Phasors & Voltage Nodes

ECE 211 Circuits I

Lecture #25

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

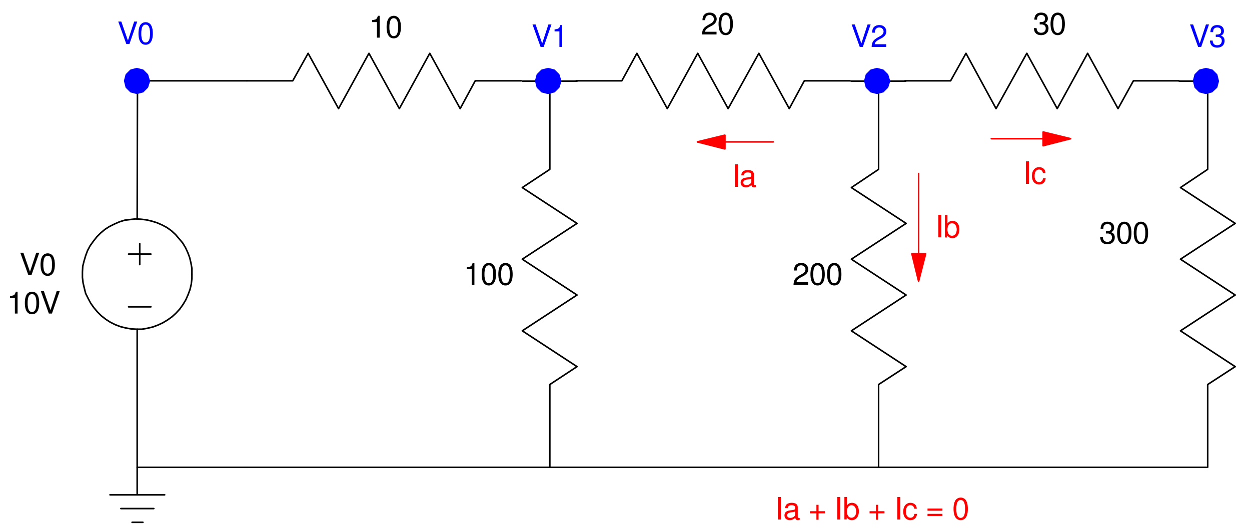
Voltage Nodes:

One tool that's useful in solving a given circuit is *voltage nodes*

- Given a circuit with N unknown voltages
- Write N equations for N unknowns

Each voltage node equation uses conservation of current

- The net current flowing from a node must be zero



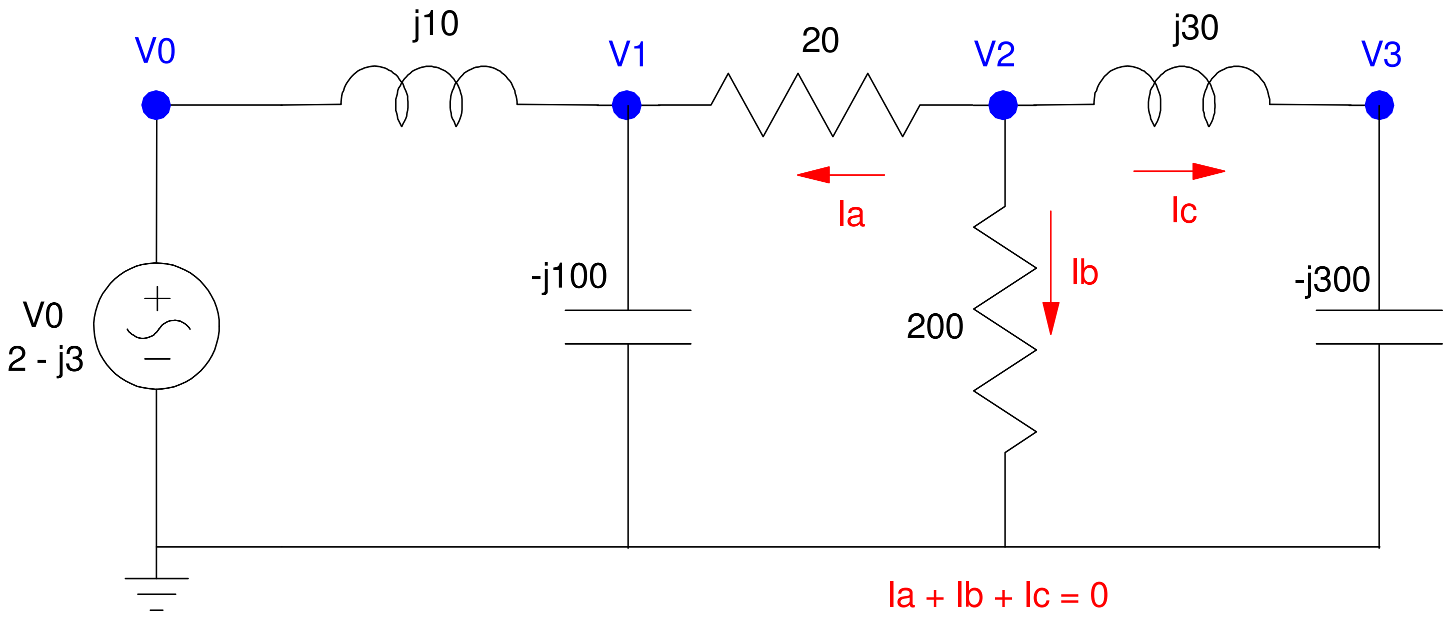
Voltage Nodes with Phasors

Voltage Nodes works for DC signals

- Node equations use real numbers

Voltage Nodes works for AC signals

- You now need to use phasors and complex numbers



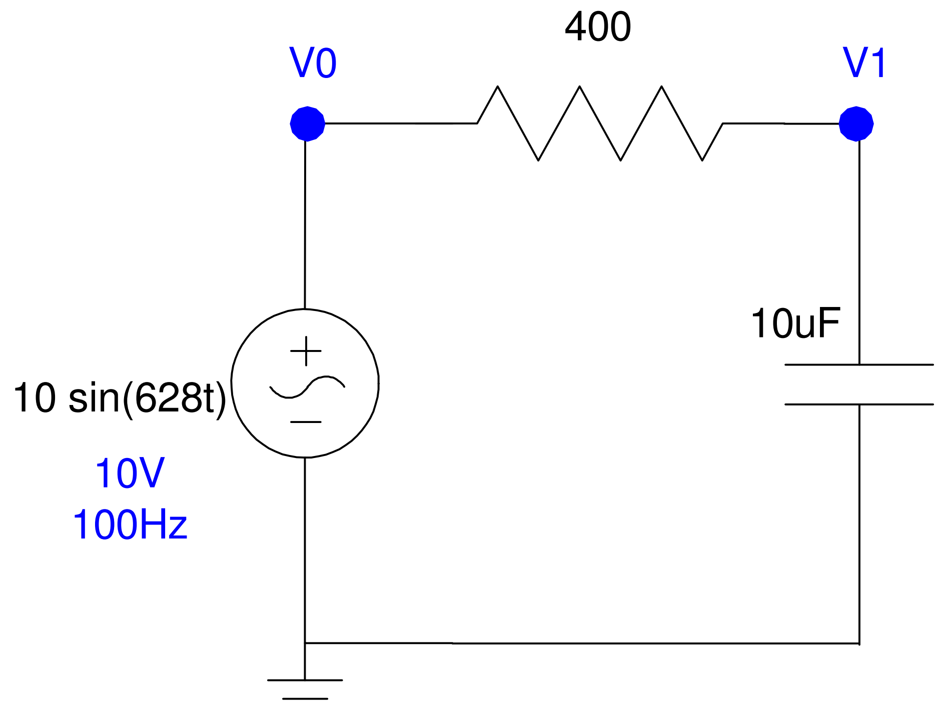
Phasor Voltages & Impedances

Recall: The phasor representation of voltages and impedances are:

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Current	$i(t) = a\cos(\omega t) + b\sin(\omega t)$	$I = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example: RC Circuit

Determine the node voltages for the following circuit:



Step 1: Replace the elements with their phasor values (shown in red)

Voltage Source

$$a\cos(\omega t) + b\sin(\omega t) \rightarrow a - jb$$

$$10\sin(628t) \rightarrow 0 - j10$$

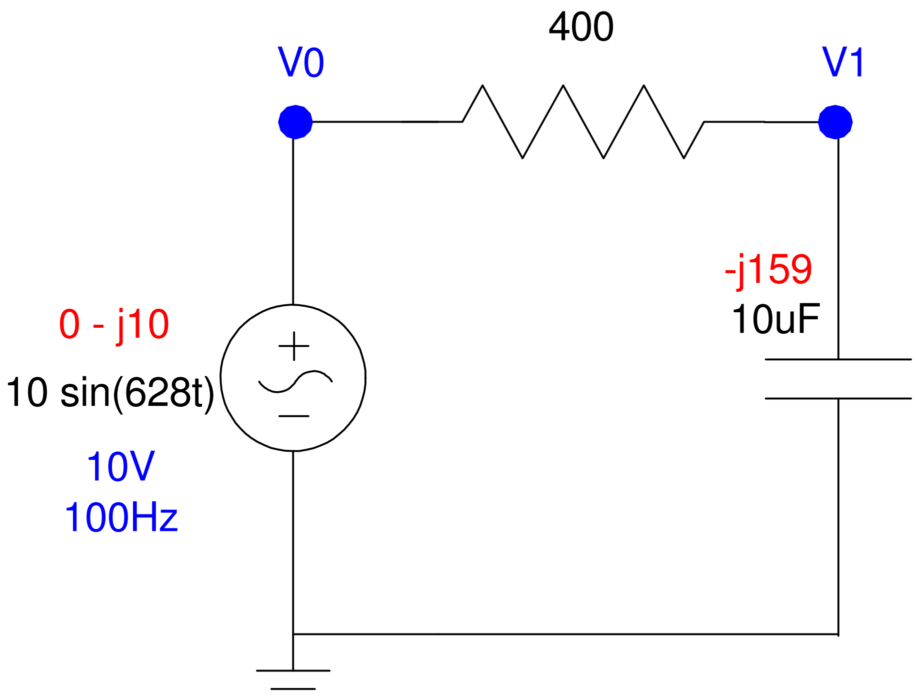
Resistors

$$R \rightarrow R$$

Capacitors

$$C \rightarrow \frac{1}{j\omega C}$$

$$10\mu F \rightarrow -j159.24\Omega$$



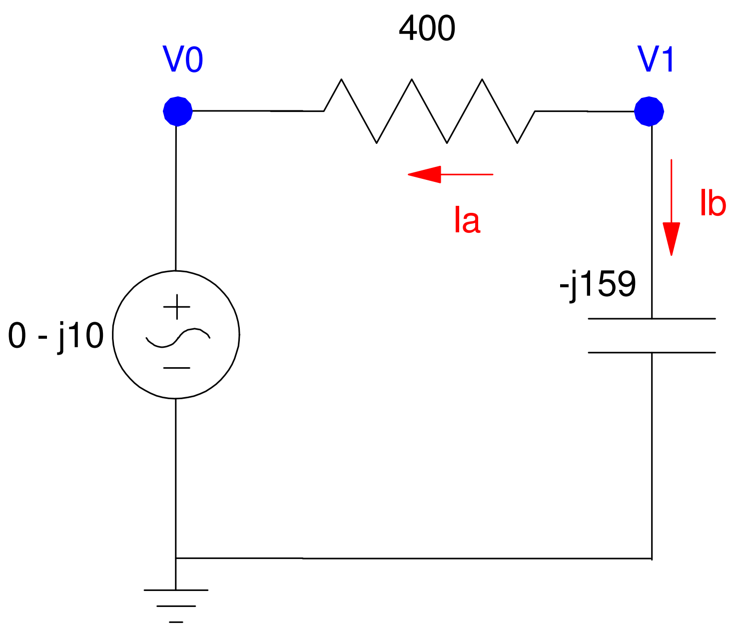
Step 2: Write N equations for N unknowns

Node V0

$$V_0 = -j10$$

Node V1

$$\left(\frac{V_1 - V_0}{400}\right) + \left(\frac{V_1}{-j159}\right) = 0$$



Solve

$$V_0 = -j10$$

$$\left(\frac{-1}{400}\right) V_0 + \left(\frac{1}{100} + \frac{1}{-j159}\right) V_1 = 0$$

Using Matlab

In matrix form

$$\begin{bmatrix} 1 & 0 \\ \left(\frac{-1}{400}\right) & \left(\frac{1}{400}+\frac{1}{-j159}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \end{bmatrix}$$

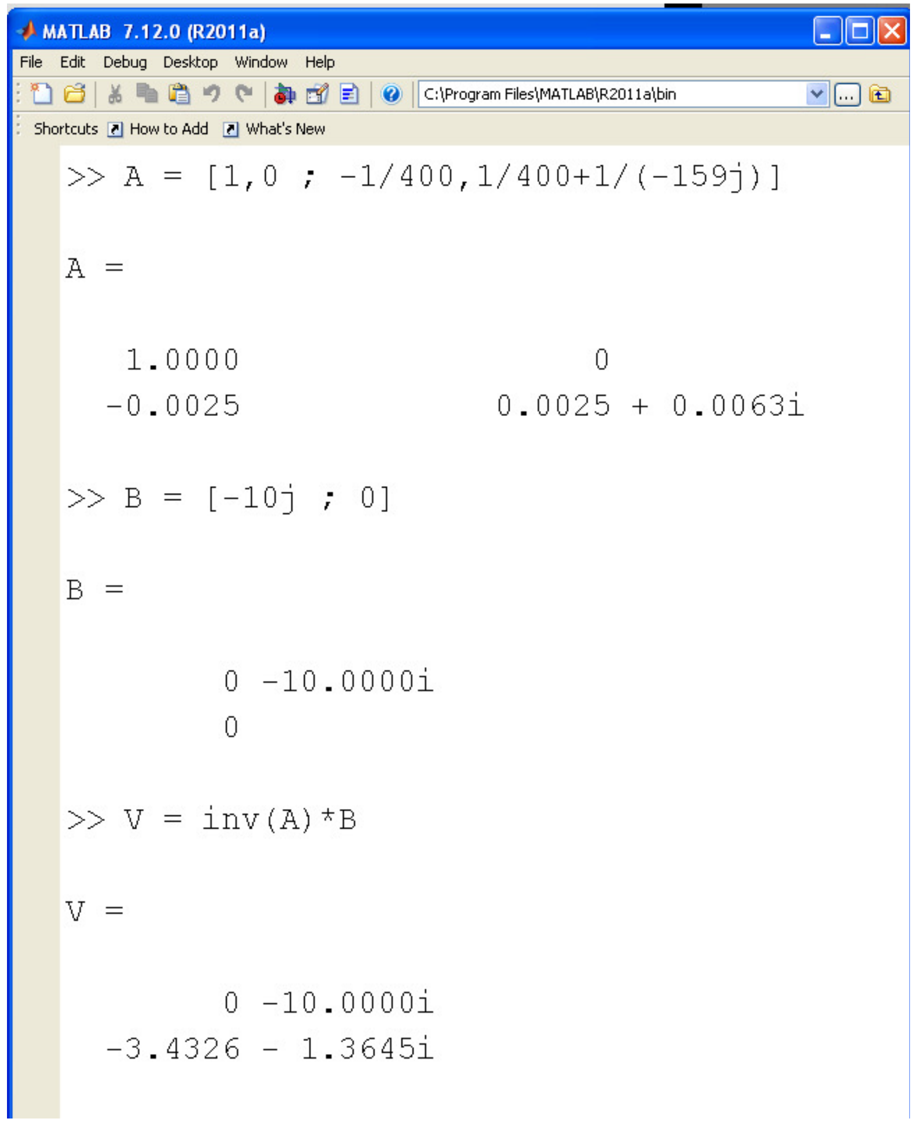
Result

$$V_0 = -j10$$

$$V_1 = -3.4326 - j1.3646$$

Translating

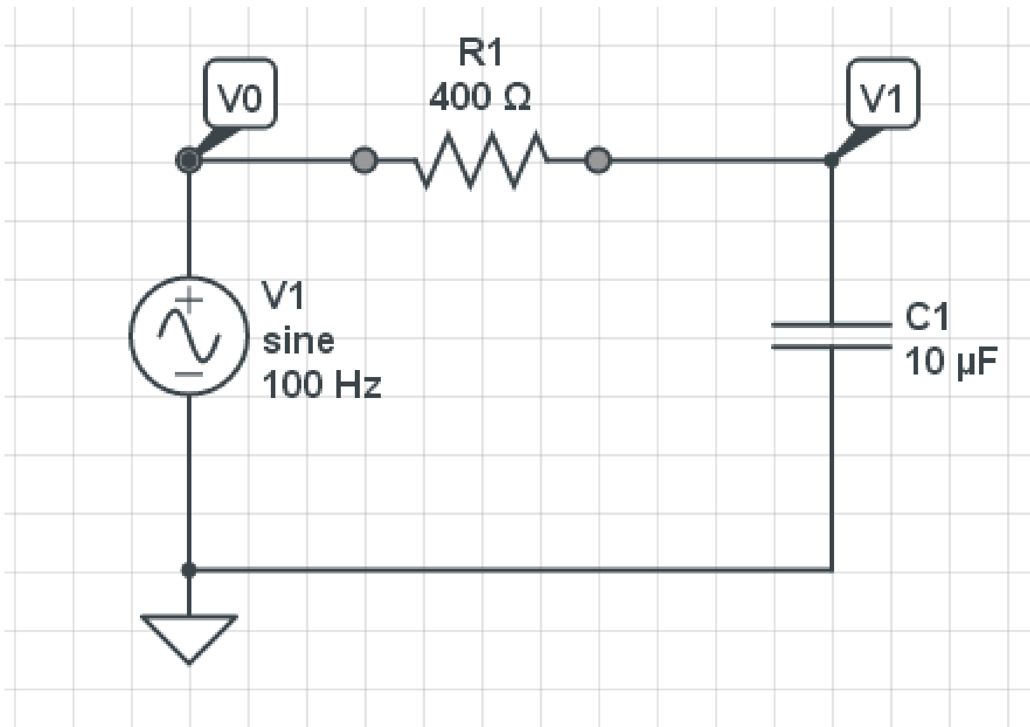
$$v_1(t) = -3.43\cos(628t) \\ +1.36\sin(628t)$$



Verify with CircuitLab

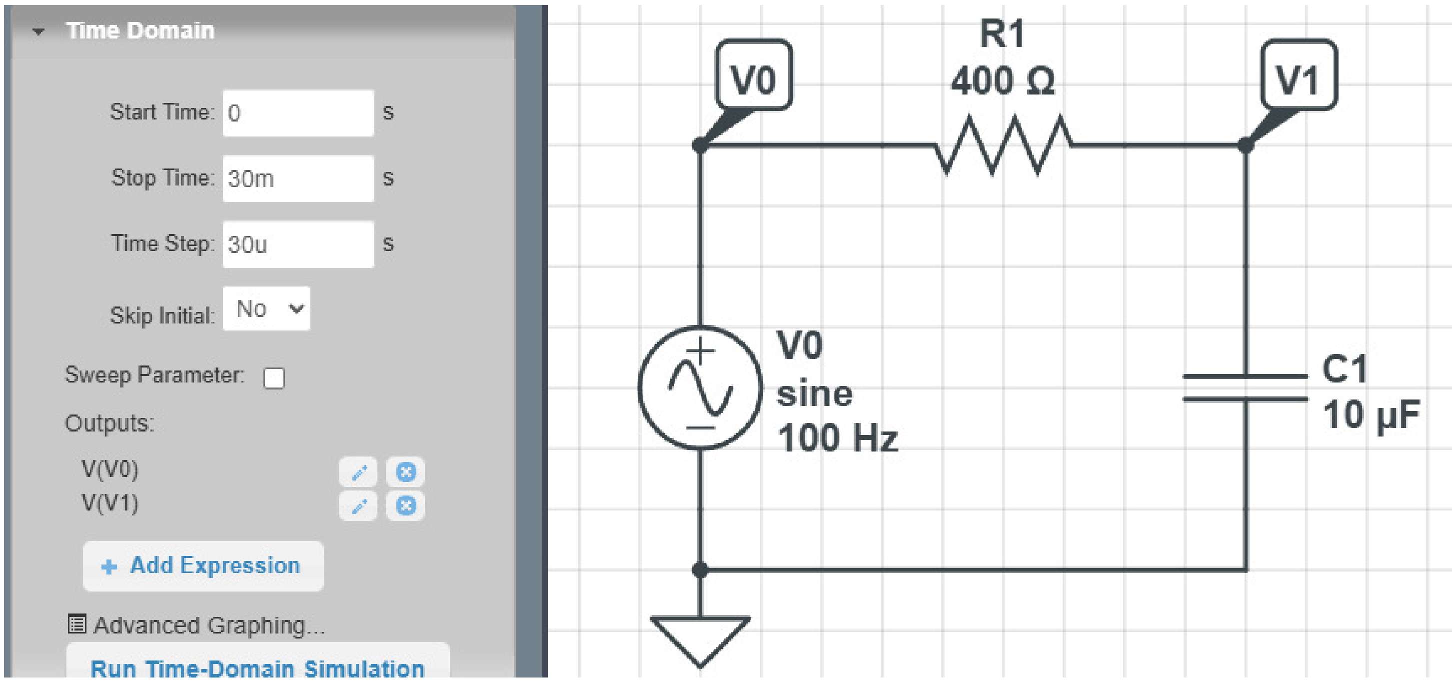
Make the input a sine wave with

- no DC offset
- 10V amplitude
- 100Hz (628 rad/sec)



Run a transient response for

- 30ms (3 cycles)
- 10us step size



CircuitLab Result

- Polar form is easier with CircuitLab

V0 (blue waveform)

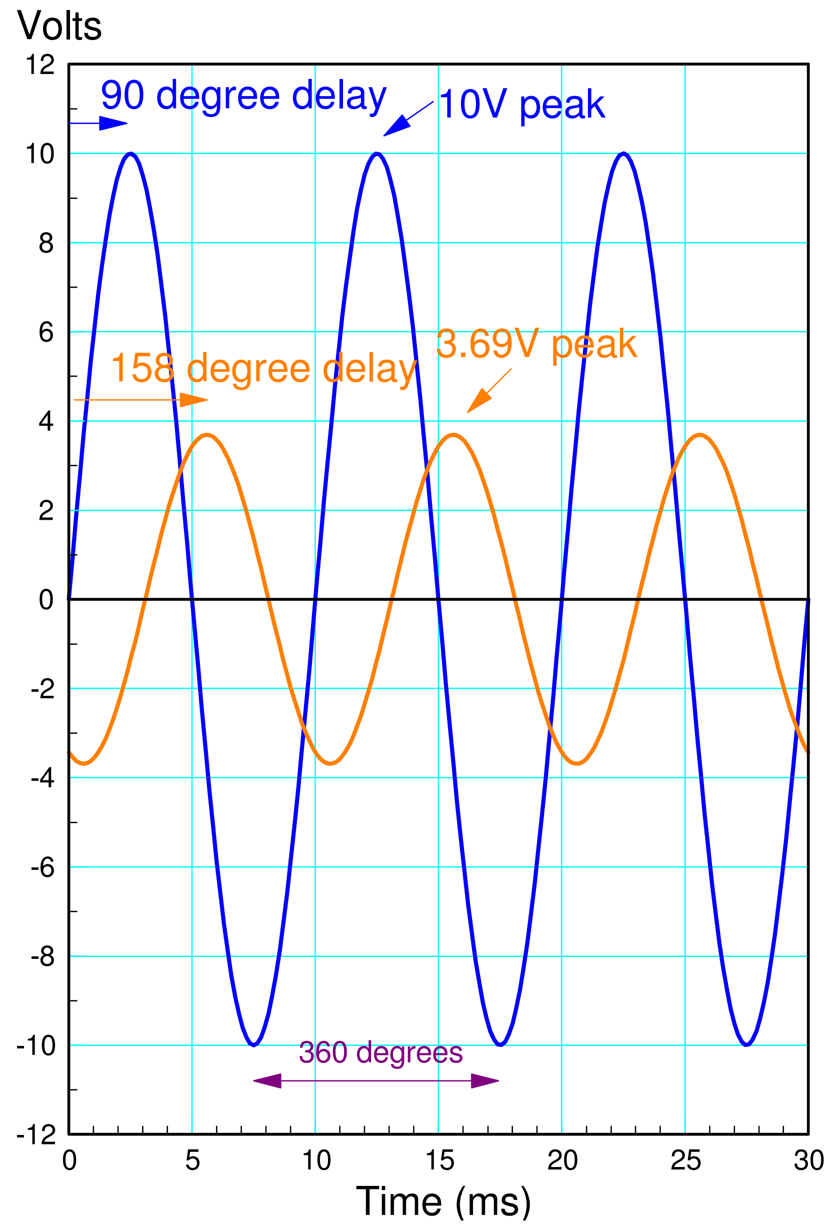
$$V_0 = 0 - j10$$
$$= 10\angle -90^0$$

- Peak = 10V
- Delay is 90 degrees

V1 (orange waveform)

$$V_1 = -3.43 - j1.36$$
$$= 3.69\angle -158^0$$

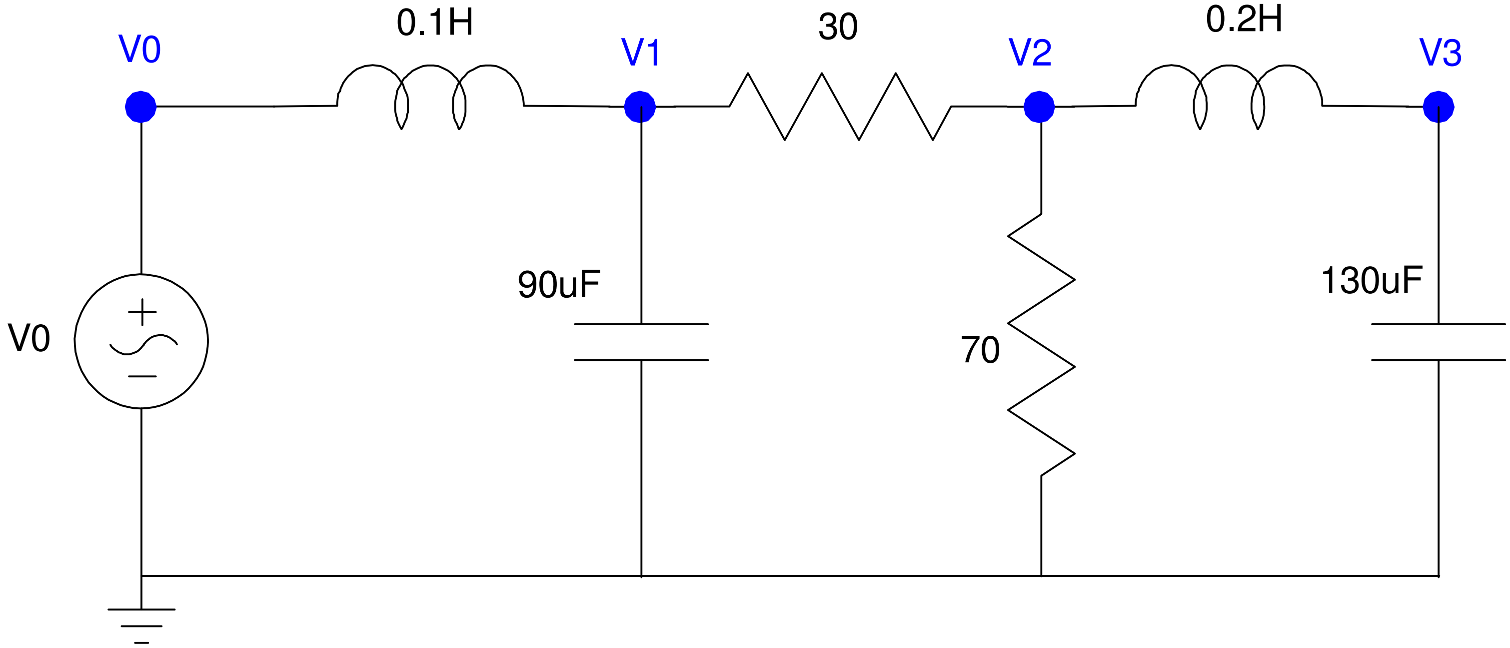
- Peak is 3.69V
- Delay is 158 degrees



Example 2: 3-Stage RC Circuit

Find the voltages for the following circuit when the input is

$$V_0 = 7\cos(100t) + 3\sin(100t)$$



Step 1: Express in Phasor form

- (shown in red)

$$V_0 = 7\cos(100t) + 3\sin(100t)$$

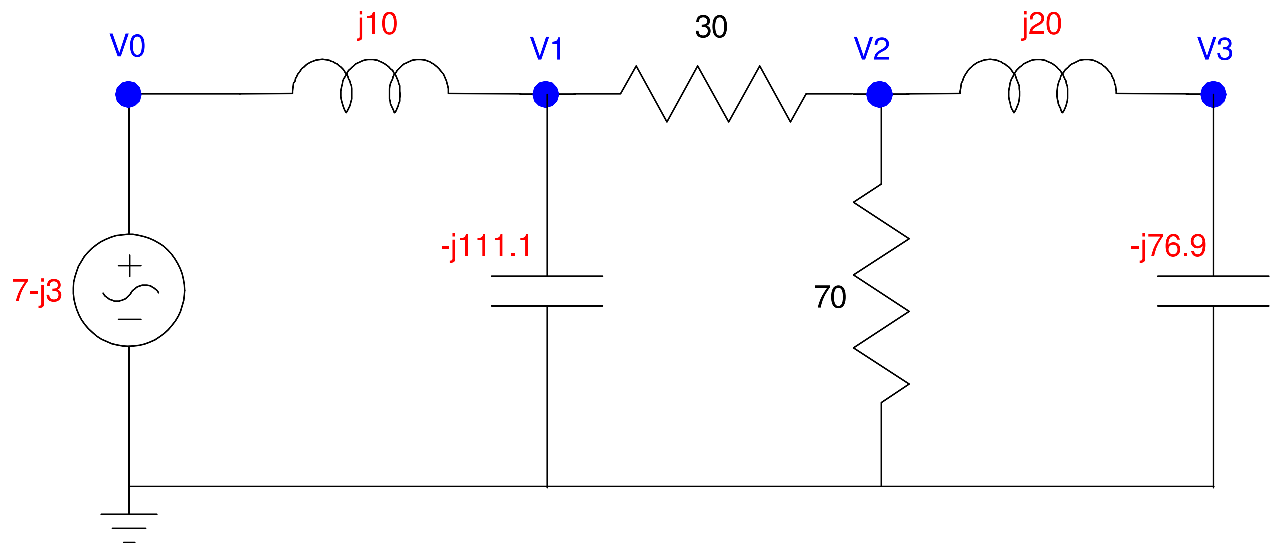
$$= 7 - j3$$

$$0.1H \to j\omega L = j10$$

$$0.2H \to j\omega L = j20$$

$$90\mu F \to \frac{1}{j\omega C} = -j111.1$$

$$130\mu F \to \frac{1}{j\omega C} = -j76.9$$



Step 2: Write N equtions for N unknowns

Voltage source

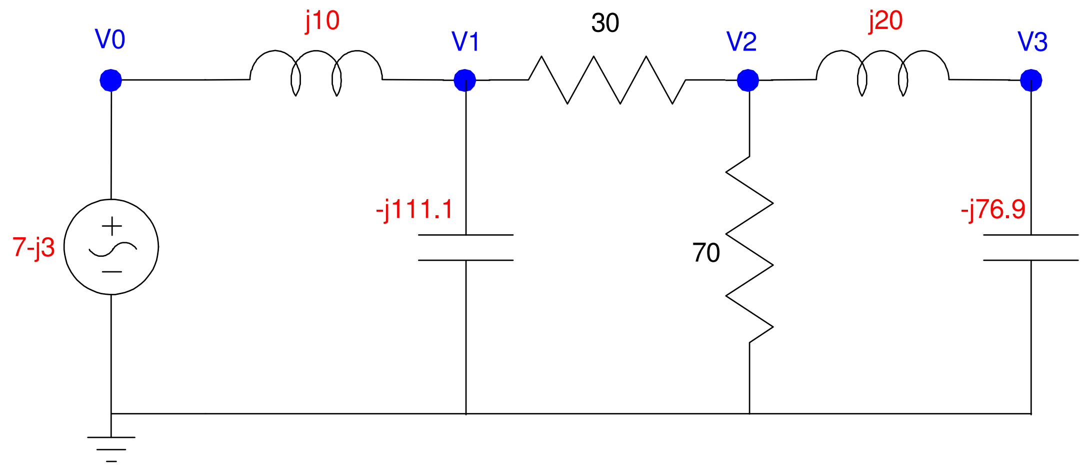
$$V_0 = 7 - j3$$

Node equations

$$\left(\frac{V_1 - V_0}{j10}\right) + \left(\frac{V_1}{-j111.1}\right) + \left(\frac{V_1 - V_2}{30}\right) = 0$$

$$\left(\frac{V_2 - V_1}{30}\right) + \left(\frac{V_2}{70}\right) + \left(\frac{V_2 - V_3}{j20}\right) = 0$$

$$\left(\frac{V_3 - V_2}{j20}\right) + \left(\frac{V_3}{-j76.9}\right) = 0$$



Step 3: Solve

Group terms

$$V_0 = 7 - j3$$

$$\left(\frac{V_1 - V_0}{j10}\right) + \left(\frac{V_1}{-j111.1}\right) + \left(\frac{V_1 - V_2}{30}\right) = 0$$

$$\left(\frac{V_2 - V_1}{30}\right) + \left(\frac{V_2}{70}\right) + \left(\frac{V_2 - V_3}{j20}\right) = 0$$

$$\left(\frac{V_3 - V_2}{j20}\right) + \left(\frac{V_3}{-j76.9}\right) = 0$$

becomes

$$V_0 = 7 - j3$$

$$-\left(\frac{1}{j10}\right) V_0 + \left(\frac{1}{j10} + \frac{1}{-j111.1} + \frac{1}{30}\right) V_1 - \left(\frac{1}{30}\right) V_2 = 0$$

$$-\left(\frac{1}{30}\right) V_1 + \left(\frac{1}{30} + \frac{1}{70} + \frac{1}{j20}\right) V_2 - \left(\frac{1}{j20}\right) V_3 = 0$$

$$-\left(\frac{1}{j20}\right) V_2 + \left(\frac{1}{j20} + \frac{1}{-j76.9}\right) V_3 = 0$$

Place in matrix form

$$V_0 = 7 - j3$$

$$-\left(\frac{1}{j10}\right)V_0 + \left(\frac{1}{j10} + \frac{1}{-j111.1} + \frac{1}{30}\right)V_1 - \left(\frac{1}{30}\right)V_2 = 0$$

$$-\left(\frac{1}{30}\right)V_1 + \left(\frac{1}{30} + \frac{1}{70} + \frac{1}{j20}\right)V_2 - \left(\frac{1}{j20}\right)V_3 = 0$$

$$-\left(\frac{1}{j20}\right)V_2 + \left(\frac{1}{j20} + \frac{1}{-j76.9}\right)V_3 = 0$$

becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{j10}\right) & \left(\frac{1}{j10} + \frac{1}{-j111.1} + \frac{1}{30}\right) & \left(\frac{-1}{30}\right) & 0 \\ 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30} + \frac{1}{70} + \frac{1}{j20}\right) & \left(\frac{-1}{j20}\right) \\ 0 & 0 & \left(\frac{-1}{j20}\right) & \left(\frac{1}{j20} + \frac{1}{-j76.9}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} (7 - j3) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve

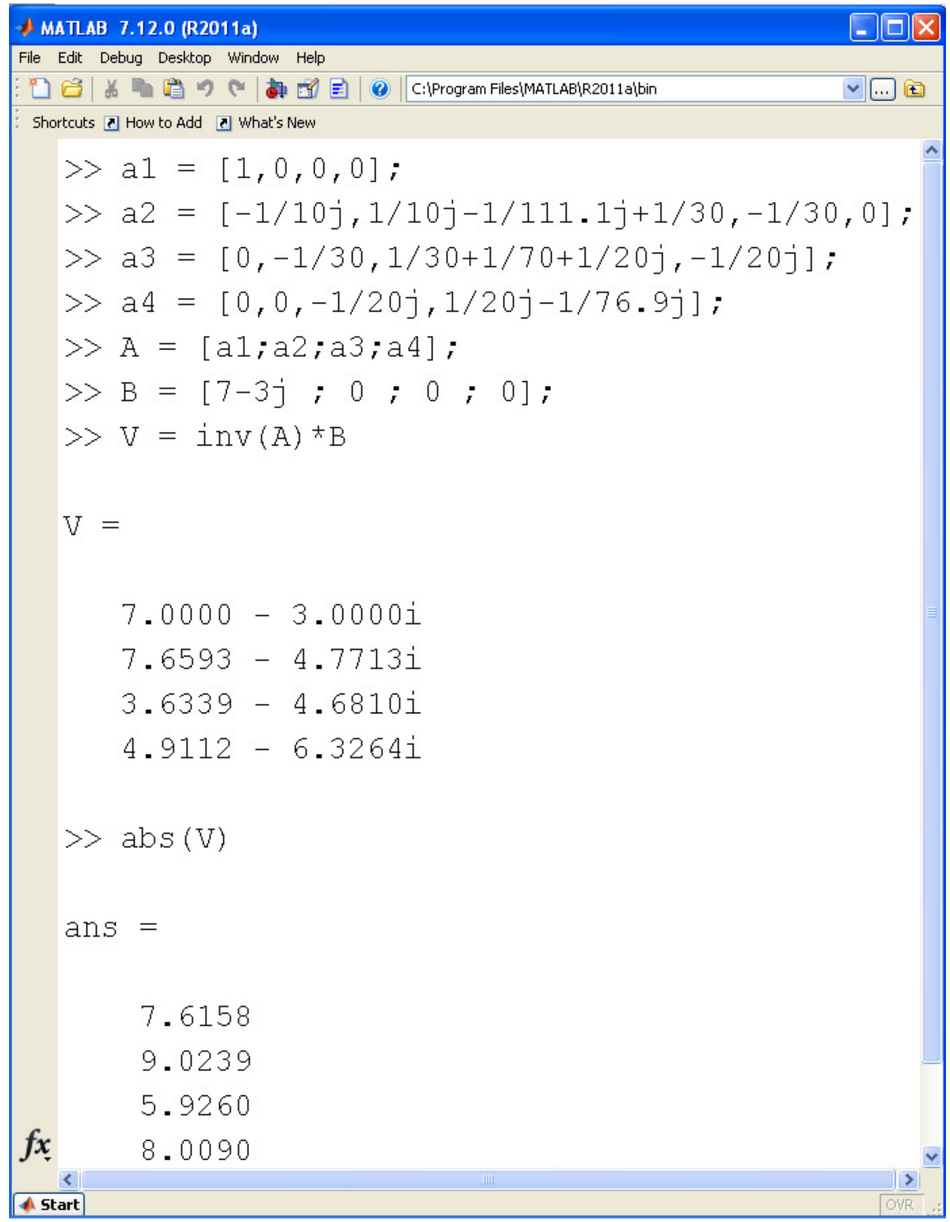
- Matlab is *really* useful here

V0(t) = 7 cos(100t) + 3 sin(100t)

V1(t) = 7.65cos(100t)+4.77sin(100t)

V2(t) = 3.63cos(100t)+4.68sin(100t)

V3(t) = 4.91cos(100t)+6.32sin(100t)



Check in CircuitLab

V0 is 100 rad/sec

$$f = \frac{\omega}{2\pi} = 15.9155Hz$$

V0(t) in polar form

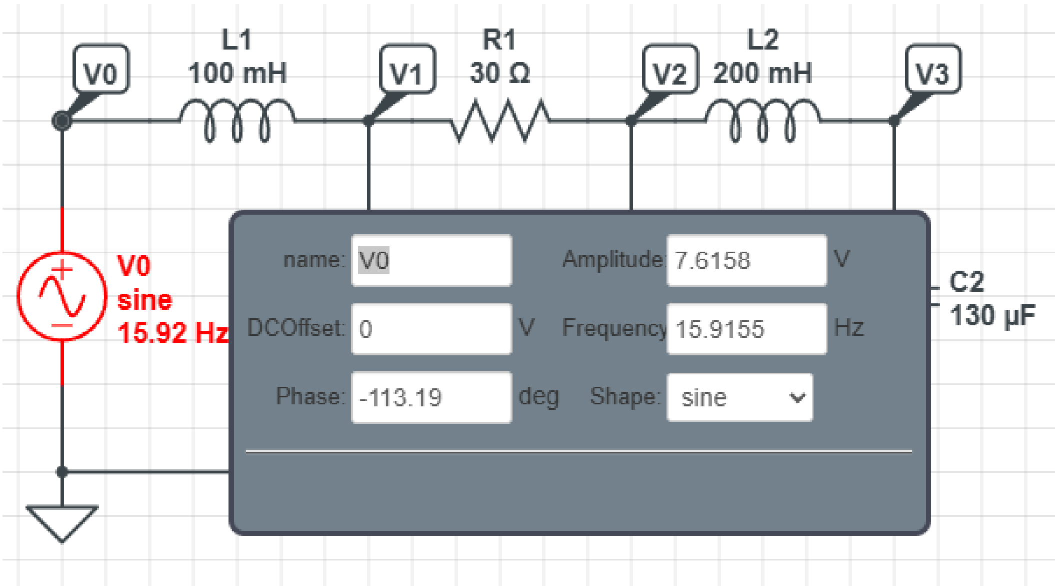
$$V_0 = 7 - j3$$

$$= 7.6158\angle - 23.1986^0$$

Switching to sine waves

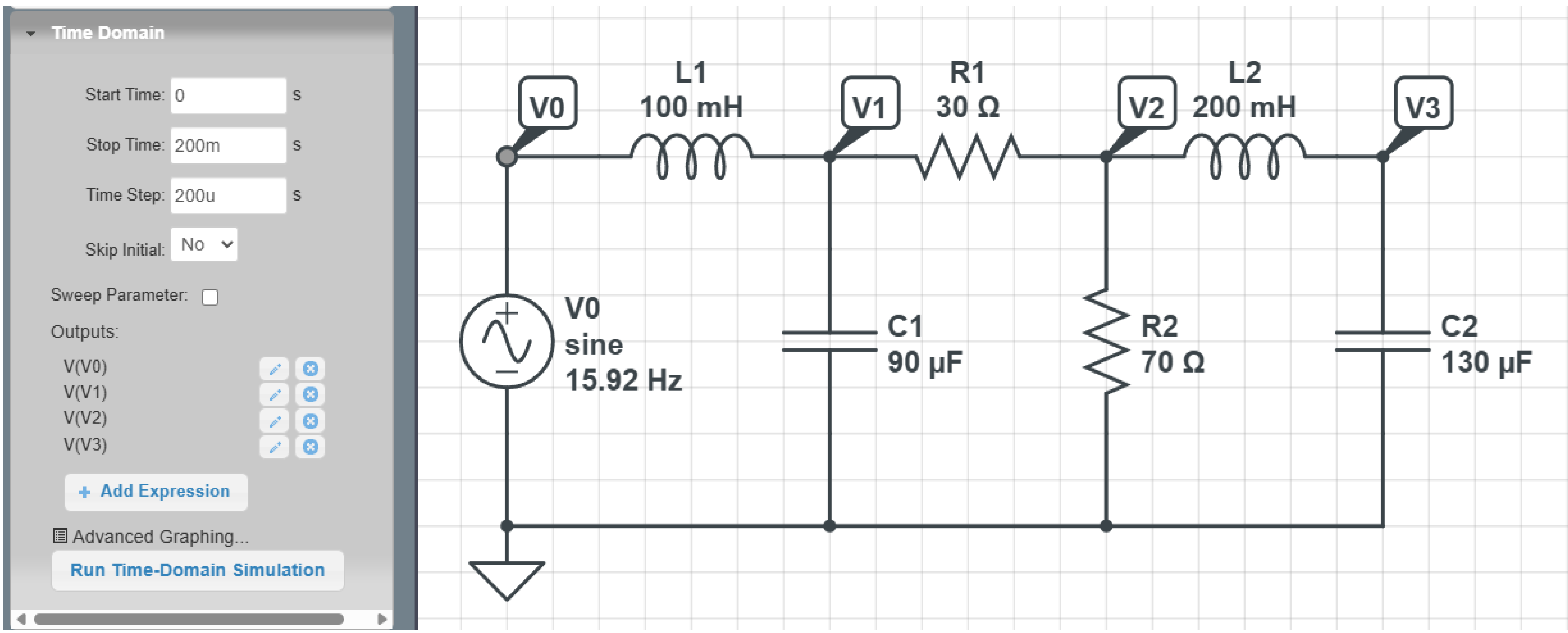
- Cosine is default, but
- CircuitLab uses sine waves

$$= 7.6158\angle - 113.1986$$



Simulate for 3 cycles

- Let the transients die out
- 15.9Hz
- 3 cycles = 200ms



Results match calculations

V0p = 7.615V

- matlab: 7.6158

V1p = 8.995V

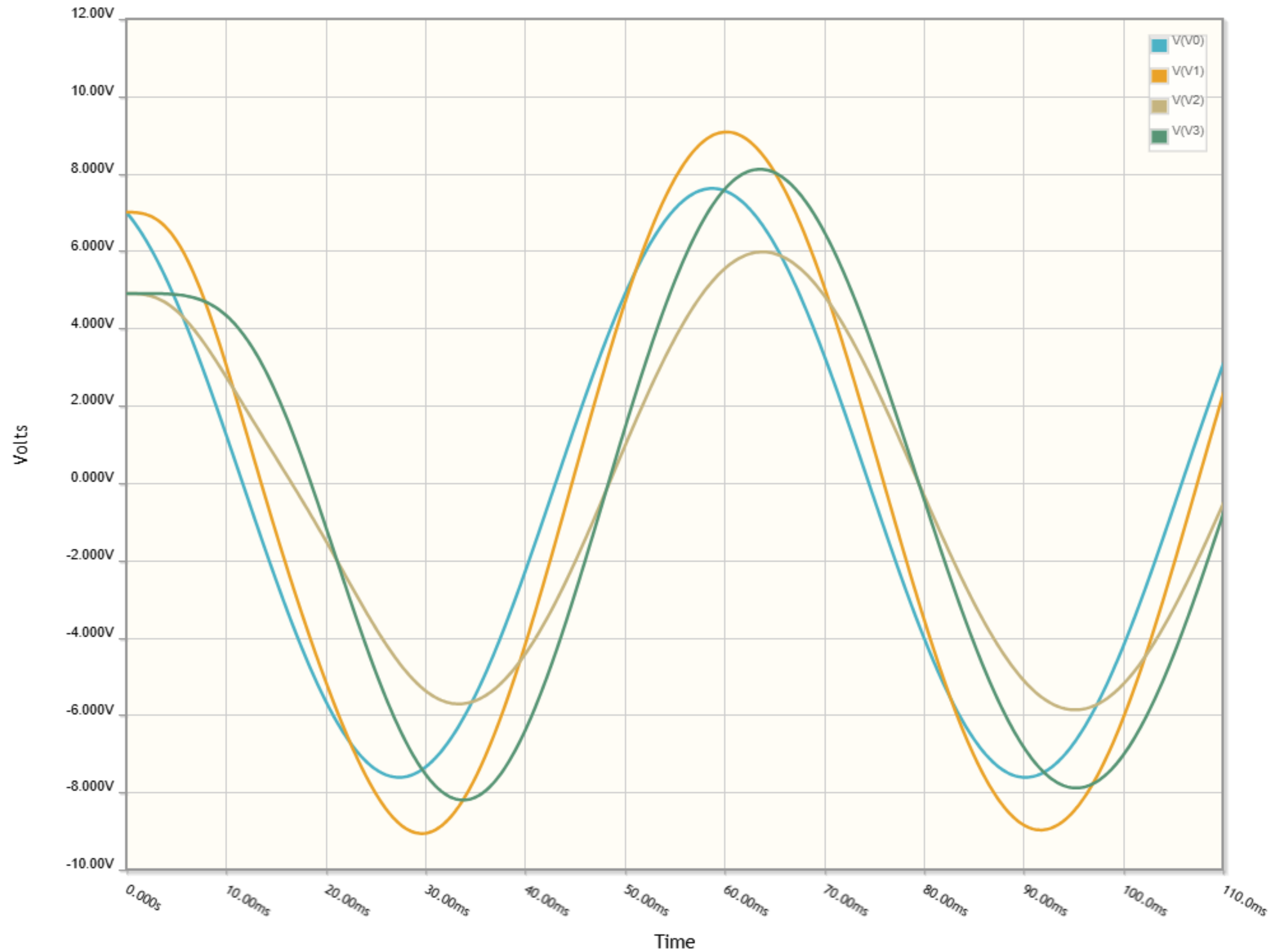
- matlab: 9.0239

V2p = 5.837V

- matlab 5.9260

V3p = 7.936V

- matlab 8.0090



What happens if you change the frequency?

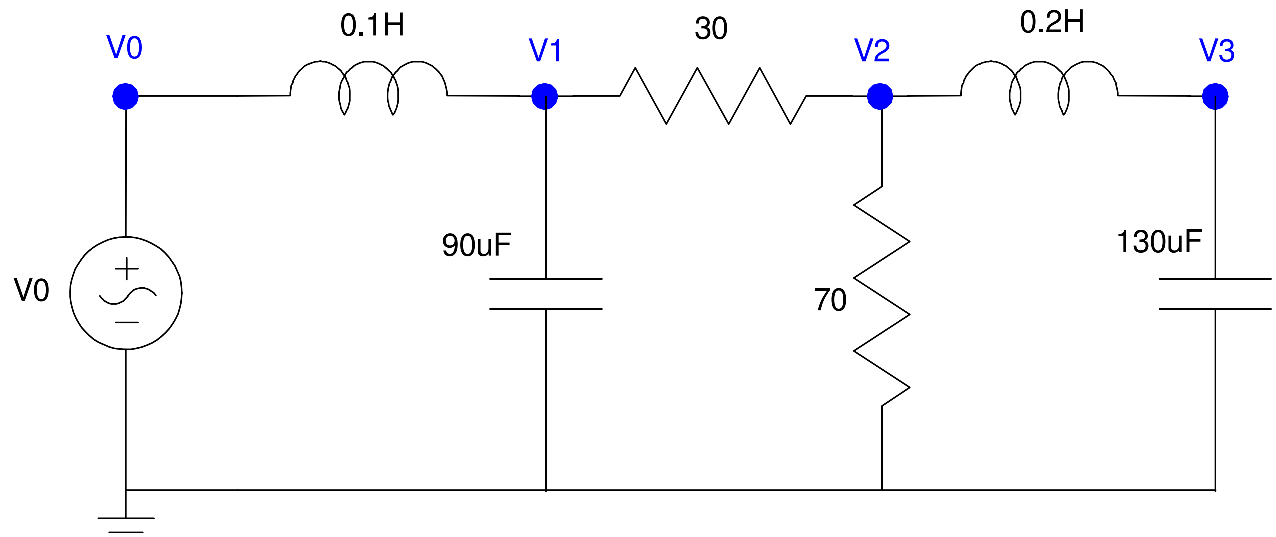
$$V_0 = 7\cos(200t) + 3\sin(200t)$$

Completely new problem

- R stays the same
- L changes
- C changes

Start over

- Each frequency gives a different answer



Summary

Voltage nodes works the same

- for DC inputs
- for AC inputs

The only difference is you're now dealing with complex numbers.

Matlab is able to solve N equations for N unknowns

- with real numbers, and
- with complex numbers

This is where Matlab really shines