Thevenin Equivalents

Dependent Sources & Max Power Transfer

ECE 211 Circuits I

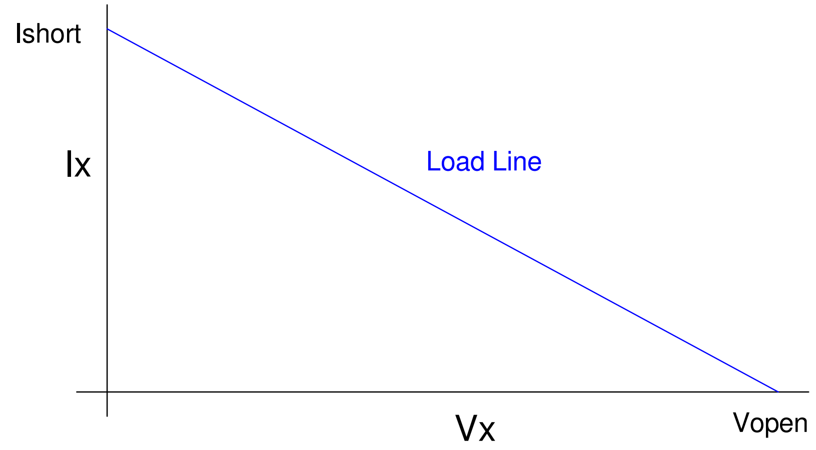
Lecture #11

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Thevenin & Norton Equivalents

Any linear circuit's output follows a load line

- Ix varies linearly with Vx

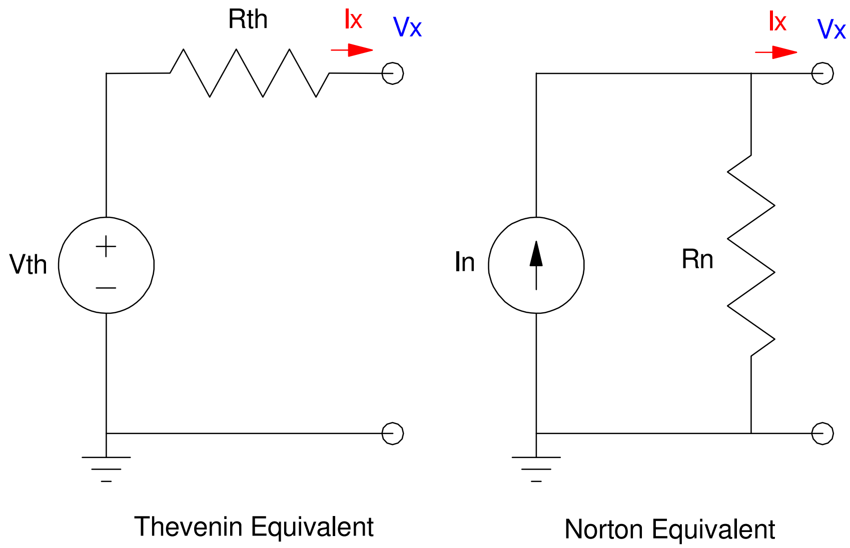


As far as the load is concerned

- All circuits with the same load line look alike

The simplest circuit to generate a given load line is

- Thevenin Equivalent
  - Voltage Source & Resistor in Series
- Norton Equivalent
  - Current Source & Resistor in Parallel

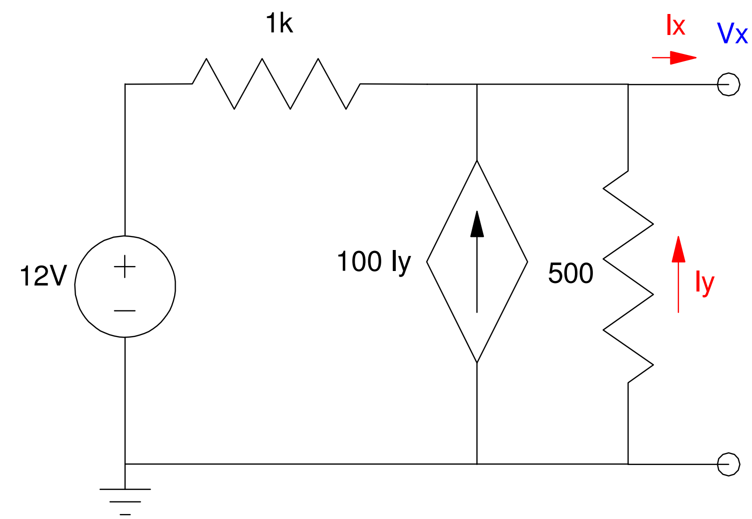


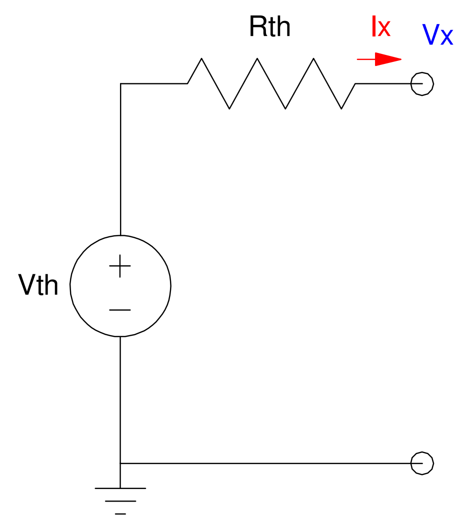
Previous Lecture

- Find the Thevenin & Norton equivents
- For resistor circuits

This Lecture

- Thevenin for Circuits with Dependent Sources
- Max Power Transfer





Thevenin Equivalent

Types of Dependent Sources

Dependent sources are

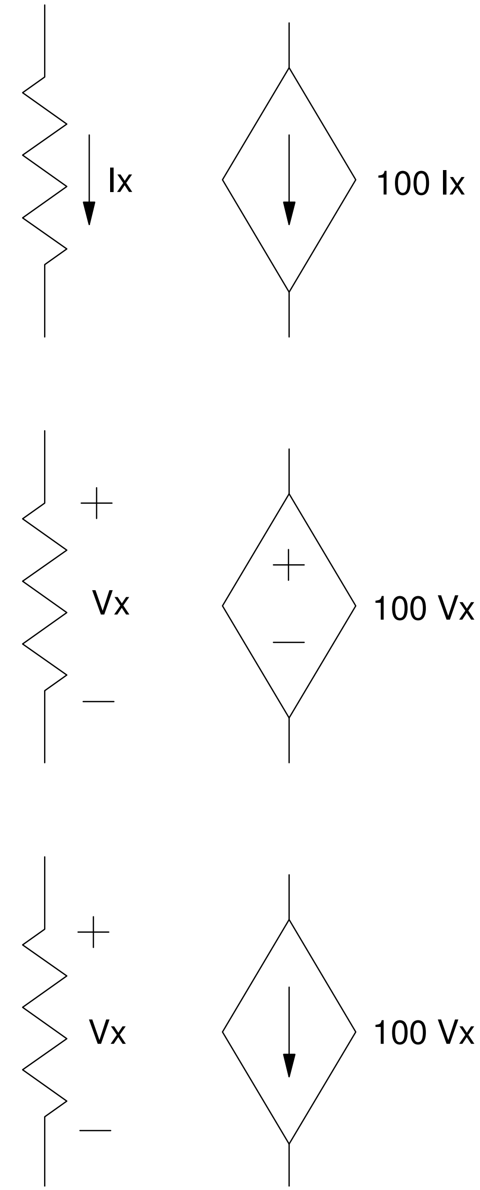
- Sources (voltage of current)
- Which depend upon another variable
- Dependent sources complicate analysis

Example:

- Current-controlled current source
  - BJT transistor
- Voltage-controlled voltage source
  - Amplifier
- Voltage-controlled current source
  - MOSFET

Devices which do this are covered in Electronics

- For this class, just treat these as sources

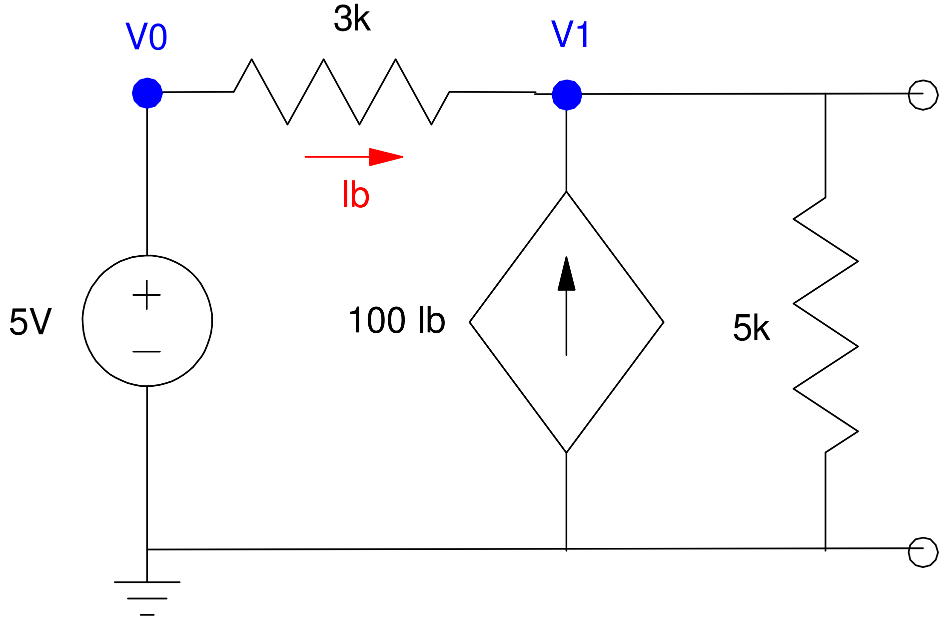


Case 1: Current-Controlled Current Sources

- BJT Transistor

Find the Thevenin equivalent for the following circuit

- V(open) = Vth
- I(short) = In
- Rth



V(open) = Vth

- Nothing changes with dependent sources
- Find the open-circuit voltage.

Write the voltage node equation at V1

$$I_b = \left(\frac{5 - V_1}{3k}\right)$$

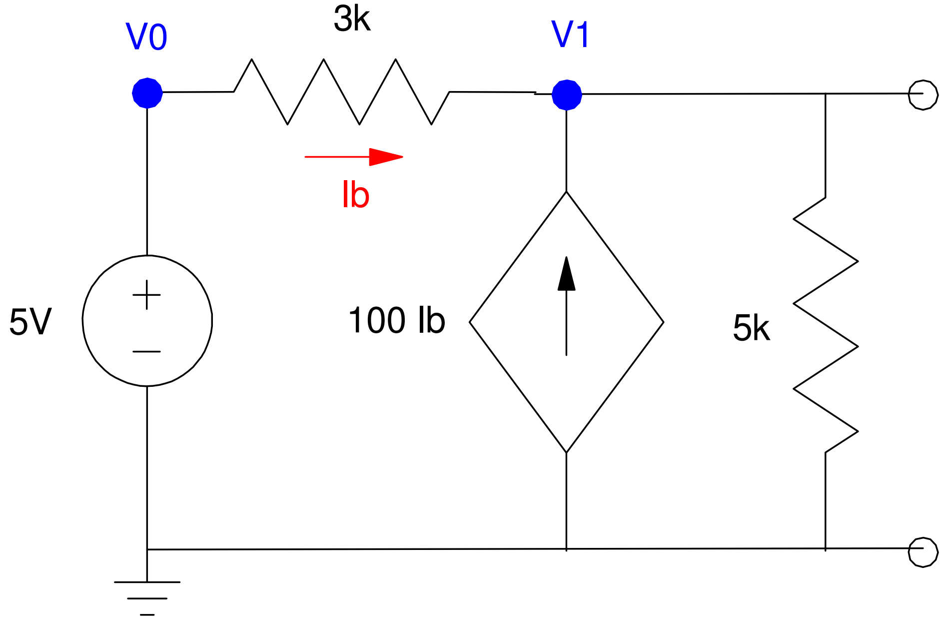
$$-I_b - 100 I_b + \left(\frac{V_1}{5k}\right) = 0$$

Substitute and solve

$$V_1 = \left(\frac{\left(\frac{101}{3k}\right)}{\left(\frac{101}{3k}\right) + \left(\frac{1}{5k}\right)}\right) 5V = 4.9705V$$

This is V_{th}.

V(open) = Vth = 4.9705V



I(short) = In

- Short the output
- Compute the current through the short

By observation

- V0 = 5V
- V1 = 0V

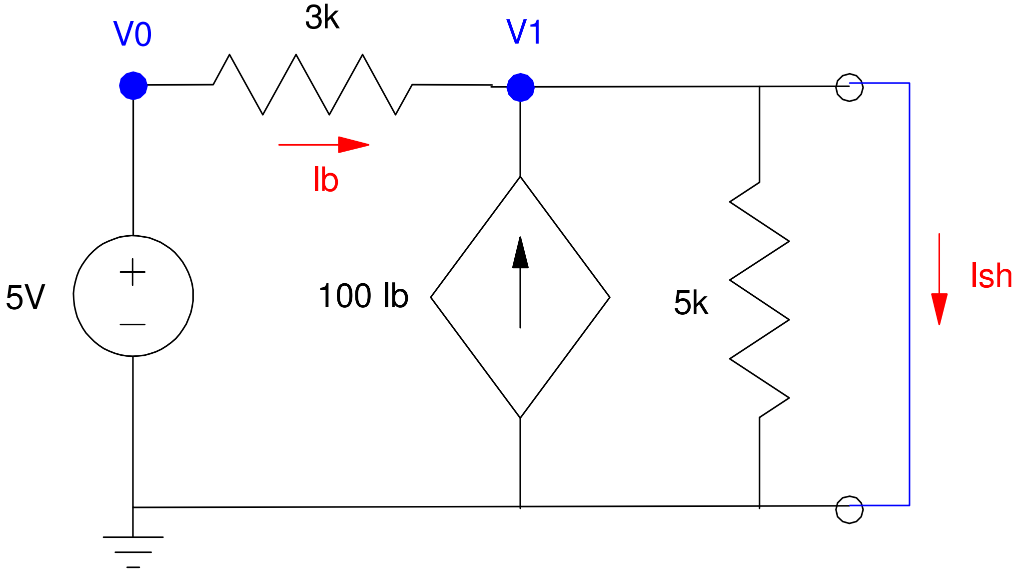
Conservation of Current at V1

$$I_b = \left(\frac{5V - 0V}{3k}\right) = 1.667mA$$

$$-I_b - 100I_b + \left(\frac{V_1}{5k}\right) + I_{short} = 0$$

$$I_{short} = 168.33mA$$

In = I(short) = 168.33mA



R_{th}:

Option 1:

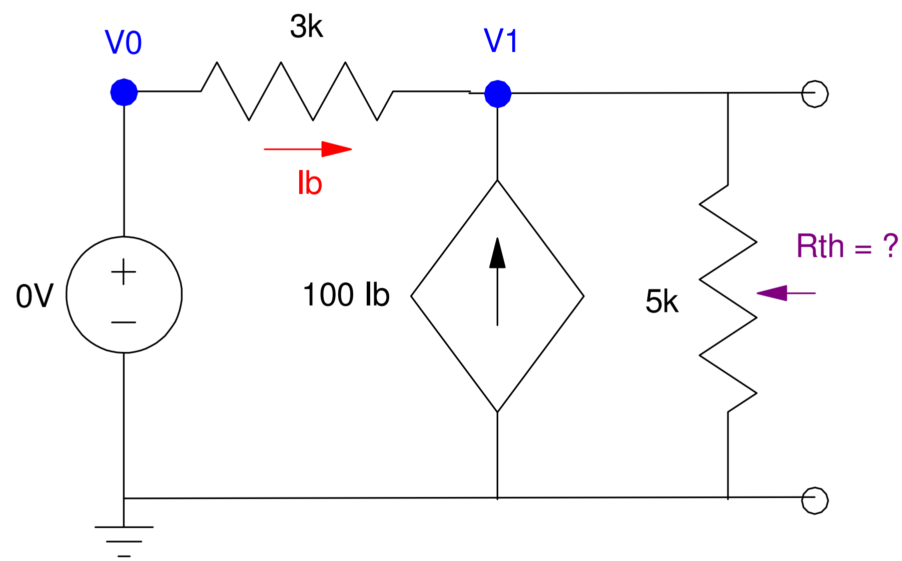
- Compute V(open) & I(short)
- Then

$$R_{th} = \frac{V_{open}}{I_{short}} = 29.53\Omega$$

Option #2:

- Turn off the sources
- Compute the resistance looking in

With the dependent source, that's not obvious



Finding Rth:

- Apply a 1V test voltage
- Compute the current draw (Ix)

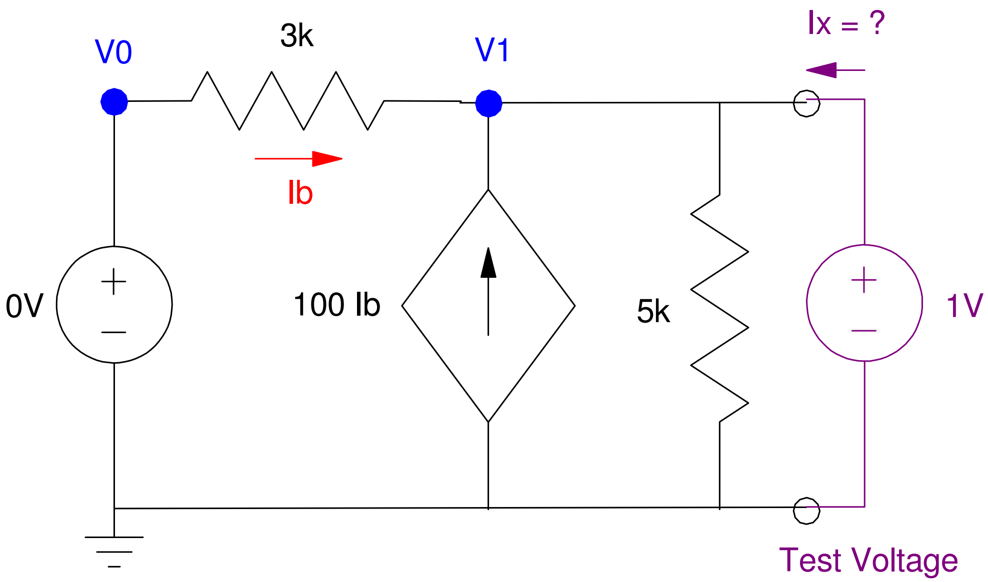
$$I_b = \left(\frac{0V - 1V}{3k}\right)$$

$$I_x = -I_b - 100 I_b + \left(\frac{1V}{5k}\right)$$

$$I_{in} = 33.87 mA$$

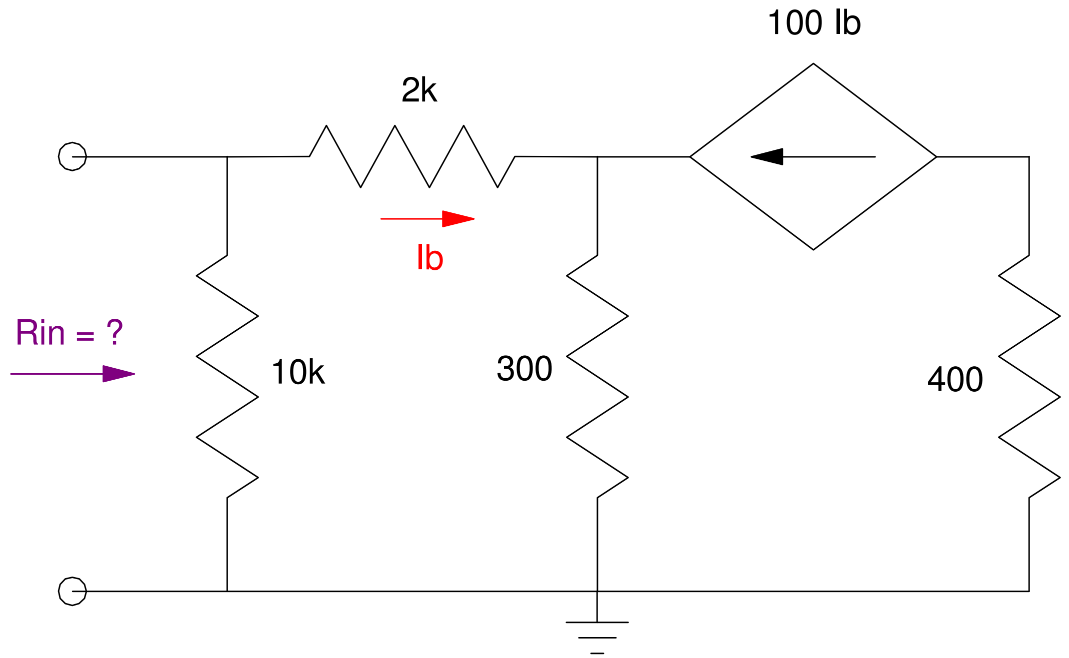
So

$$R_{th} = \frac{V_{in}}{I_{in}} = \frac{1V}{33.87 mA} = 29.53 \Omega$$



Practice Problem:

Find the impedance looking in (Rin)



Practice Problem:

- Apply a 1V source
- Compute the current draw

Equations

$$I_b = \left(\frac{1 - V_1}{2k}\right)$$

$$-I_b - 100 I_b + \frac{V_1}{300} = 0$$

Solving

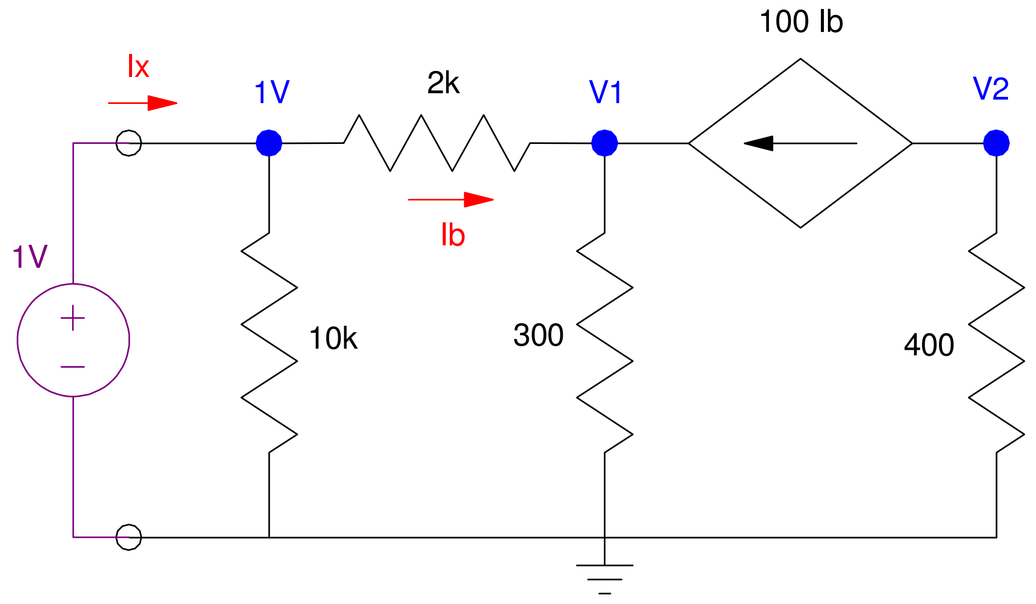
$$V_1 = 0.93808V$$

$$I_x = \left(\frac{1V}{10k}\right) + \left(\frac{1 - V_1}{2k}\right)$$

$$I_x = 130.9\mu A$$

Thus

$$R_{in} = \frac{1V}{130.9\mu A} = 7635\Omega$$

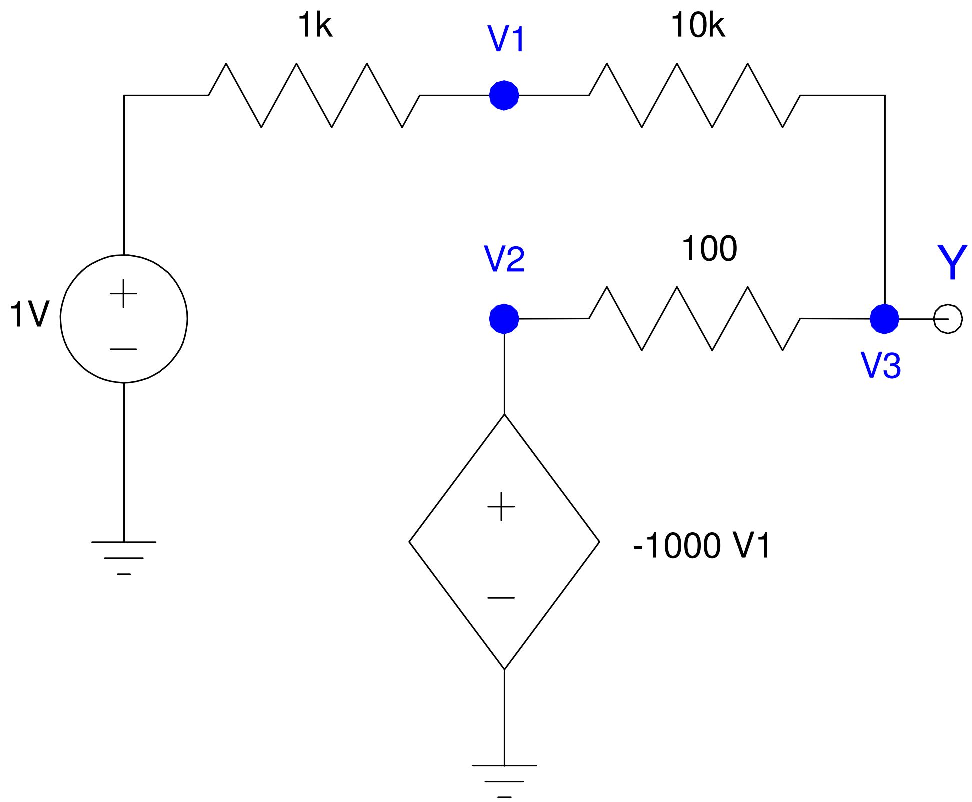


Case 2: Voltage-Controlled Voltage Source

- Op-Amp

Find the Thevenin equivalent

- V(open) = Vth
- I(short) = In
- Rth = Rn



Case 2: V(open)

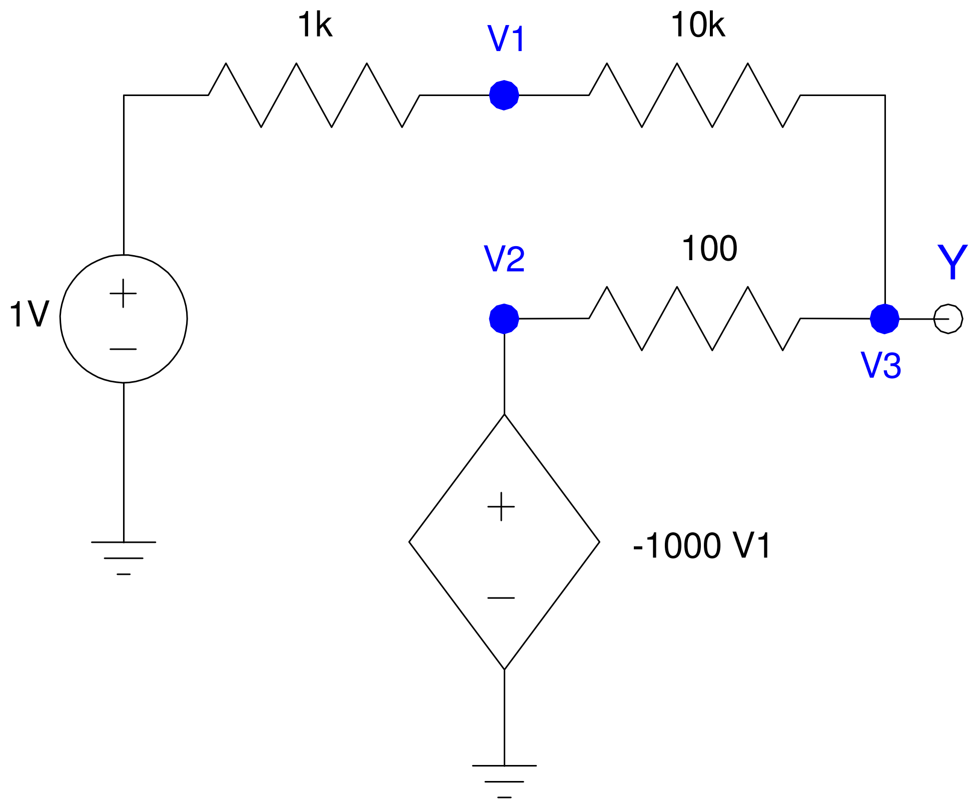
- Vth

Find V3 (open circuit voltage)

$$V_2 = -1000V_1$$

$$\left(\frac{V_1-1}{1k}\right)+\left(\frac{V_1-V_3}{10k}\right)=0$$

$$\left(\frac{V_3-V_1}{10k}\right)+\left(\frac{V_3-V_2}{100}\right)=0$$



Solve (time passes....)

```
A =
[1000,1,0;11,0,-1;-1,-100,101];
B = [0;10;0];
V =  inv(A)*B
```

V1 0.0100
V2 -9.9891
V3 -9.8901 = Vth

Case 2: I(short)

- In

Short the output

- Compute the current

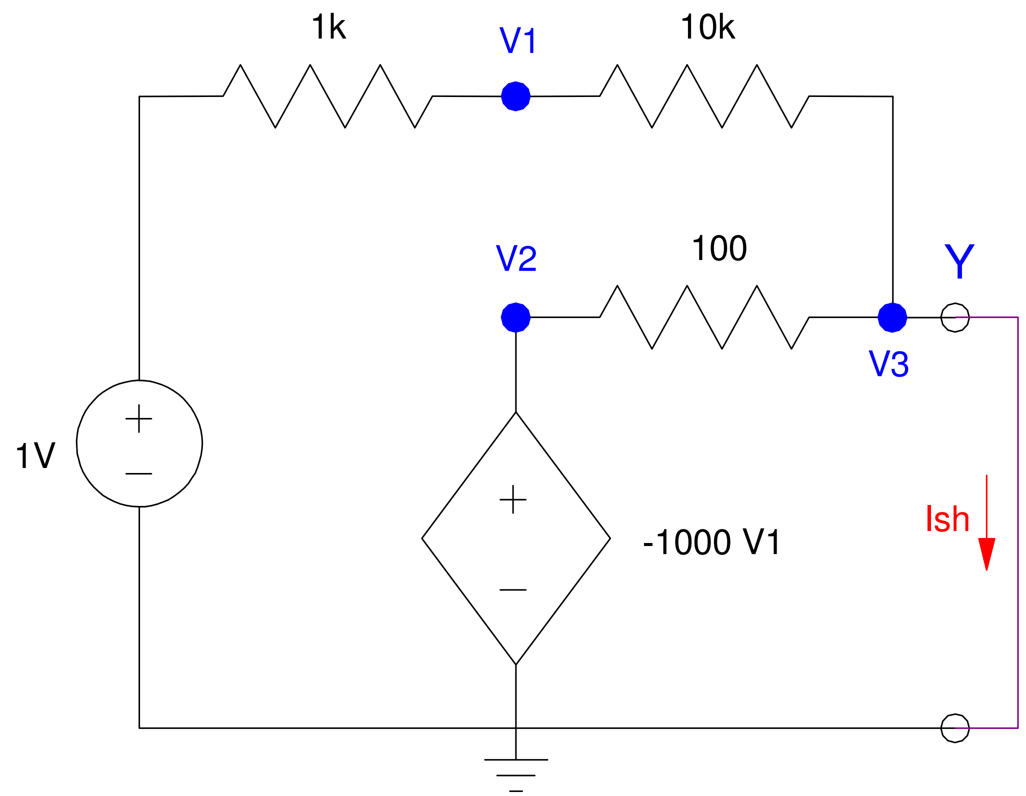
Equations

$$\left(\frac{V_1-1}{1k}\right)+\left(\frac{V_1-0}{10k}\right)=0$$

$$V_1=0.90909V$$

$$I_{sh}=\left(\frac{1V}{11k}\right)-\left(\frac{1000V_1}{100}\right)$$

$$I_{sh}=-9.0908A$$



Case 2: R_{th}:

Option 1:

$$R_{th} = \frac{V_{open}}{I_{short}} = 1.088\Omega$$

Option 2:

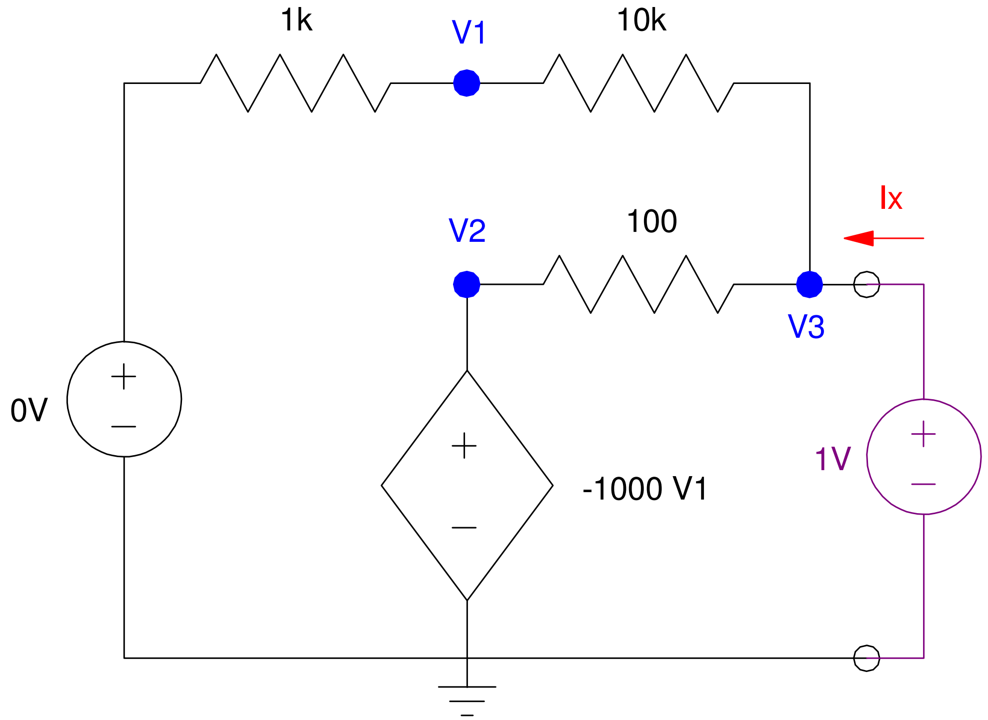
- Turn off voltage sources
- Apply a 1V test voltage
- Compute the current

$$V_1 = \left(\frac{1k}{1k+10k}\right)1V = 90.91mV$$

$$V_2 = -1000V_1 = -90.91V$$

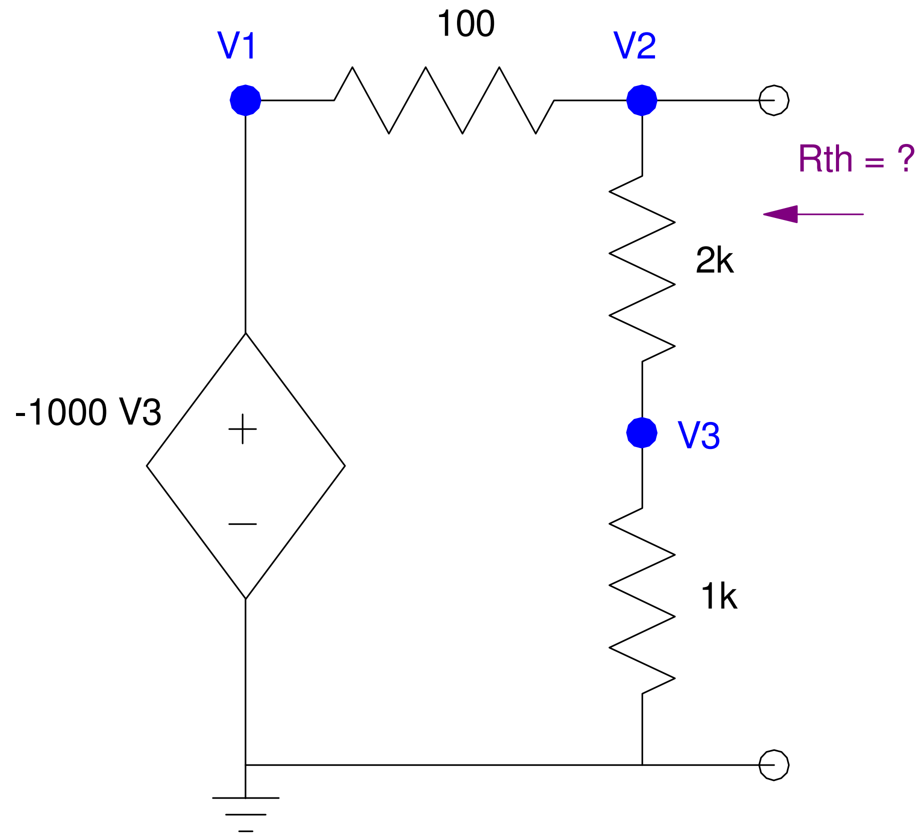
$$I_x = \left(\frac{1V}{11k}\right) + \left(\frac{1V - V_2}{100}\right) = 919.2mA$$

$$R_{th} = \frac{V_{in}}{I_{in}} = \frac{1V}{919.2mA} = 1.088\Omega$$



Practice Problem

Find the Thevenin resistance for the following circuit



Solution

- Apply a 1V test voltage
- Compute the current draw

Equations

$$V_3 = \left(\frac{1k}{1k+2k}\right) 1V$$

$$V_3 = 0.3333V$$

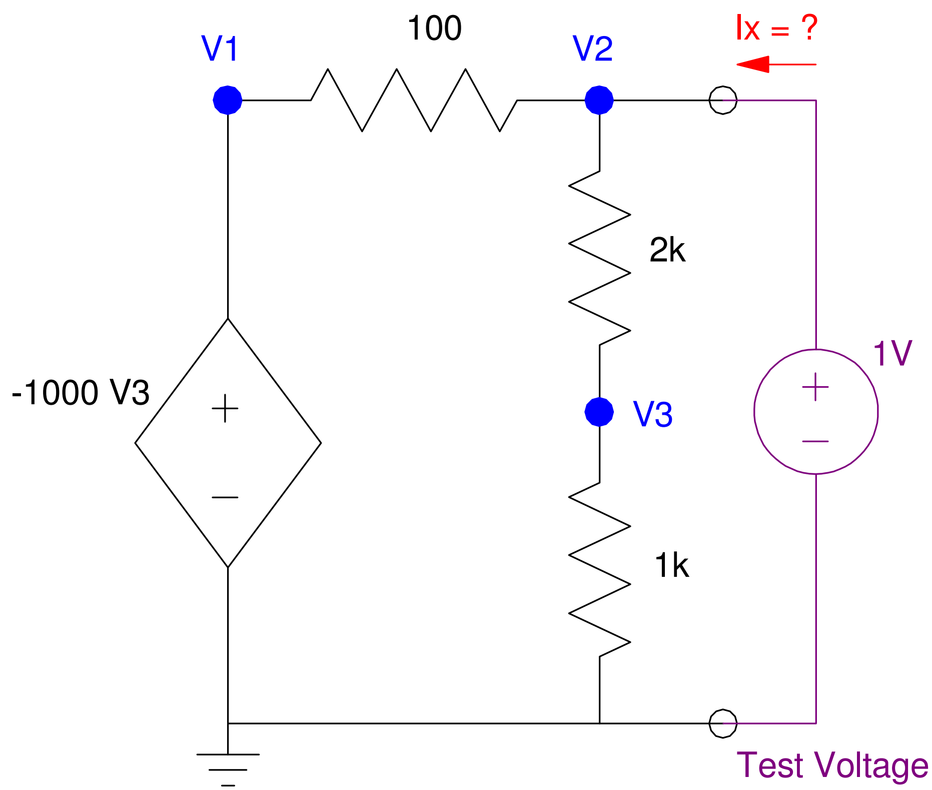
$$V_1 = -1000 V_3$$

$$V_1 = -333.3V$$

$$I_x = \left(\frac{1V - V_1}{100}\right) + \left(\frac{1V}{3k}\right)$$

$$I_x = 3.34367A$$

$$R_{th} = \frac{1V}{3.34367A} = 0.299\Omega$$

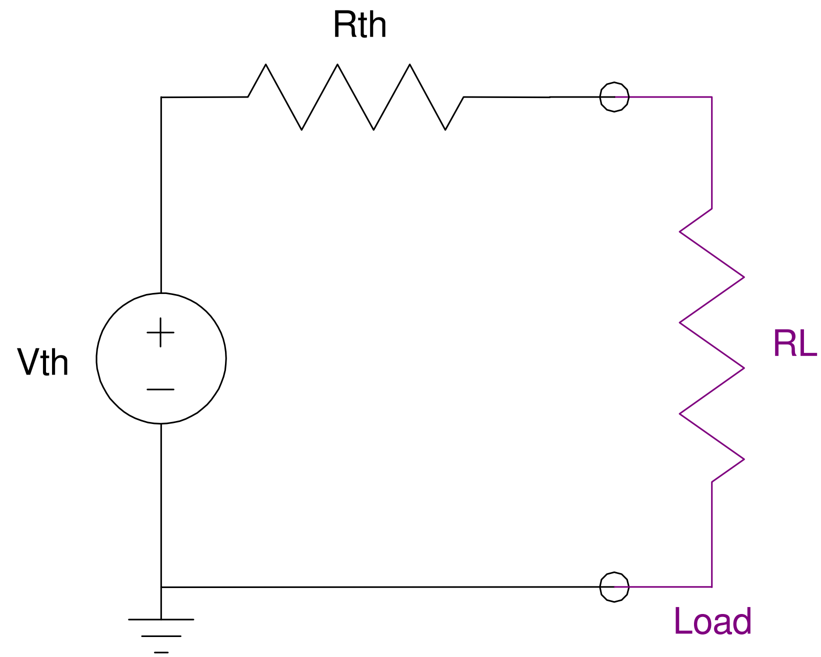


Max Power Transfer

- New Topic

What resistance (R_L) maximizes the power to the load?

- (V_{th}, R_{th}) models a solar panel.
  - What load maximizes the power the solar cell produces?
- (V_{th}, R_{th}) models a stereo.
  - What speaker (R_L) maximizes the output power?



Note:

- $R_L = 0 \Rightarrow P = 0$
  - V = 0
- $R_L = \infty \Rightarrow P = 0$
  - I = 0

Max power is in the range $0 < R_L < \infty$

Case 1: R_{th} is fixed.

Calculations

$$I = \left(\frac{V_{th}}{R_{th}+R_L}\right)$$

$$P = I^2 R_L$$

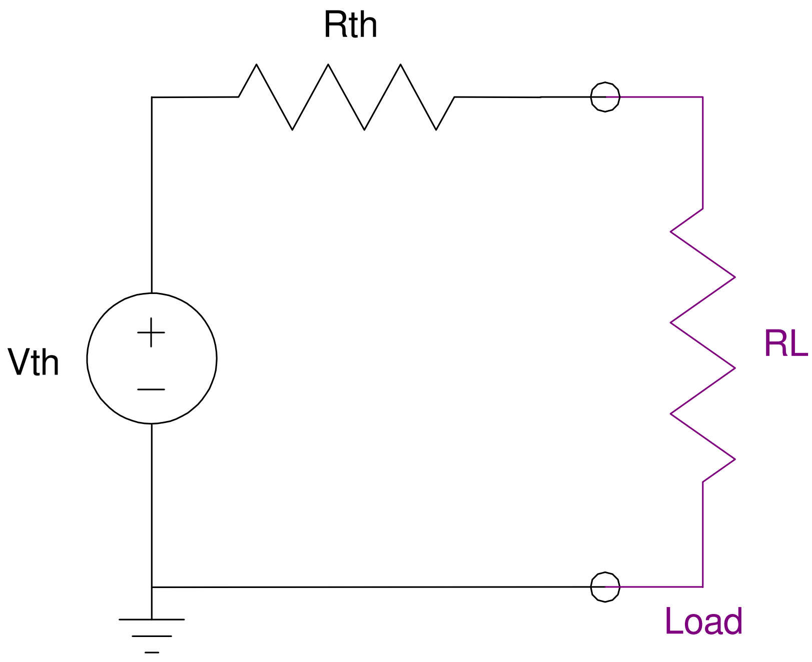
$$P = \left(\frac{R_L}{(R_{th}+R_L)^2}\right) V_{th}^2$$

$$\frac{d}{dR_L}\left(\frac{R_L}{(R_{th}+R_L)^2}\right) = 0$$

$$(R_L + R_{th})(R_{th} - R_L) = 0$$

$R_L = R_{th}$ *maximum*

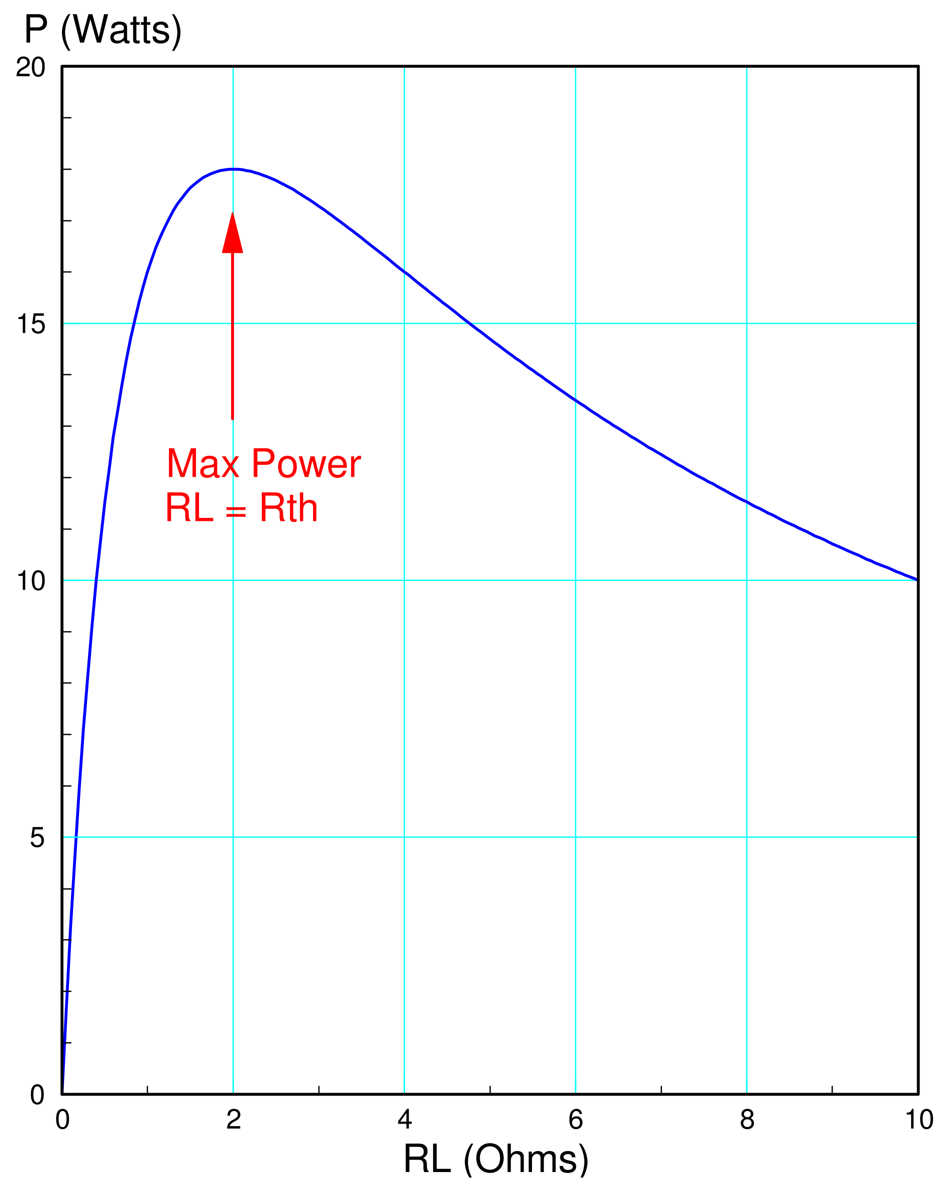
$R_L = -R_{th}$ *minimum*



Matlab Calculations

- Let V_{th} = 12V and R_{th} = 2 Ohms:

```
>> Vth = 12;
>> Rth = 2;
>> RL = [0:0.05:10]';
>> I = Vth ./ (Rth + RL);
>> V = RL ./ (Rth + RL) * Vth;
>> P = V .* I;
>> plot(RL, P);
```



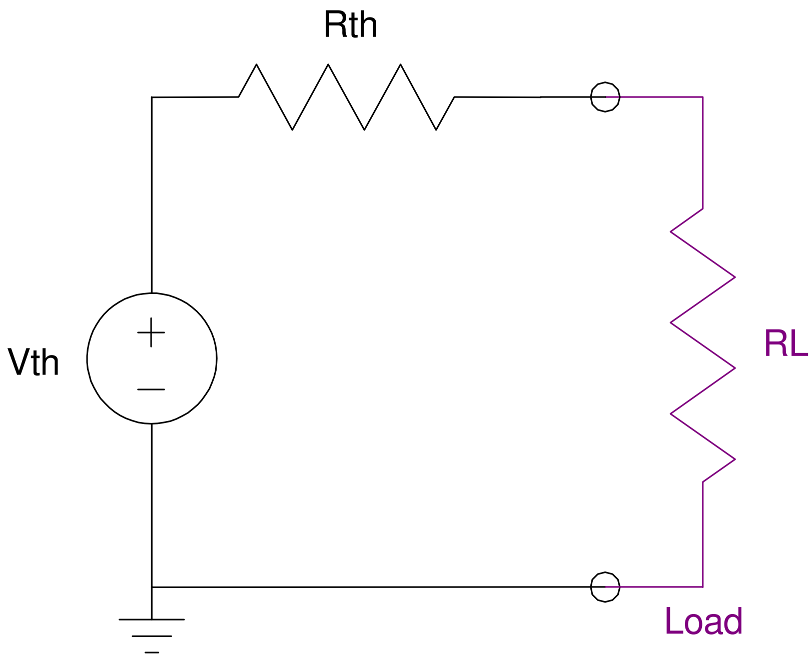
Note

- RL = Rth for max power
- At that point, you're at 50% efficiency

Case 2: R_L is fixed.

Find R_{th} to maximize the power to the load.

$$P = \left(\frac{R_L}{(R_{th}+R_L)^2}\right) V_{th}^2$$



Find the maximum with respect to Rth

$$\frac{dP}{dR_{th}} = 0 = \left(\frac{-2(R_{th}+R_L)R_L}{(R_{th}+R_L)^3}\right) V_{th}^2$$

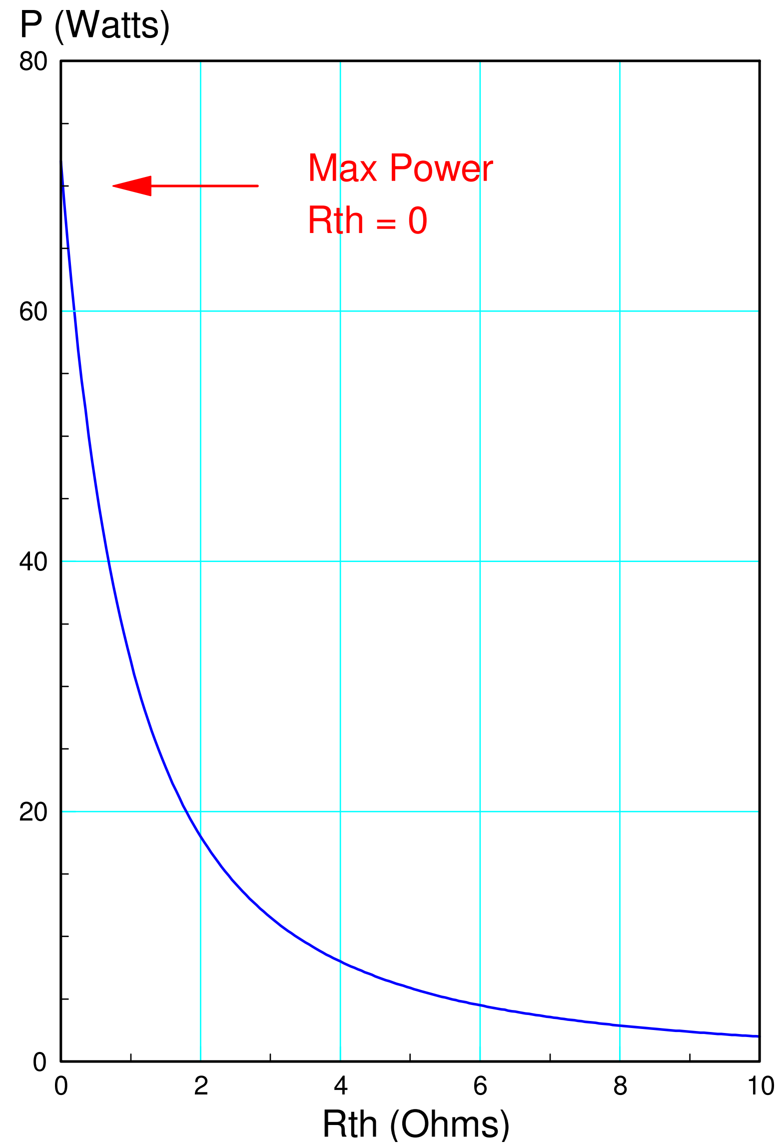
Maximum is when $R_{th} = -R_L$

- Make Rth as small as possible
  - Zero is good
  - Negative Rth is better

Matlab Calculations

- V_{th} = 12V and R_L = 2 Ohms

```
>> Vth = 12;
>> RL = 2;
>> Rth = [0:0.05:10]';
>> I = Vth ./ (Rth + RL);
>> V = RL ./ (Rth + RL) * Vth;
>> P = V .* I;
>> plot(Rth, P);
```



Case 3: Find R_L to maximize efficiency

Efficiency is the power to the load divided by the total power dissipated.

$$eff = \left(\frac{P_{Load}}{P_{total}}\right)$$

or

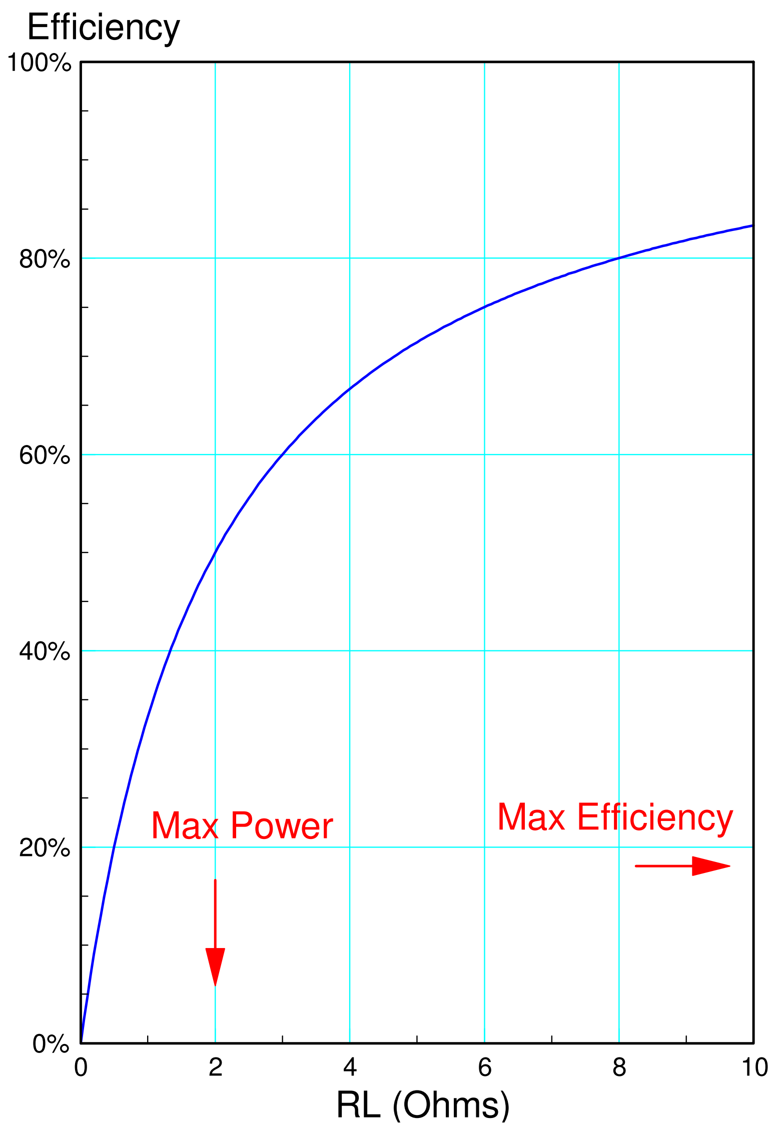
$$eff = \frac{I^2 R_L}{I^2 (R_{th} + R_L)} = \left(\frac{R_L}{R_L + R_{th}}\right)$$

Max efficiency is

- R_L = infinity
- Efficiency = 100%
- Power = 0

That's one of the problems of delivering power

- For high efficiency, you want $R_L >> R_{th}$
- For maximum power, you want $R_L = R_{th}$



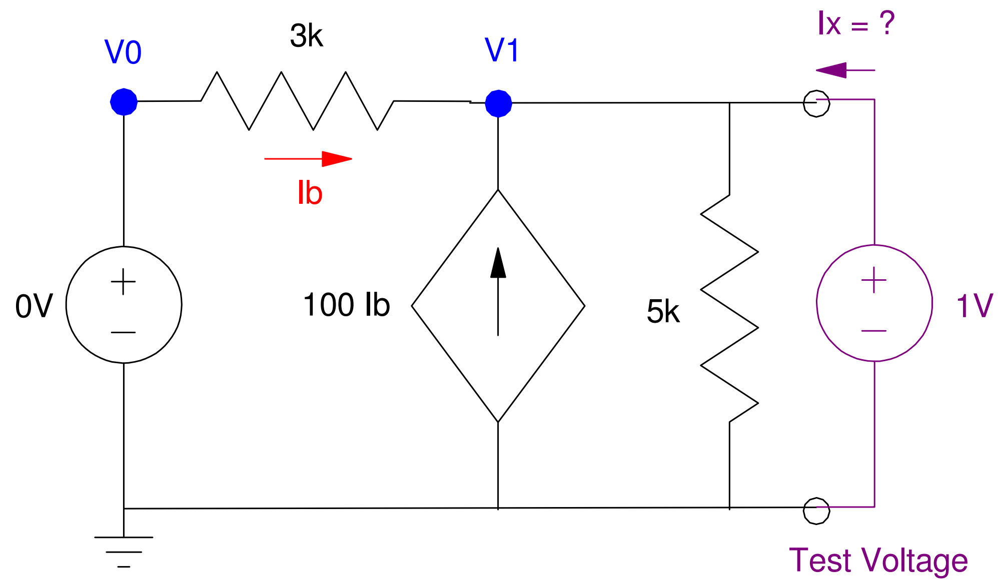
Summary

Dependent sources complicate calulations for Rth

Trick:

- Turn off independent sources
  - Same as before
- Apply a 1V test voltage
  - New trick
- Compute the current draw, Ix

$$R_{th} = \frac{1V}{I_x}$$



The Thevenin resistance is important for calculating the max-power to a load

- Max power is when RL = Rth
- Efficiency = 50% at that point