Mutual Inductance

ECE 211 Circuits I

Lecture #24

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Introduction

Previous lectures looked at capacitors and inductors. Inductors are slightly different.

Two inductors in a circuit can affect each other

- Termed mutual inductance

Sometimes this is bad

- This creates cross-talk
- Signals on one wire appear on another wire where they shouldn't

Sometimes this is good

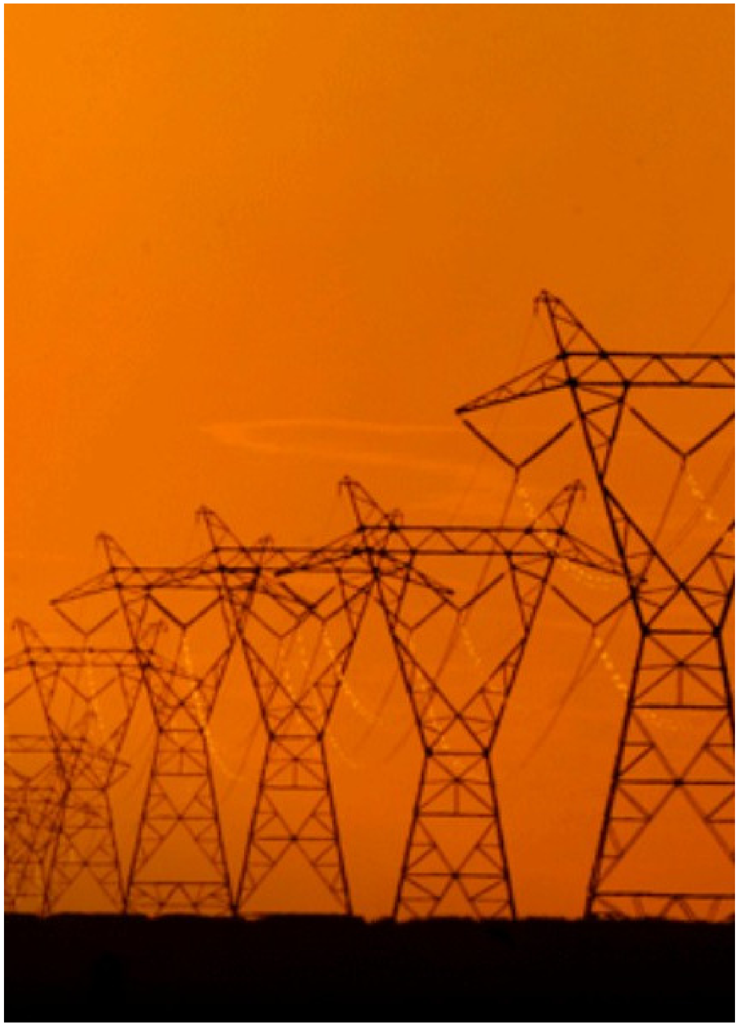
- Mutual inductance is how electric motors work
- Mutual inductance is how transformers work
- Mutual inductance is how utilities are able to transmit over long distances

This Lecture

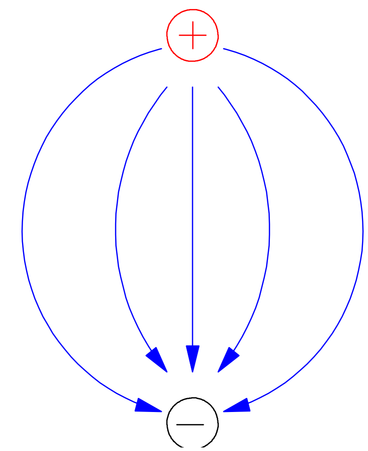
In this lecture, we'll look at:

- Mutual inductance along with how to model it.
- Transformers: an electronic device which uses mutual inductance to change AC voltage levels
- Modeling a transformer using phasors from our last lecture, and
- Using transformers to transmit power over long distances.

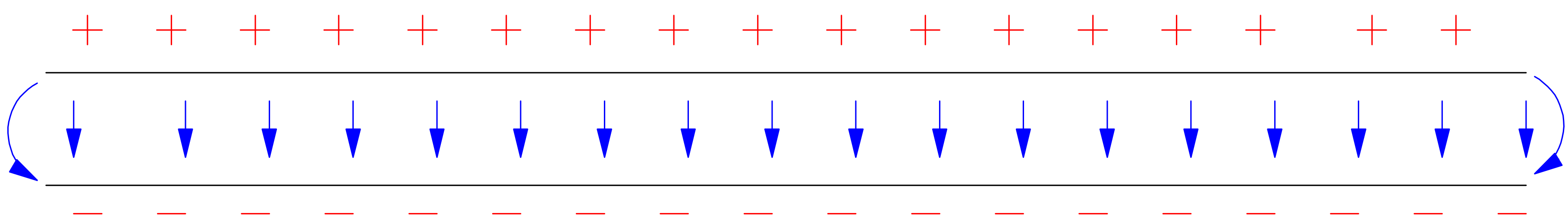
The last item is important. Mutual inductance and transformers are how our power grid works. With them, utilities are able to transmit large amounts of energy thousands of kilometers at more than 90% efficiency.



Capacitors

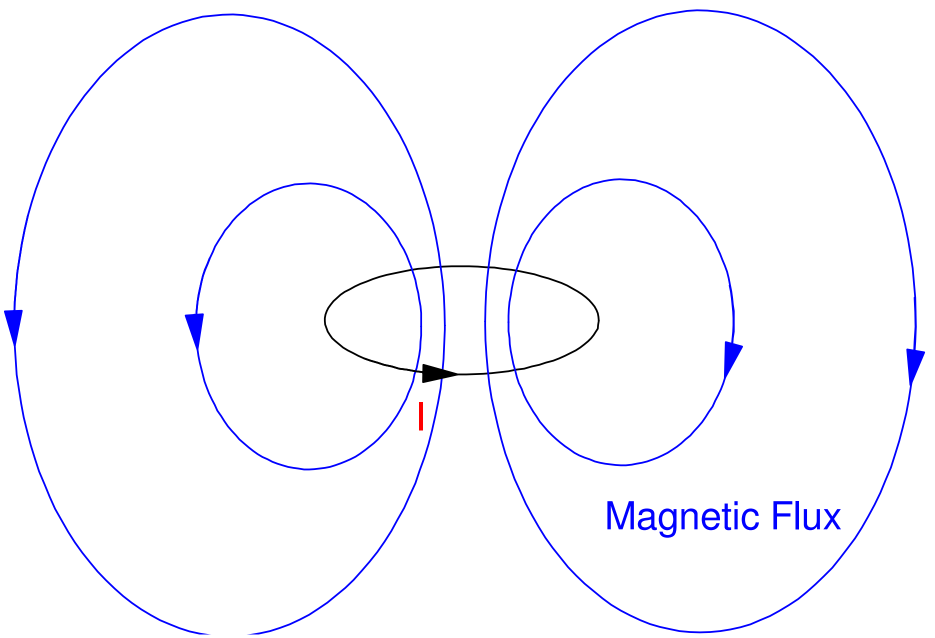


- Electric field lines have a starting point and an ending point.
- In a parallel plate capacitor, some of the field lines will extend beyond the plates of the capacitor. If these field lines overlap with another capacitor's field lines, there will be coupling. Since the spacing is so small, this coupling is effectively zero.



Inductors

- Magnetic field do *not* have a start or an end (magnetic monopoles do not exist)
- Magnetic fields are difficult to contain
- If the field lines of two inductors overlap, the two inductors affect each other
- This is called *mutual inductance*

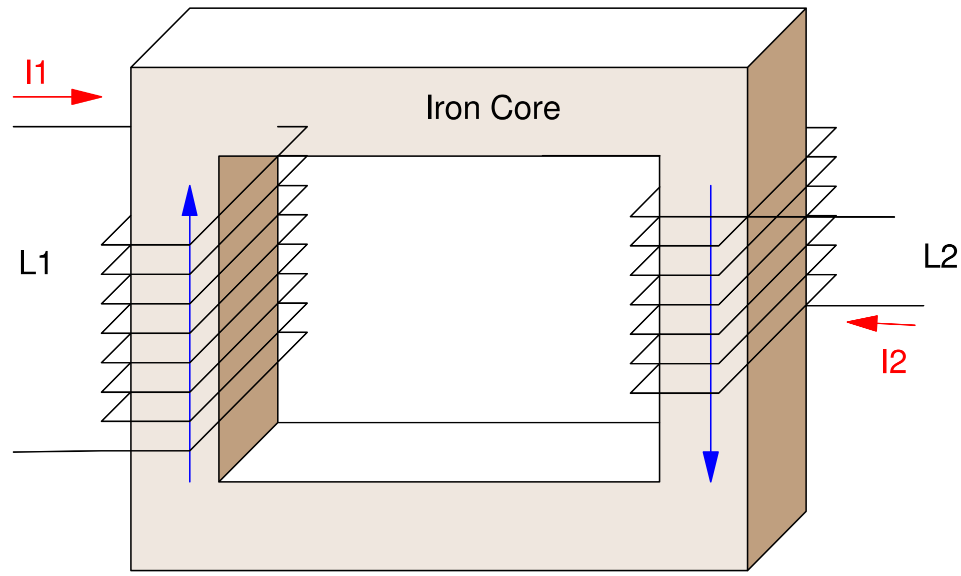


Mutual Inductance

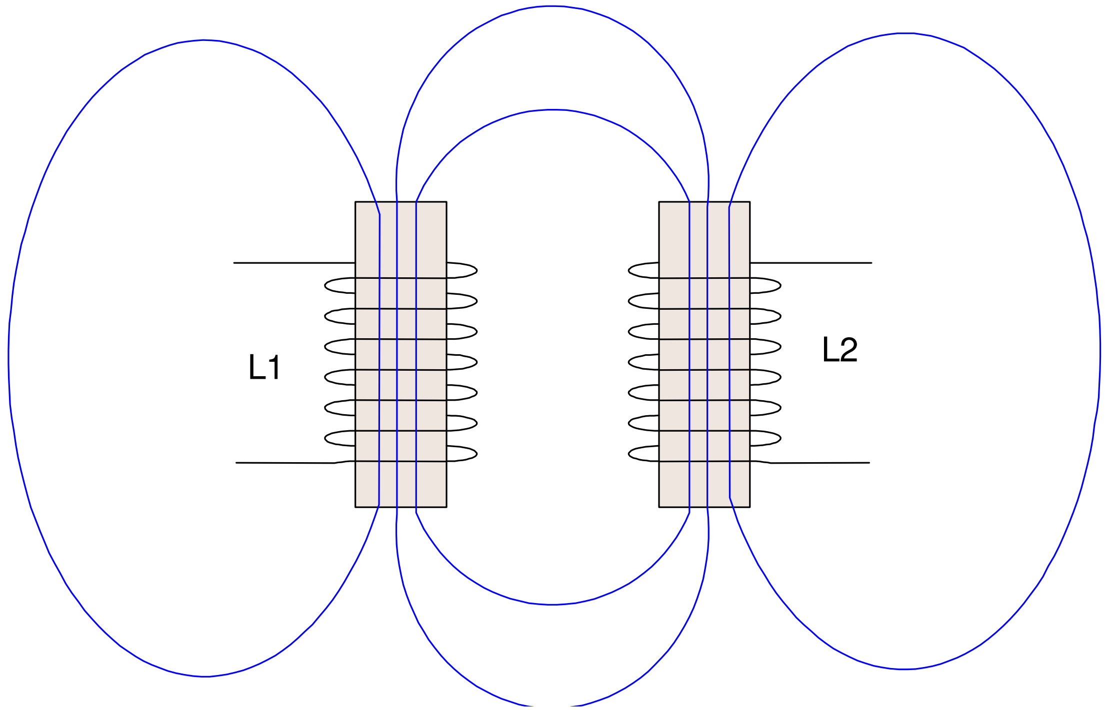
- Coupling between two inductors

Let 'k' be the percentage of the field lines shared by two inductors.

- k=1: all of the magnetic flux is shared (example: transformer)

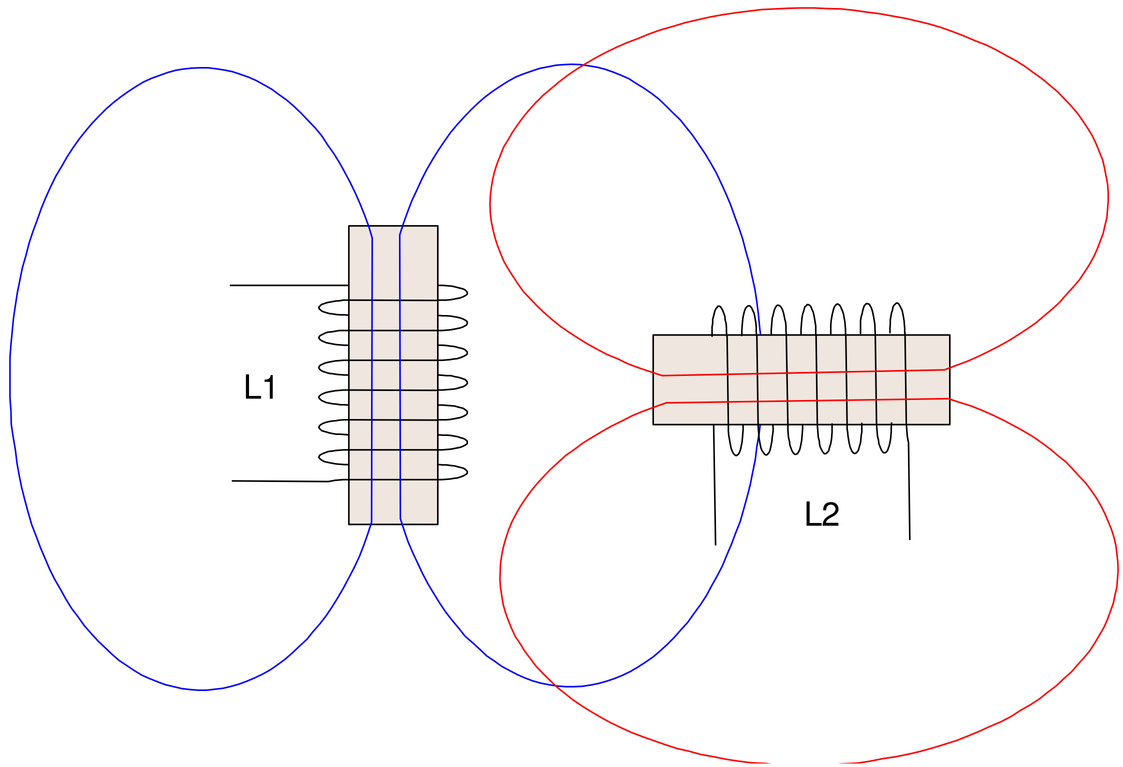


- $0 < k < 1$: Two transformers are close to each other
  - Some of the magnetic flux lines will be shared
  - Magnetic field strength drops off as distance cubed



- k = 0: Two inductors are far apart or are perpendicular

This makes circuit board design with inductors somewhat annoying: the placement of the inductors can have a significant influence on how the circuit behaves.

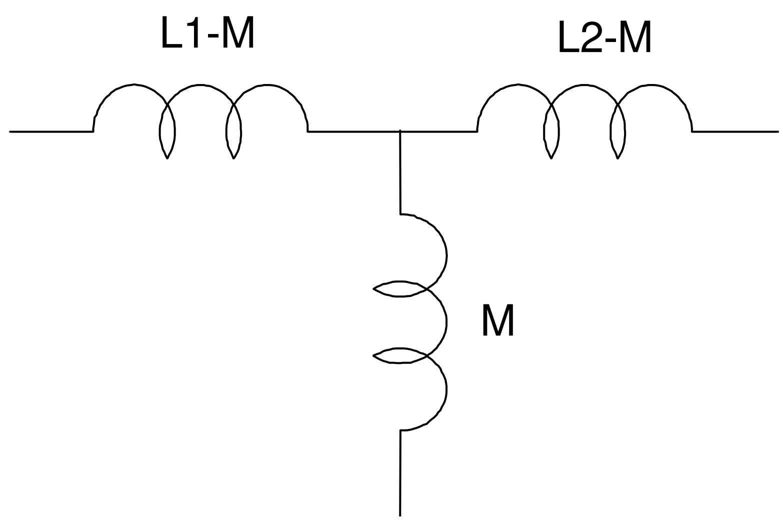


Electric Model for Mutual Inductance

If 'k' is the percentage of field lines that are common between two inductors, the mutual inductance, M, is

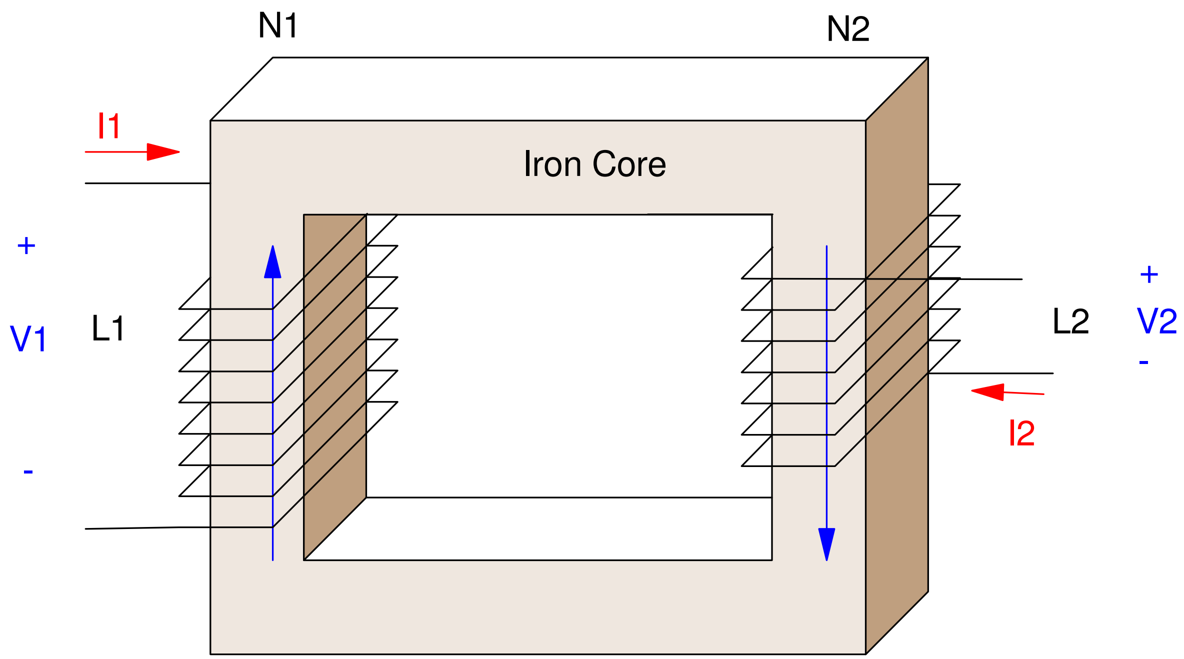
$$M = k\sqrt{L_1 L_2}$$

The T-circuit model for two couples inductors is then



Usage of Mutual Inductance: Transformers

- Two inductors which share a common iron core
- Most of the magnetic flux from one inductor going through the other inductor.
  - $k \approx 1$



Transformer with N1 turns on inductor L1 and N2 turns on inductor L2

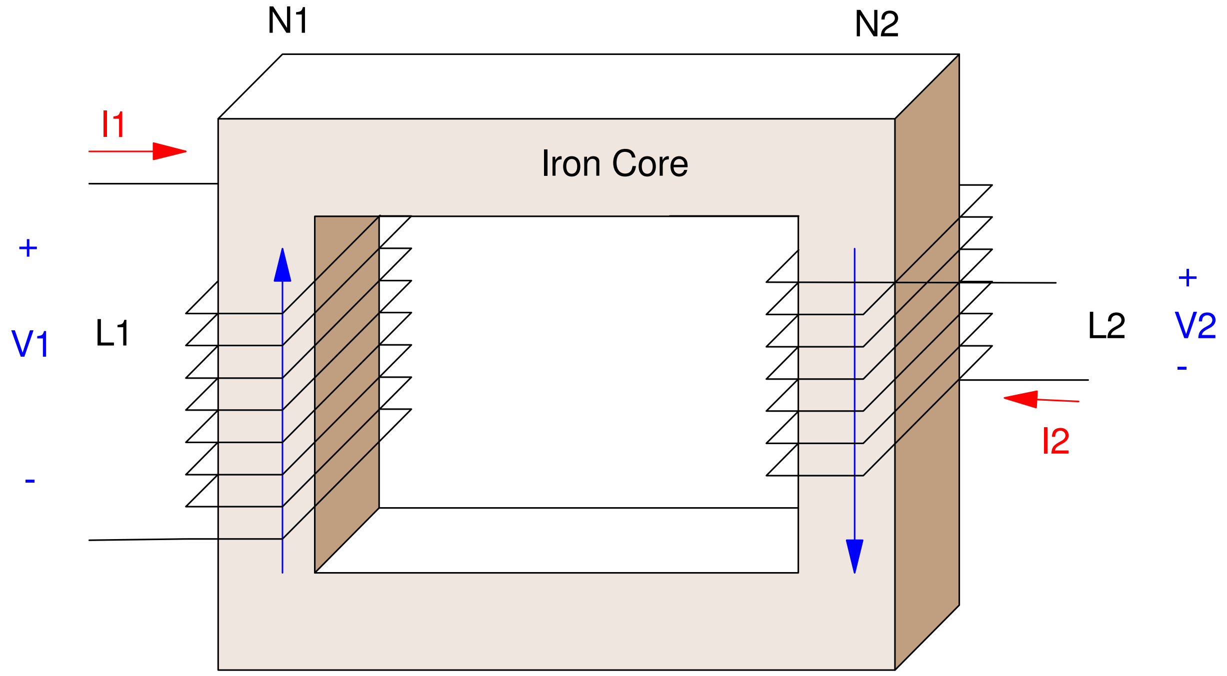
Transformer Equations

Since the magnetic field is the same for both inductors

$$\phi_1 = \phi_2$$

$$N_1 I_1 = N_2 I_2$$

$$I_2 = \left(\frac{N_1}{N_2}\right) I_1$$



Power must balance

$$P_1 = P_2$$

$$V_1 I_1 = V_2 I_2$$

This results in

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

Transformers are a convenient way to change voltages and currents.

Utility Transformers

Transformers allow you to convert one voltage to another

- Example: 25kVA transformer
- Primary: 13,200V
- Secondary: 120V / 240V
- Weight: 470lb
- 98.95% efficient
- $3000 (ebay)

Transformers are large, heavy, and expensive

- Only iron has a high permiability
- Iron supports limited magnetic fields per unit volume
- Hence, transformers use *lots* or iron

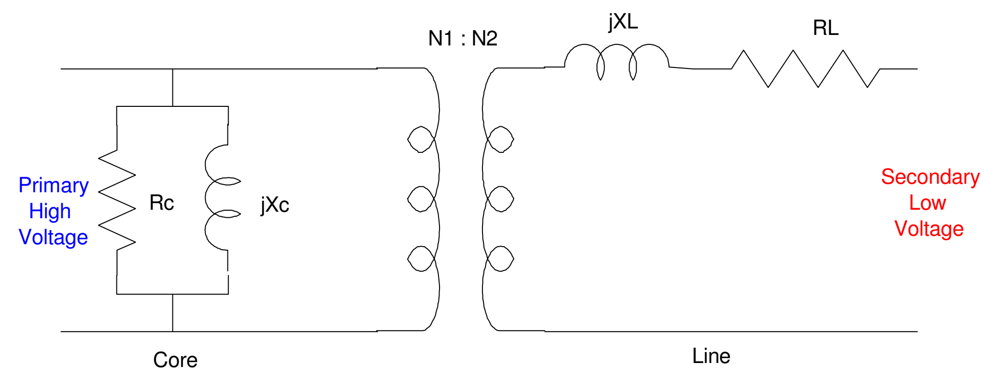
Transformer Modeling

Transformers are inherently inductors

- They store energy in their magnetic fields
- Model with L

Transformers also have losses

- Copper losses proportional to current
- Core losses just from being powered up
  - Eddy current losses
  - Hysteresis losses
- Model with R



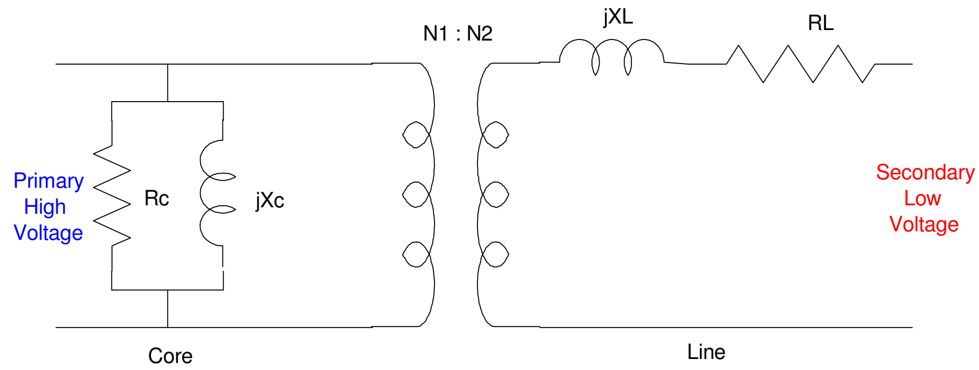
Transformer models reflect both of these

Transformer Testing

- The line and core model can be measured with two tests

Open-Circuit Test

- Leave the secondary side open
- Provide rated voltage to the primary side
- Measure Volts, Amps, Watts
- Compute Rc, jXc



Equations

- $pf = \left(\frac{Watts}{|V|\cdot|A|}\right)$
- $Z_c = \left(\frac{|V|}{|A|}\right)\angle\arccos(pf)$
- $\frac{1}{Z_c} = \frac{1}{R_c} + \frac{1}{jX_c}$

Transformer Testing

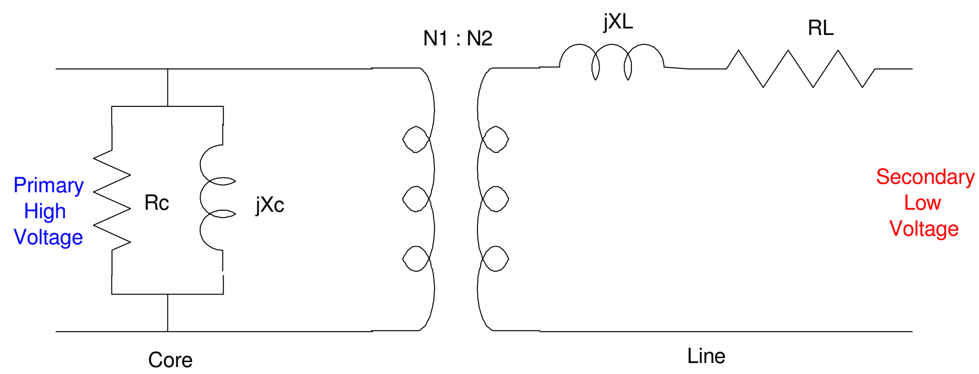
Short-Circuit Test

- Short the primary side
- Provide a **low** voltage to the secondary side
- Measure Volts, Amps, Watts

Equations

- $pf = \left(\frac{Watts}{|V|\cdot|I|}\right)$

$$Z_L = \left(\frac{|V|}{|I|}\right)\angle \arccos(pf)$$



Open-Circuit Test

Test	Volts	Amps	Watts
Open-Circuit	Vp = 13,200V	Ip = 0.020A	250 Watts
Short-Circuit	Vs = 12V	Is = 50A	500W

Determine the Rc & jXc

$$P = V \cdot I \cdot pf$$

$$250W = 13.2kV \cdot 20mA \cdot pf$$

$$pf = 0.9470$$

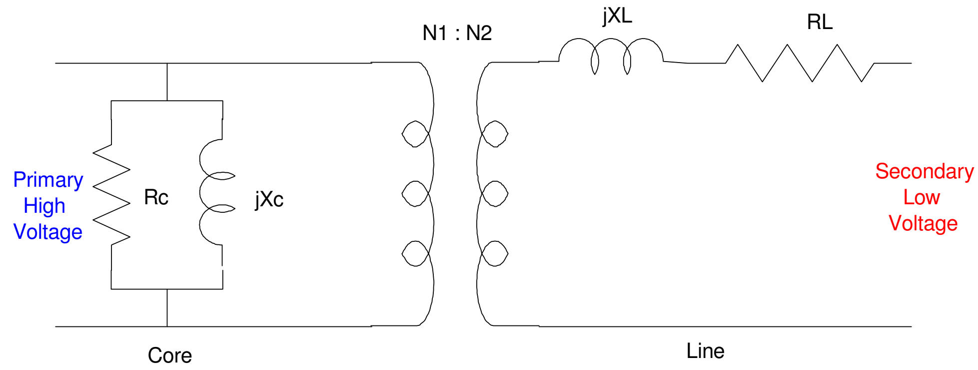
$$Z = \left(\frac{V}{I}\right) \angle \arccos(pf)$$

$$Z = 660k\angle 18.74^0$$

$$\frac{1}{Z} = \frac{1}{R_c} + \frac{1}{jX_c}$$

$$R_c = 697k\Omega$$

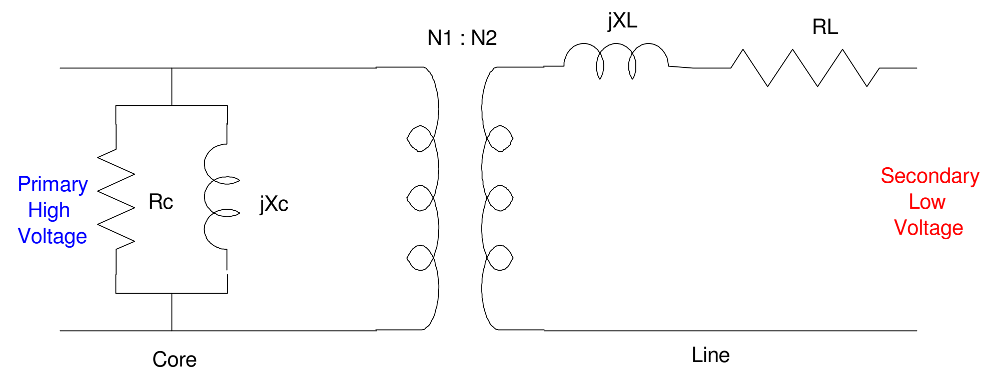
$$jX_c = j2.054M\Omega$$



Short Circuit Test

Test	Volts	Amps	Watts
Open-Circuit	Vp = 13,200V	Ip = 0.020A	250 Watts
Short-Circuit	Vs = 12V	Is = 50A	500W

Determine RL and jXL



$$pf = \left(\frac{500W}{12V \cdot 500A}\right) = 0.0833$$

$$Z_L = \left(\frac{12V}{50A}\right) \angle \arccos(0.08333)$$

$$Z_L = 0.24 \angle 85.22^0$$

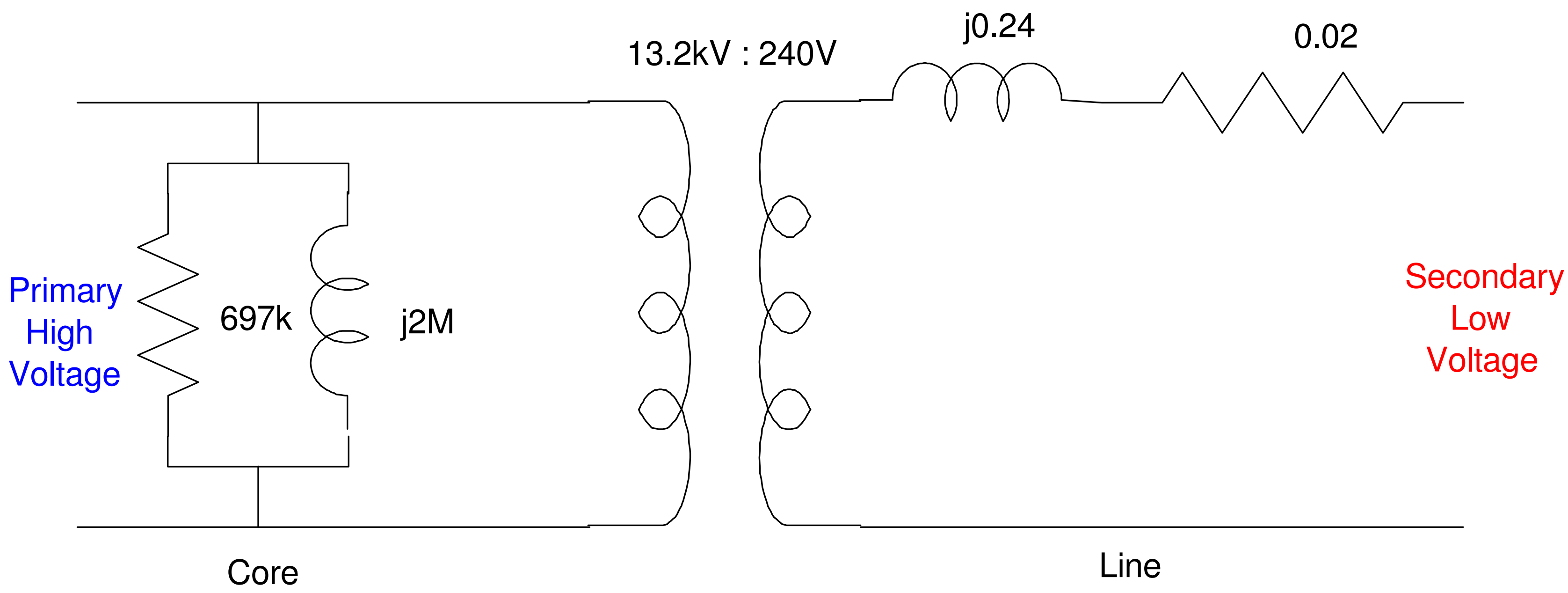
$$Z_L = R_L + jX_L$$

$$R_L = 0.020\Omega$$

$$jX_L = j0.239\Omega$$

Transformer Model

The net result is a model of this transformer



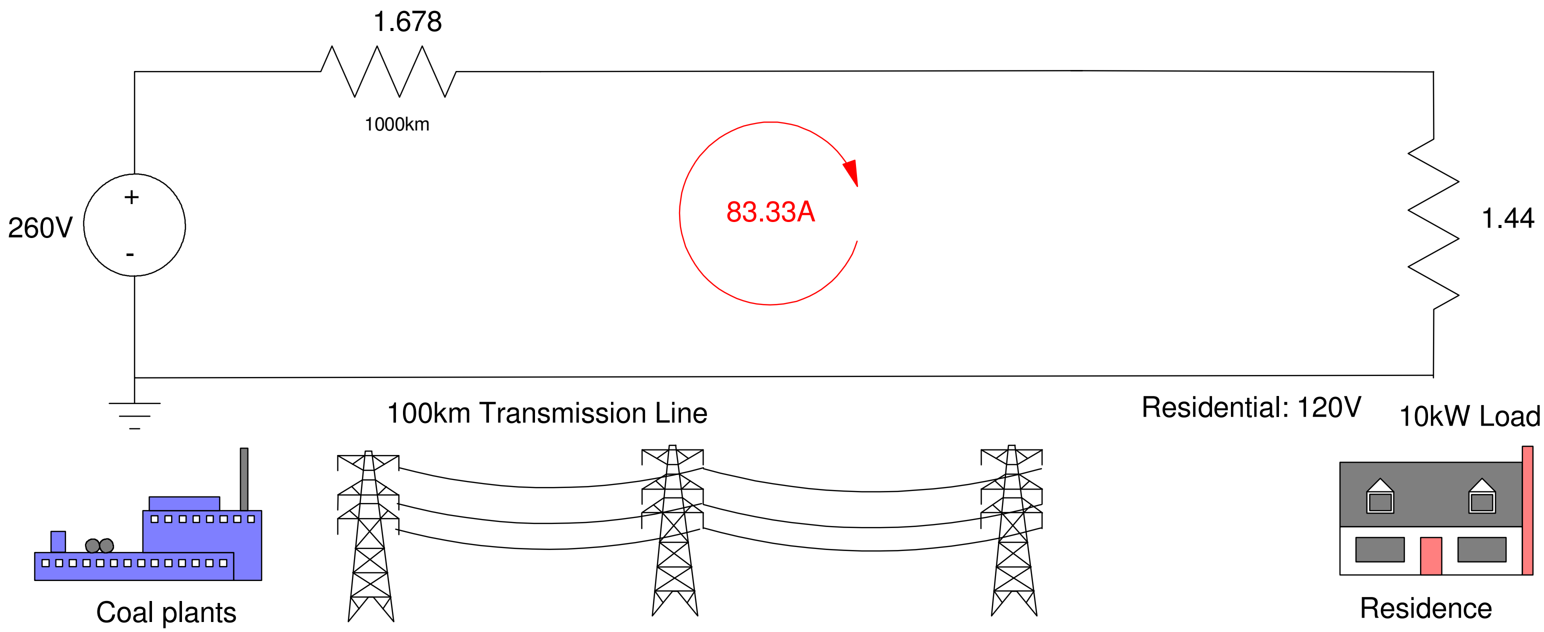
Transformer Uses: Power Grid

- Transformers allow you to transmit power over large distances
- Transformers only work at AC (they don't work at DC)
- This is why the power grid operates at 60Hz
- Eletroboom has a good video on this
    - https://youtu.be/S7C5sSde9e4?si=xloxxTr35nTrWEPd

Problem: Transmit power 1000km

Transmit power from western North Dakota to Minneapolis (1000km)

- Load = 1.44 Ohms (10kW @ 120V)
- RL = 1.678 Ohms (1000km of 1cm dia copper wire)



DC Solution:

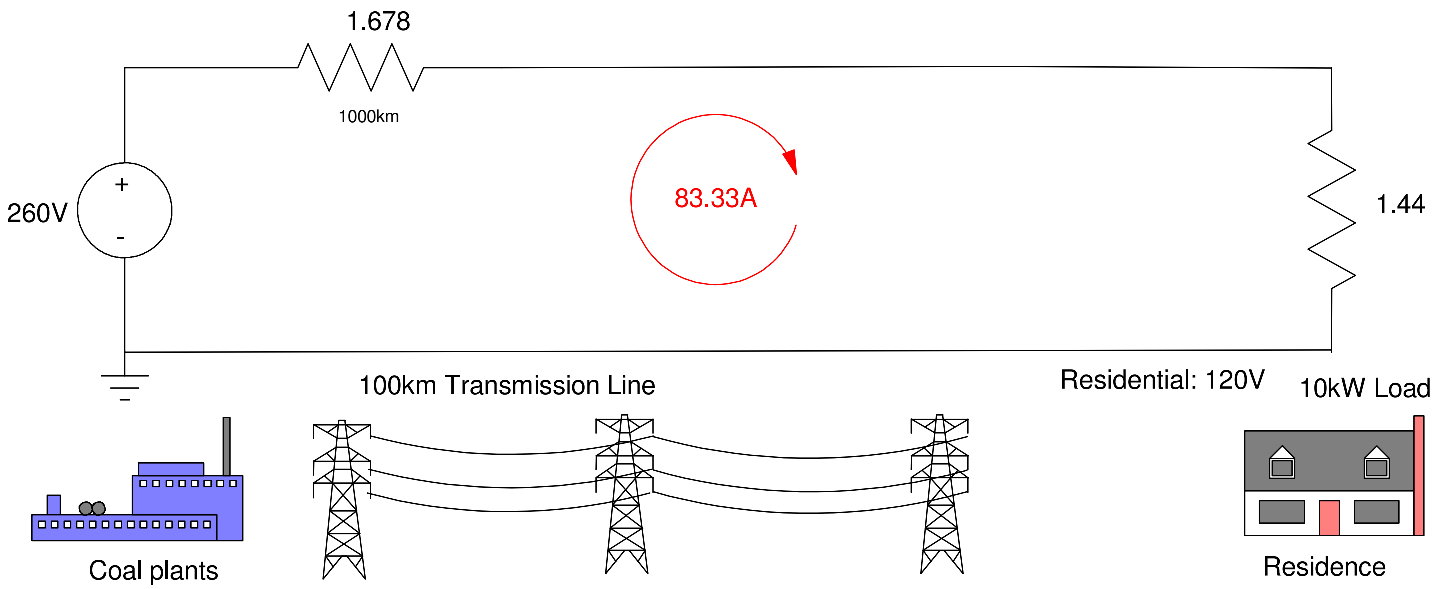
Power Calculations:

$$P_{line} = I^2R = (83.33A)^2 \cdot 1.678\Omega = 11.65kW$$

$$P_{load} = I^2R = (83.33A) \cdot 1.44\Omega = 10kW$$

The efficiency is 46.2%

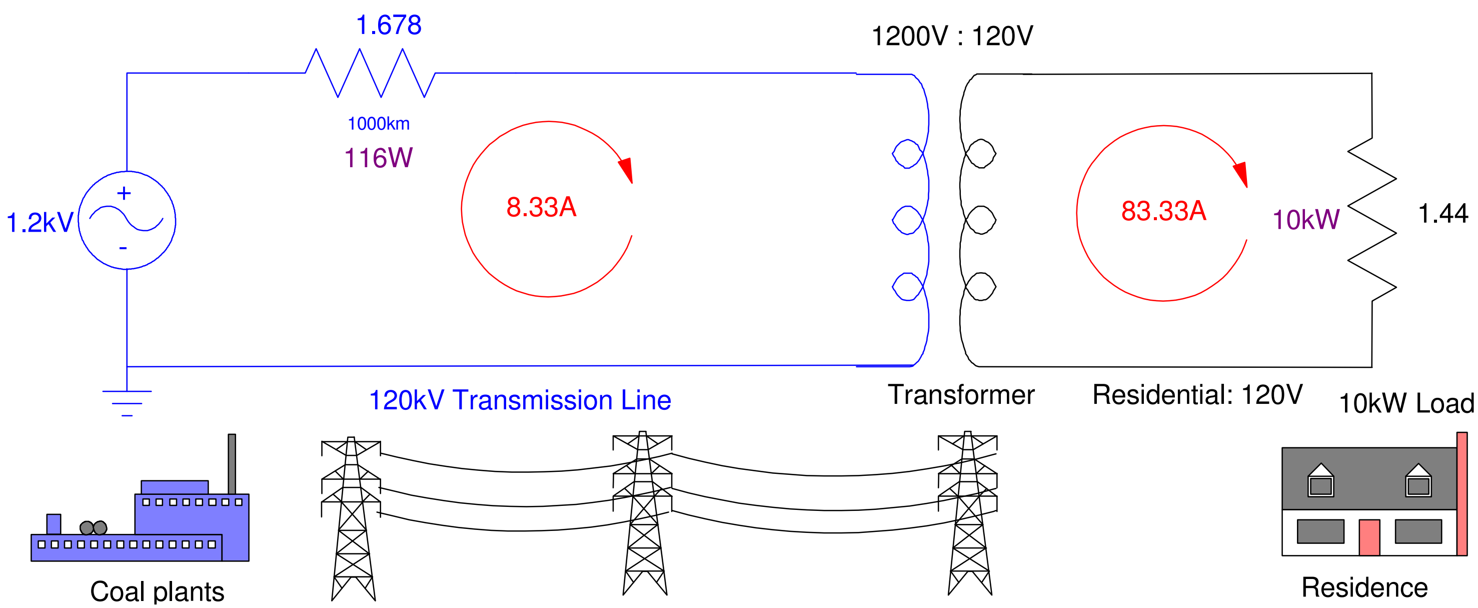
$$\eta = \left(\frac{\text{power to load}}{\text{total power}}\right) = \left(\frac{10kW}{10kW+11.65kW}\right) = 0.462$$



AC Solution:

Use a 10:1 transformers to bump up the voltage to 1.2kV

- 83.33A goes to the customer (10kW)
- 8.333A goes through the transmission line (10x smaller)
  - Line losses are reduced 100x
  - Efficiency increases to 98.8%



Transformers also change impedances

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1$$

$$I_2 = \left(\frac{N_1}{N_2}\right) I_1$$

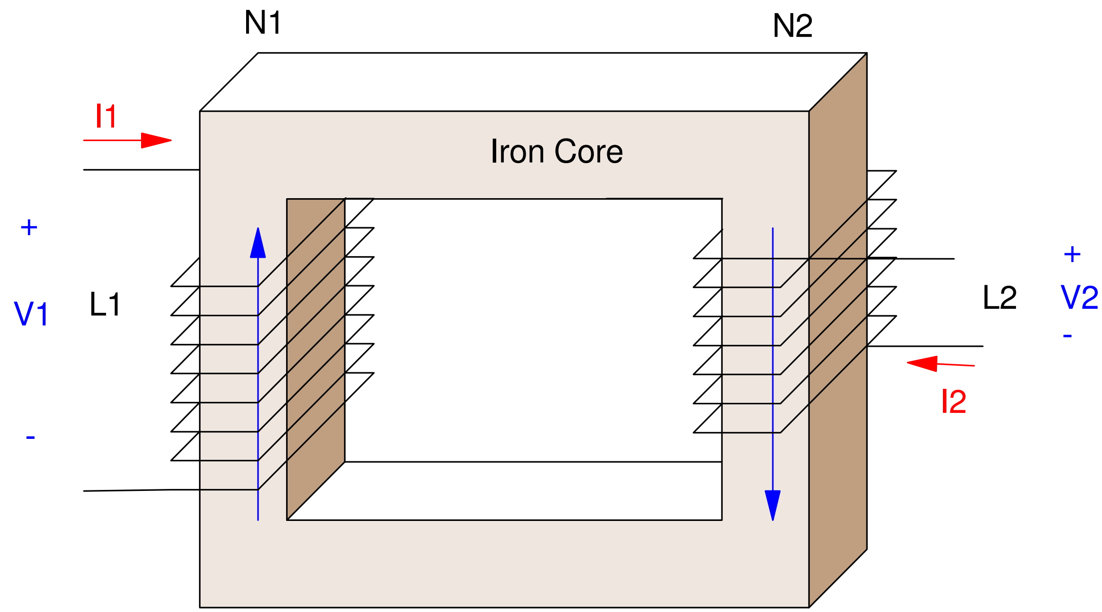
If you add a ressitance to V2

$$R_2 = \frac{V_2}{I_2}$$

As seen from size 1

$$R_2 = \frac{\left(\frac{N_2}{N_1}\right) V_1}{\left(\frac{N_1}{N_2}\right) I_1}$$

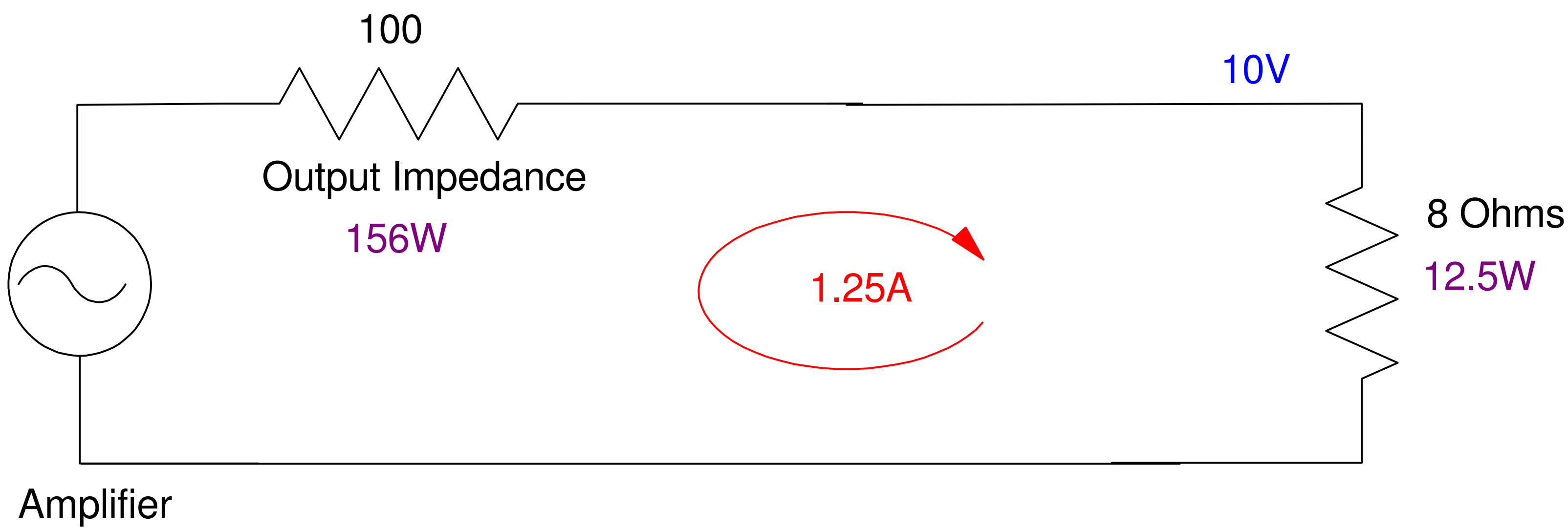
$$R_2 = \left(\frac{N_2}{N_1}\right)^2 R_1$$



Transformers in Stereos:

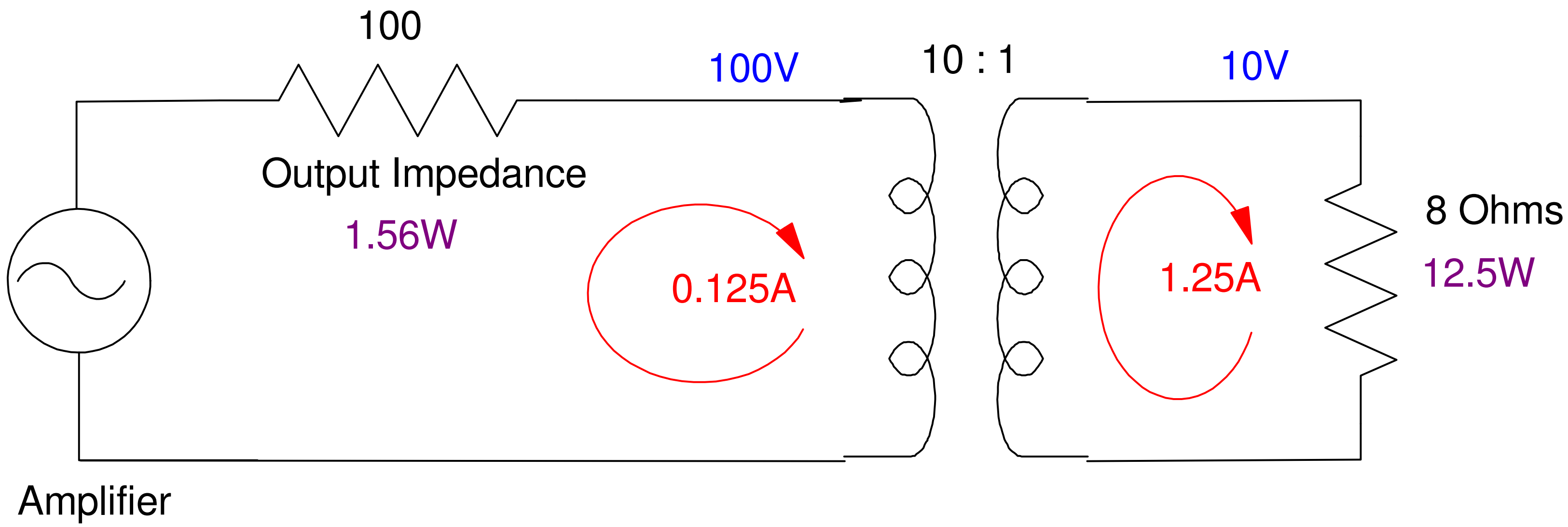
Problem: Drive an 8-Ohm speaker with a stereo with a 100 Ohm output impedance

Solution 1: Efficiency = 7%



Solution 2: Use a 10:1 transformer

- 8-Ohm speaker now looks like an 800 Ohm speaker
- Efficiency increases to 89%
- To get 10V at the speaker, the amplifier has to output 100V



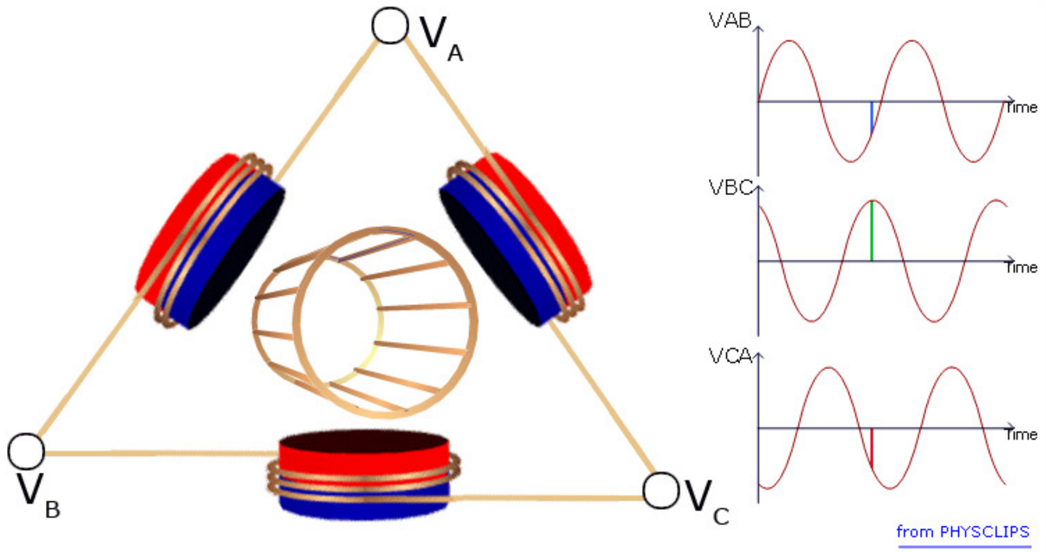
Transformers in Motors: AC Induction Motor

At the heart of an AC induction motor is a stator

- N : 1 transformer

A rotating field in the stator induces current in the rotor

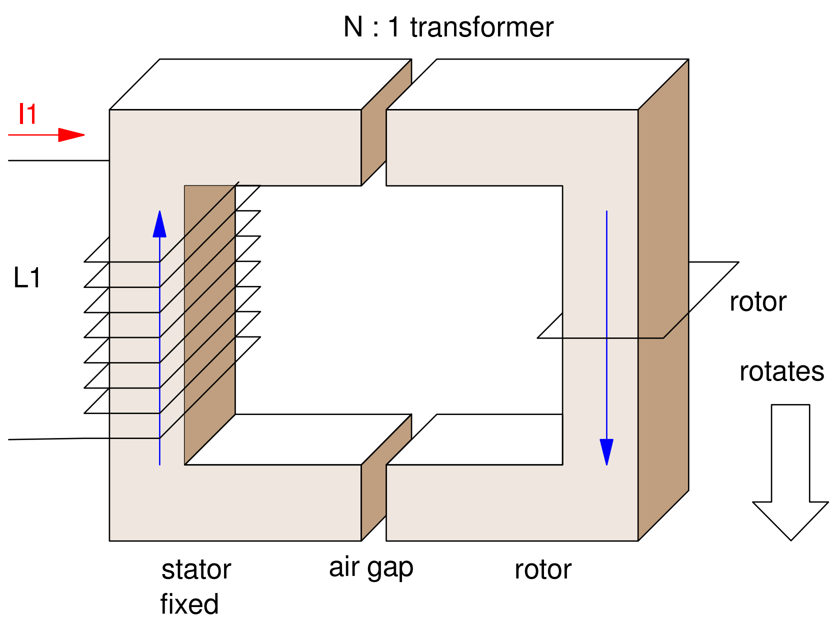
- Which produces a magnetic field
- Which created torque and causes the motor to spin



http://www.animations.physics.unsw.edu.au/jw/electricmotors.html

Modeling an AC Induction Motor

- Essentially a N:1 transformer where the secondary side rotates
- The electrical model for an AC induction motor is almost identical to the electrical model for a transformer.
- This will be covered in more detail in ECE 331 Energy Conversion.



Summary

Inductors store energy in magnetic fields

- The magnetic fields of different inductors can overlap
- The percentage of fields that overlab is the *mutual inductance*

Mutual inductance can be bad

- Causes cross-talk between signals

Mutual inductance can be good

- How motors work
- How transformers work

Transformers are the basis of our power grid

- They allow you to change AC voltages & currents easily
- They allow you to transmit power thousands of km