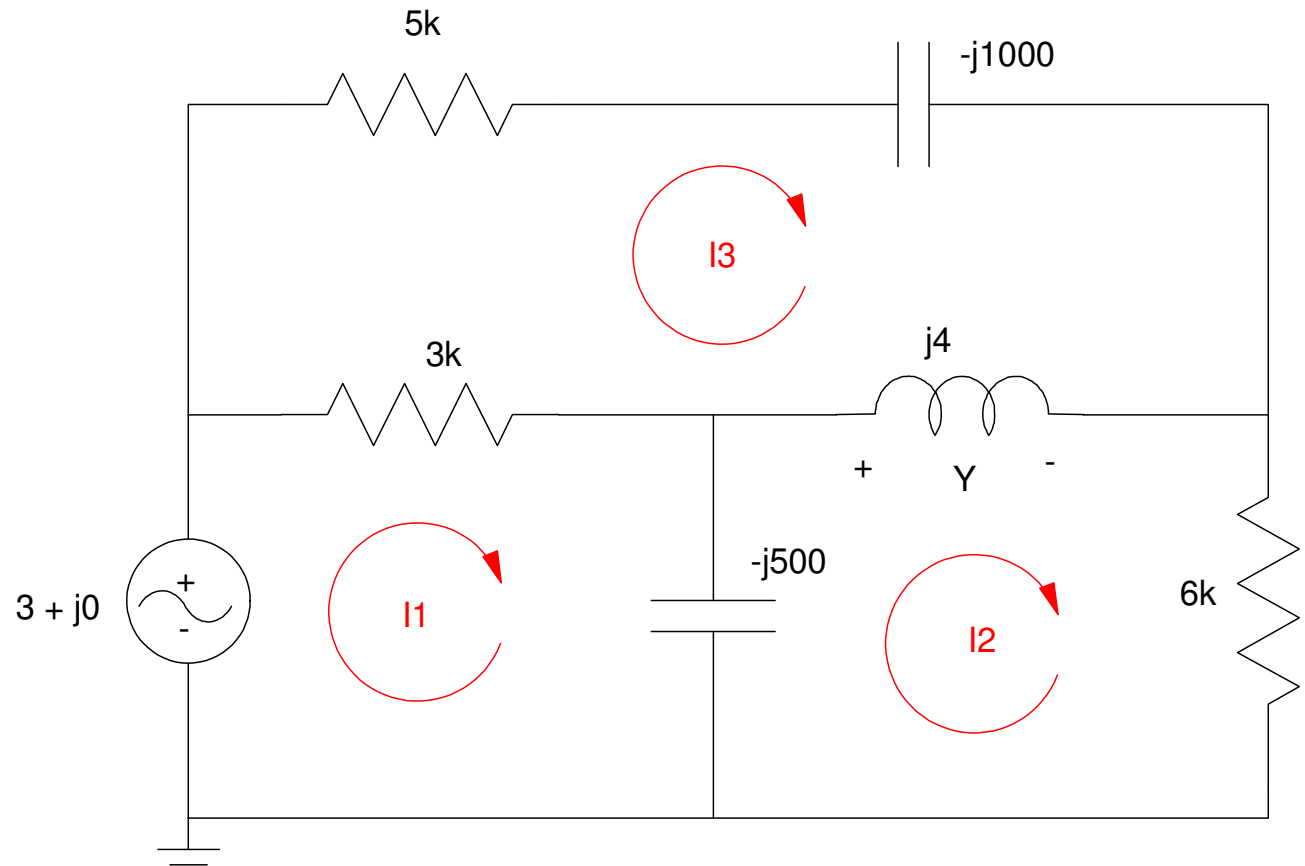
$$Z = \frac{1}{j\omega C} = -j1000$$

$$C = 2\mu\text{F}$$

$$Z = \frac{1}{j\omega C} = -j500$$

$$L = 4\text{mH}$$

$$Z = j\omega L = j4$$



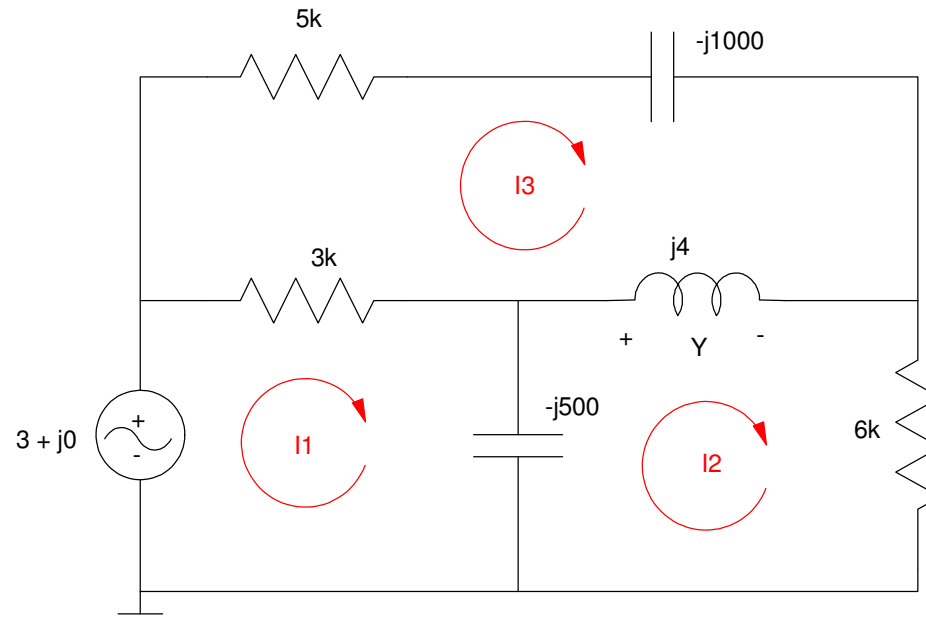
Step 2: Write the current loop equations

3 equations for 3 unknowns

$$-3 + 3000(I_1 - I_3) - j500(I_1 - I_2) = 0$$

$$-j500(I_2 - I_1) + j4(I_2 - I_3) + 6000I_2 = 0$$

$$-j1000(I_3) + 5000(I_3) + j4(I_3 - I_2) + 3000(I_3 - I_1) = 0$$



Step 3: Solve

Group terms:

$$(3000 - j500)I_1 + (j500)I_2 + (-3000)I_3 = 3$$

$$(j500)I_1 + (6000 - j496)I_2 + (-j4)I_3 = 0$$

$$(-3000)I_1 + (-j4)I_2 + (8000 - j996)I_3 = 0$$

$$\begin{aligned} A = & \begin{bmatrix} 3000-j*500, & j*500, & -3000 \\ j*500, & 6000-j*496, & -j*4 \\ -3000, & -j*4, & 8000-j*996 \end{bmatrix}; \end{aligned}$$

$$I_mA = \text{inv}(A) * [3; 0; 0] * 1000$$

$$I1 \text{ (mA)} \quad 1.4008 + 0.4598i$$

$$I2 \text{ (mA)} \quad 0.0475 - 0.1125i$$

$$I3 \text{ (mA)} \quad 0.4962 + 0.2342i$$

Step 4: Convert back to time

In phasor form:

$$I_1 \text{ (mA)} \quad 1.4008 + 0.4598i$$

$$I_2 \text{ (mA)} \quad 0.0475 - 0.1125i$$

$$I_3 \text{ (mA)} \quad 0.4962 + 0.2342i$$

In the time-domain

$$i_1(t) = 1.4008 \cos(1000t) - 0.4598 \sin(1000t) \text{ mA}$$

$$i_2(t) = 0.0475 \cos(1000t) + 0.1125 \sin(1000t) \text{ mA}$$

$$i_3(t) = 0.4962 \cos(1000t) - 0.2342 \sin(1000t) \text{ mA}$$

CircuitLab Results

1 cycle = 6.3ms

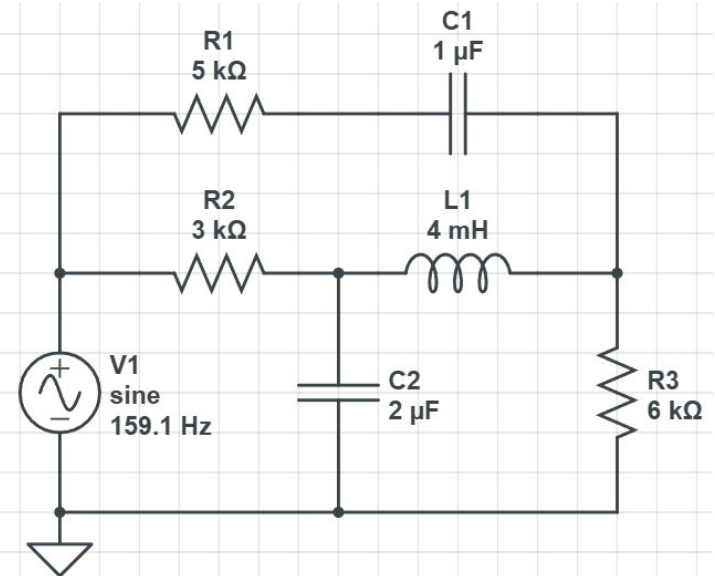
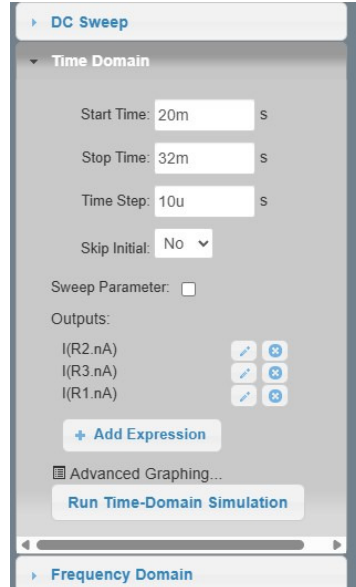
- $1 / 159.15\text{Hz}$

Plot for 2 cycles

- 12ms

Start at 20ms

- Let the transient die out
- Phasor analysis gives steady-state solution



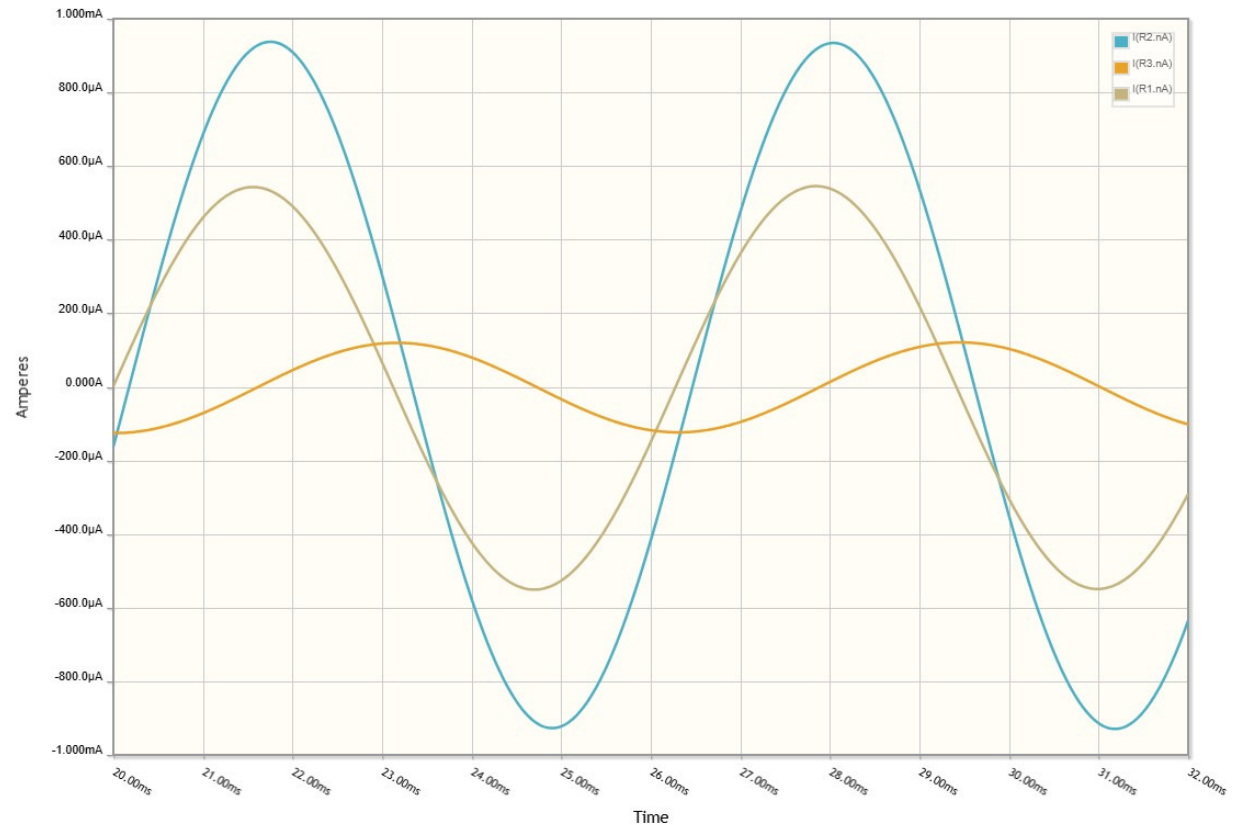
CircuitLab Result

Matlab

- $|I1| = 1.474\text{mA}$
- $|I2| = 0.122\text{mA}$
- $|I3| = 0.549\text{mA}$
- $|I1 - I3| = 0.9323\text{mA}$

CircuitLab

- $|I3| = 0.5450\text{mA}$
 - $I(R1) = I3$
- $|I2| = 0.122\text{mA}$
 - $I(R2) = I2$
- $|I1 - I3| = 0.934\text{mA}$
 - $I(R3) = I1 - I3$



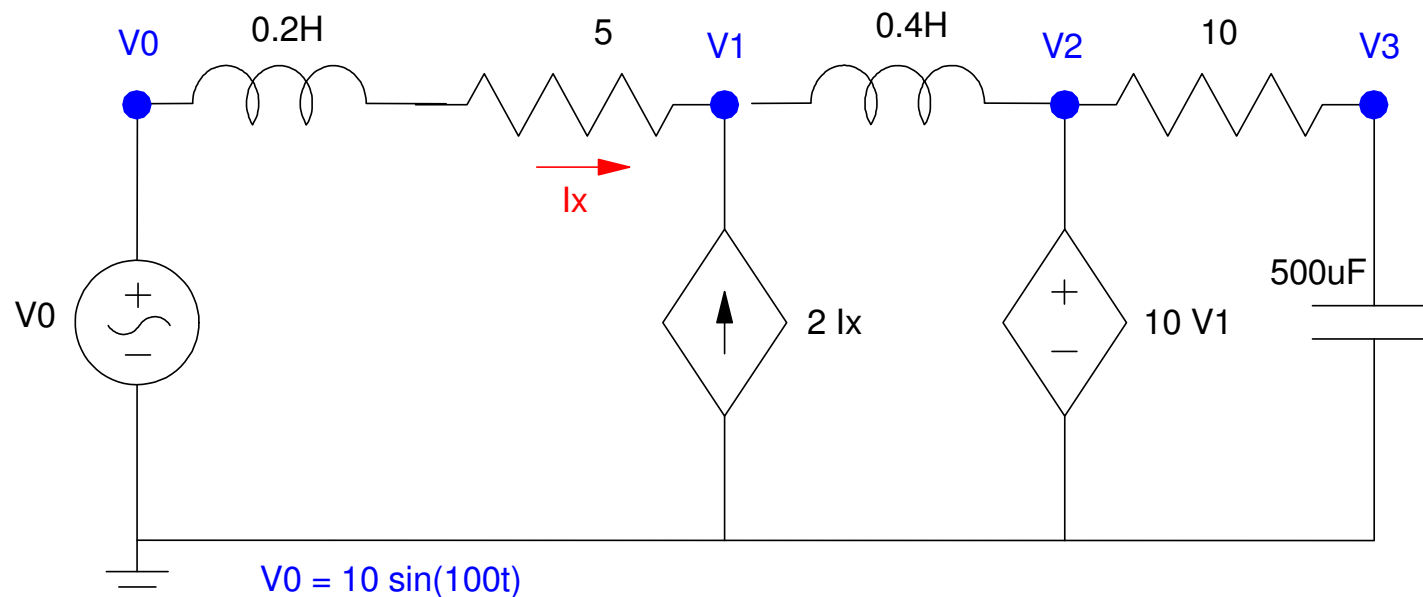
Results match

- There is an angle as well - this shows up as the phase shift in these graphs

Current Loops with Dependent Sources

Not surprisingly, it also works with dependent sources

Example: Determine the currents in the following circuit



Step 1:

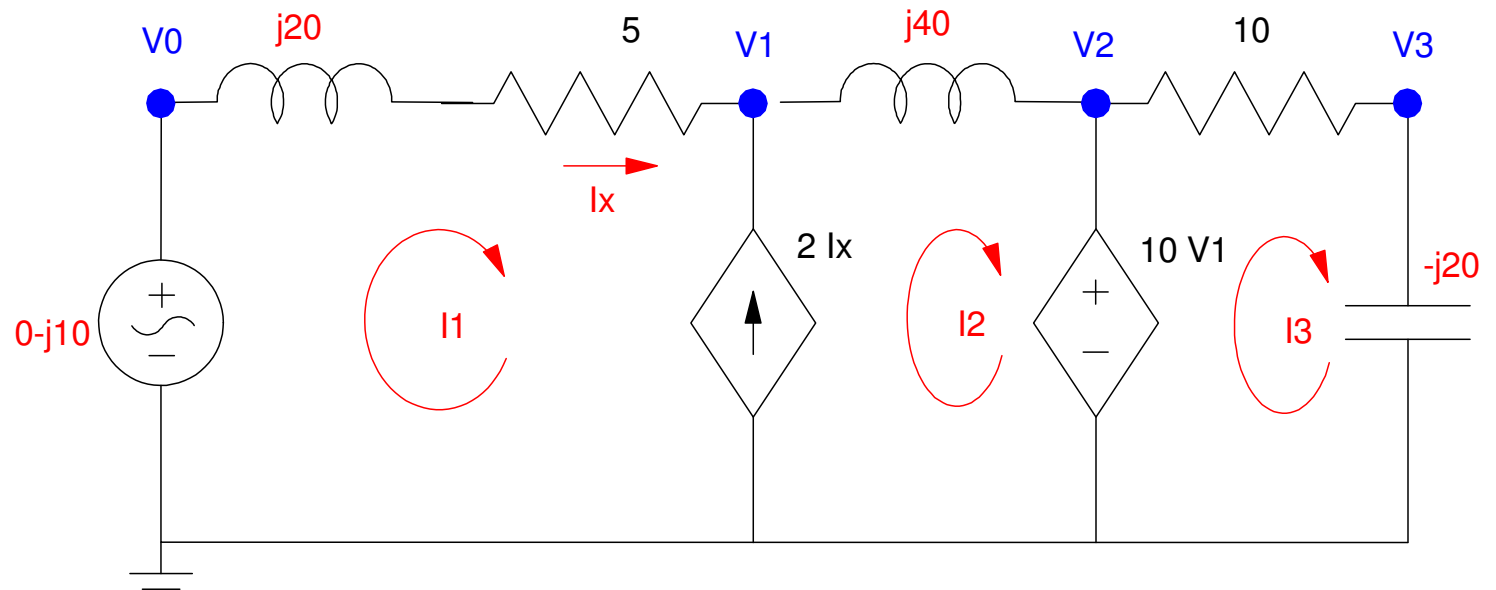
Define the loop currents

Convert to the phasor domain: (shown in blue)

$$a \cos(\omega t) + b \sin(\omega t) \rightarrow a - jb$$

$$L \rightarrow j\omega L$$

$$C \rightarrow \frac{1}{j\omega C}$$



Step 2: N equations for N unknowns

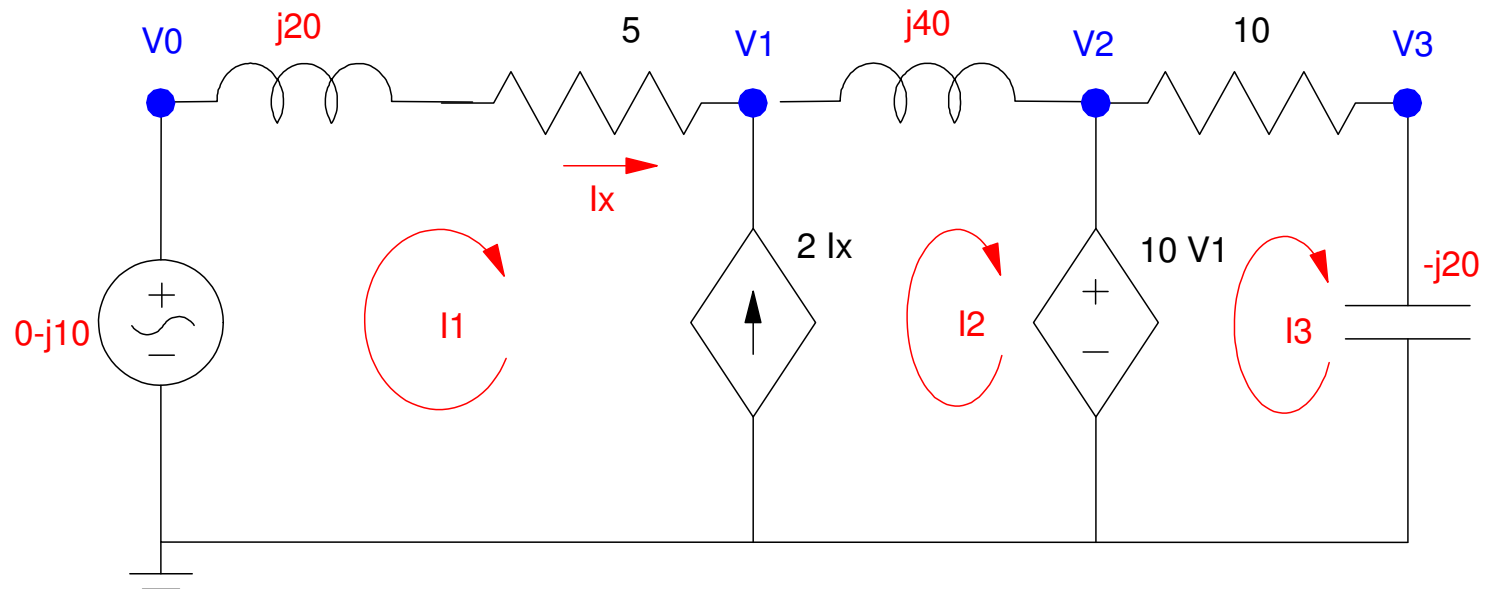
$$I_x = I_1$$

$$V_1 = j40 \cdot I_2 + (10 - j20)I_3$$

$$2I_x = I_2 - I_1$$

$$-10V_1 + 10I_3 - j20I_3 = 0$$

$$-(0 - j10) + (5 + j20)I_1 + j40I_2 + (10 - j20)I_3 = 0 \quad \text{super loop}$$



Step 3: Solve

Group terms

$$I_x - I_1 = 0$$

$$j40 \cdot I_2 + (10 - j20)I_3 - V_1 = 0$$

$$2I_x - I_2 + I_1 = 0$$

$$-10V_1 + (10 - j20)I_3 = 0$$

$$(5 + j20)I_1 + j40I_2 + (10 - j20)I_3 = 0 - j10$$

Place in matrix form

$$\begin{bmatrix} (-1) & 0 & 0 & (1) & 0 \\ 0 & (j40) & (10 - j20) & 0 & (-1) \\ (1) & (-1) & 0 & (2) & 0 \\ 0 & 0 & (10 - j20) & 0 & (-10) \\ (5 + j20) & (j40) & (10 - j20) & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_x \\ V_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -j10 \end{bmatrix}$$

Solve in Matlab

```
>> a1 = [-1,0,0,1,0];  
>> a2 = [0,40j,10-20j,0,-1];  
>> a3 = [1,-1,0,2,0];  
>> a4 = [0,0,10-20j,0,-10];  
>> a5 = [5+20j,40j,10-20j,0,0];  
>> A = [a1;a2;a3;a4;a5];  
>> B = [0;0;0;0;-10j];  
>> I = inv(A)*B
```

```
I1    -0.9600 - 0.7200i  
I2    -2.8800 - 2.1600i  
I3    -7.0400 - 1.2800i  
Ix    -0.9600 - 0.7200i  
V1   -9.6000 +12.8000i
```

Step 4: Convert back to the time domain

Phasor domain

```
I1  -0.9600 - 0.7200i
I2  -2.8800 - 2.1600i
I3  -7.0400 - 1.2800i
Ix  -0.9600 - 0.7200i
V1  -9.6000 +12.8000i
```

Time Domain

$$i_1(t) = -0.96 \cos(100t) + 0.72 \sin(100t)$$

$$i_2(t) = -2.88 \cos(100t) + 2.16 \sin(100t)$$

$$i_3(t) = 7.04 \cos(100t) + 1.28 \sin(100t)$$

Checking in CircuitLab

Matlab

- $|I1| = 1.2000$
- $|I2| = 3.6000$
- $|I3| = 7.1554$

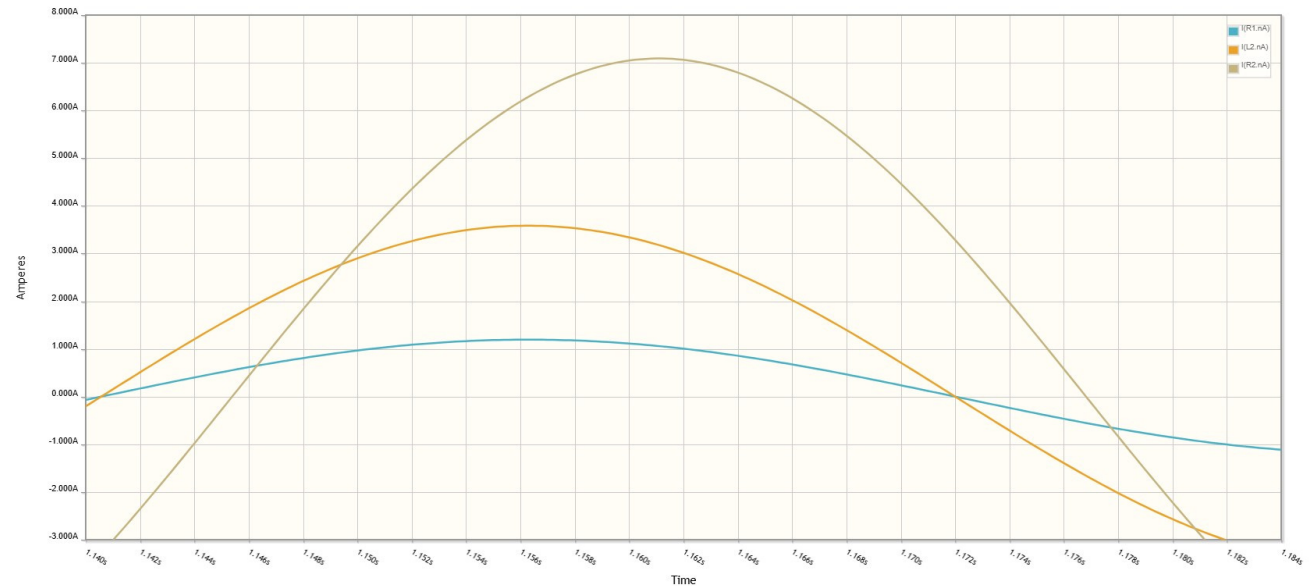
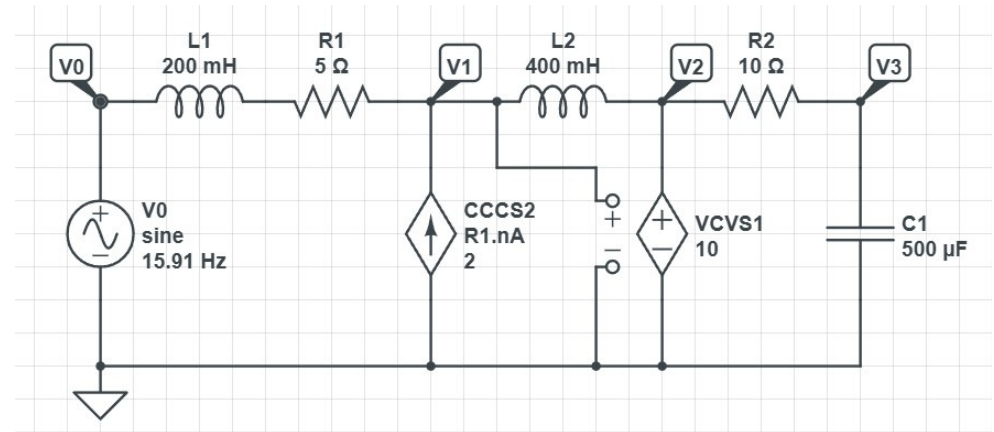
CircuitLab

- Start at $t=1$ seconds
- End at $t=1.2$ seconds
- Step Size = 0.2ms

Result

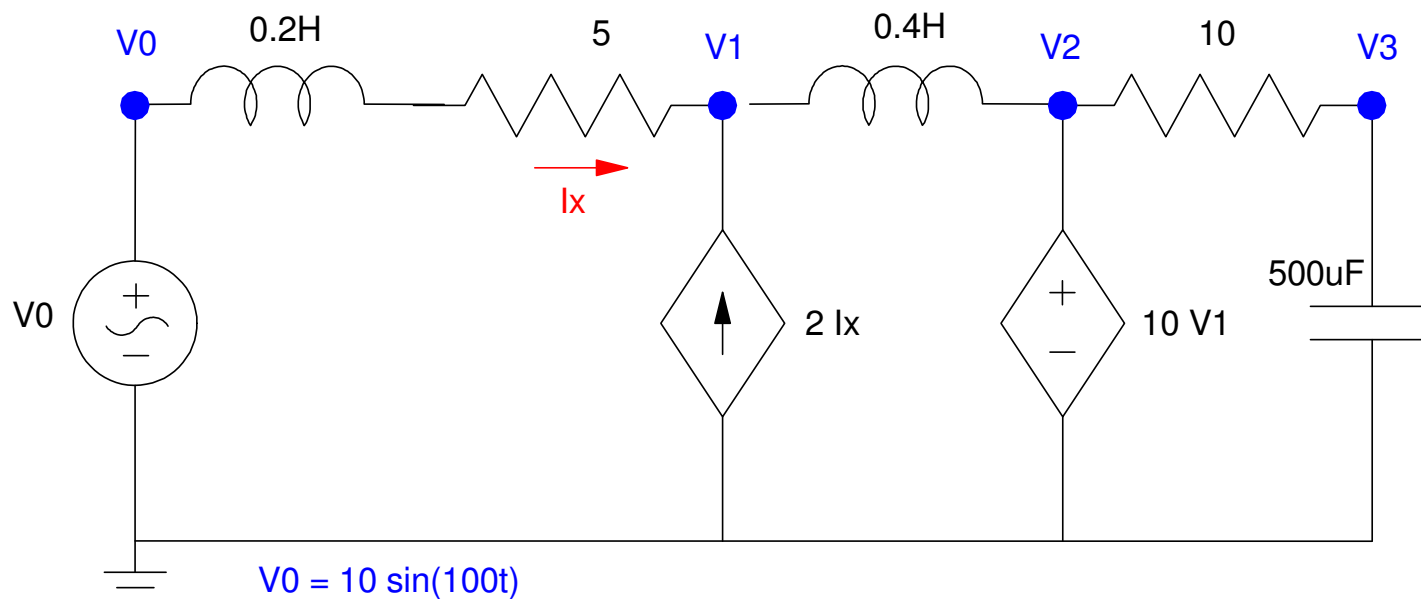
- $|I1| = 1.192\text{A}$
- $|I2| = 3.582\text{A}$
- $|I3| = 7.090\text{A}$

Matches Matlab



What happens if the frequency changes?

- Phasor impedances change
- Whole new problem



Summary

Current Loops work the same

- for DC inputs
- for AC inputs

The only difference is you're now dealing with complex numbers.

Matlab is able to solve N equations for N unknowns

- with real numbers, and
- with complex numbers

This is where Matlab really shines
