Thevenin Equivalent and Load Lines

ECE 211 Circuits I

Lecture #10

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Thevenin Equivalent

So far, we have two tools:

- Voltage Nodes: Find the voltages so that the currents balance (sum to zero)
- Current Loops: Find the currents so that the votlages balance (sum to zero)

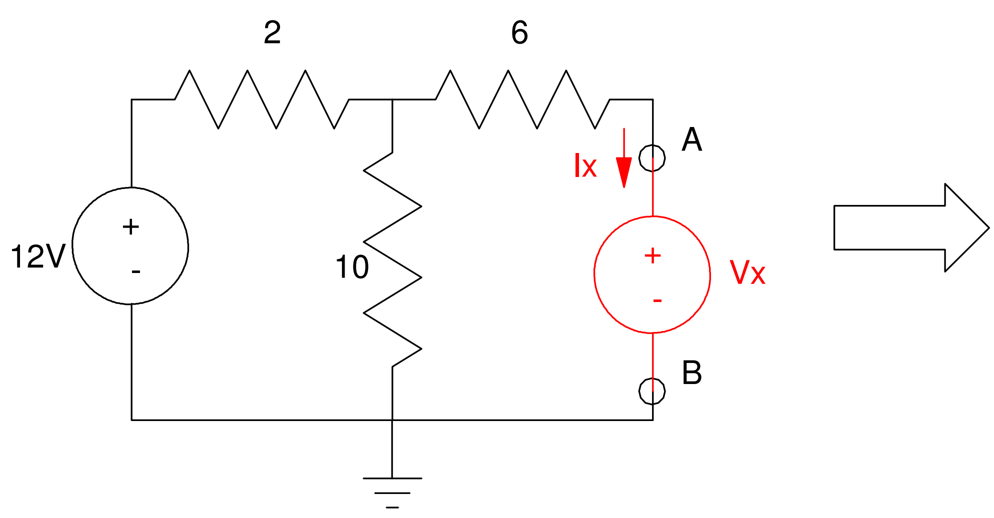
Thevenin Equivalents is tool #3

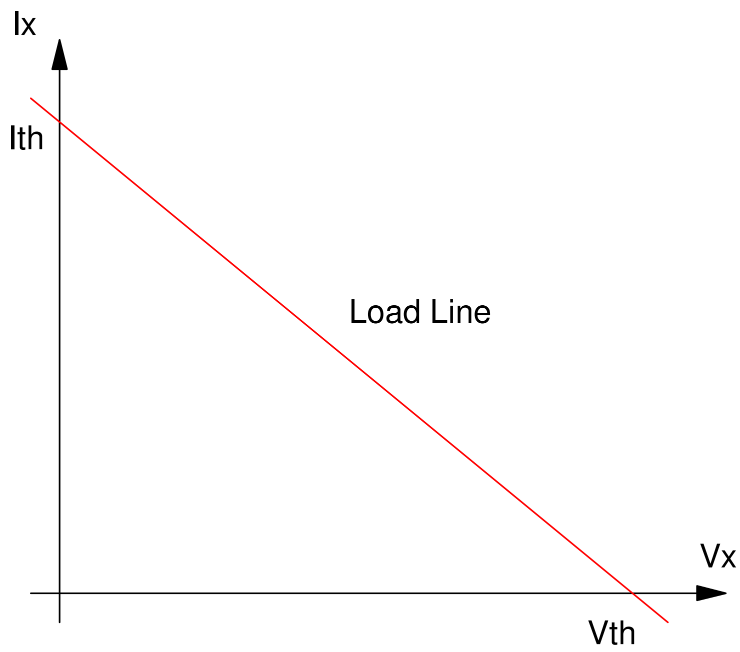
- Harder to find
- More difficult to comprehend
- Not necessary: you can solve any circuit using current loops and voltage nodes
- But, Thevenin equivalents can make some circuits much easier to analyze

Load Lines:

Idea:

- The voltage-current relationship for a linear circuit follows a straight line.
- Any circuit which produces the same load line behaves the same
- The simplest circuit which does this is
  - A voltage source & resistor (Thevenin equivalent)
  - A current source & a resistor (Norton equivalent)

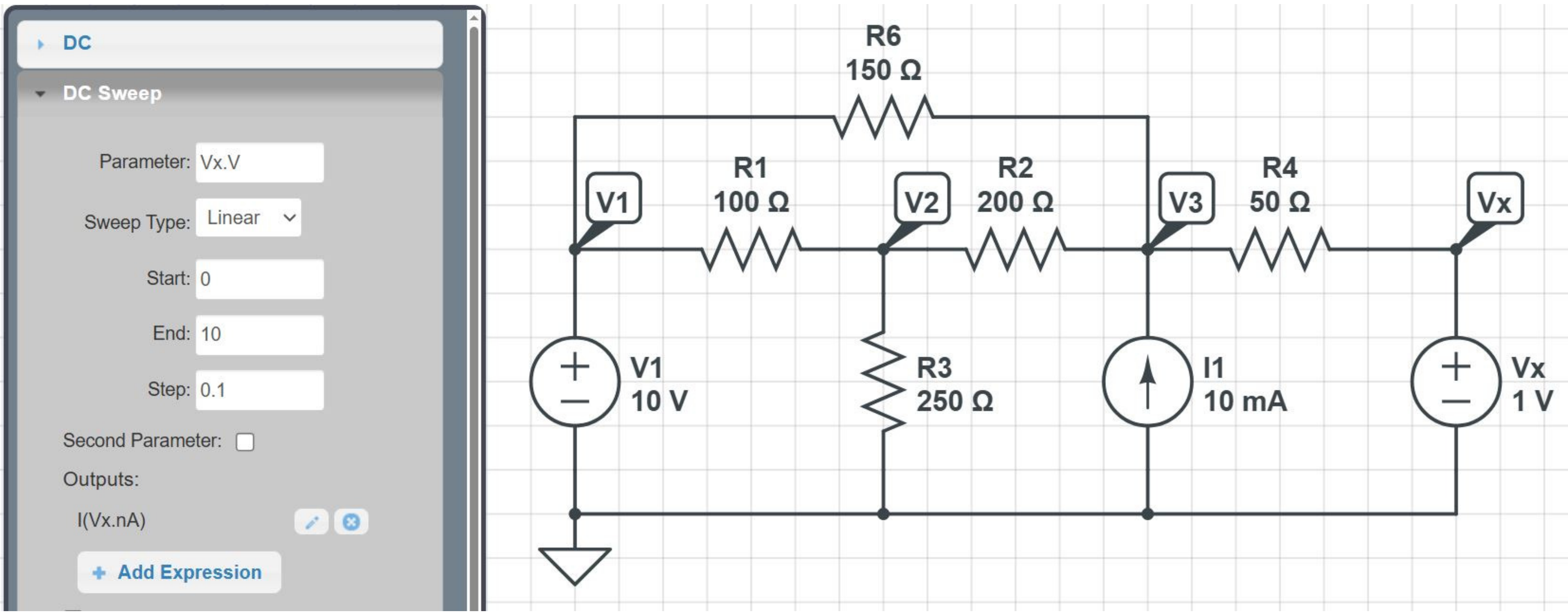




Example:

How does Ix (current out to Vx) vary as Vx varies?

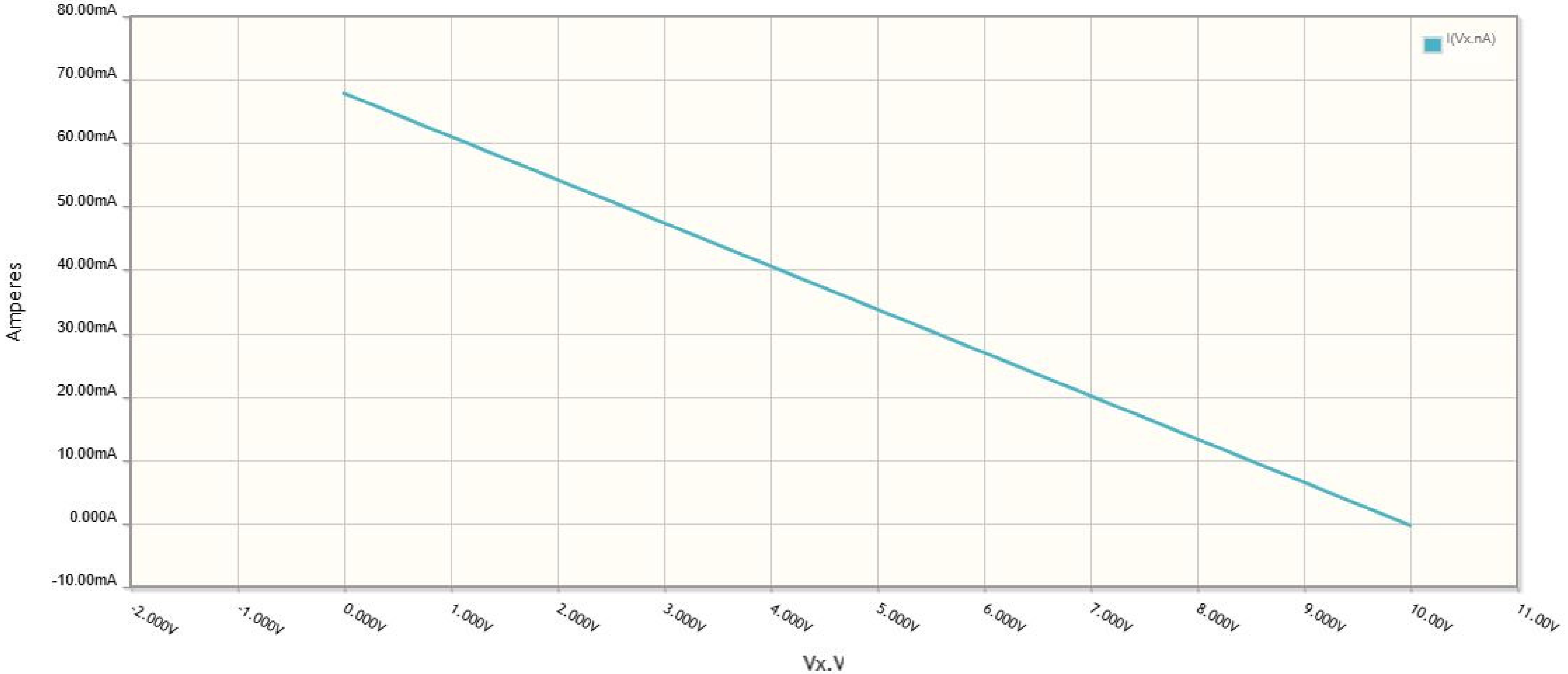
- Option 1: Measure or compute Ix as Vx varies, or
- Option 2: Do a DC Sweep as Vx varies from 0V to 10V



Result:

Vx and Ix have a linear relationship

- As Vx increases, Ix decreases
- This is termed *the load line*



Thevenin and Norton Equivalent

The load doesn't know what circuit generates the load line

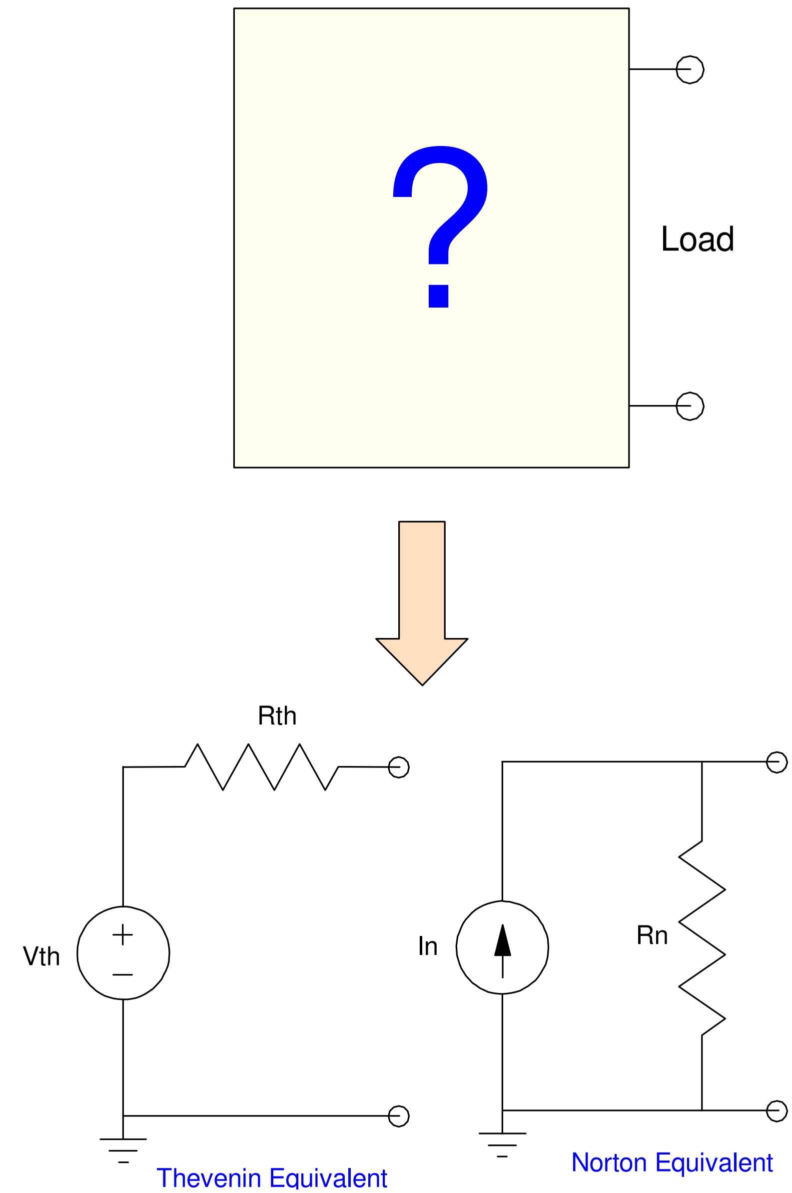
- Any circuit with the same load line looks the same as far as the load is concerned

Replace the circuit with the simplest circuit with the same load line:

- Thevenin: A voltage source and resistance in series
- Norton: A current source and a resistance in parallel

The trick is to find the values of

- Vth & Rth, or
- In & Rn



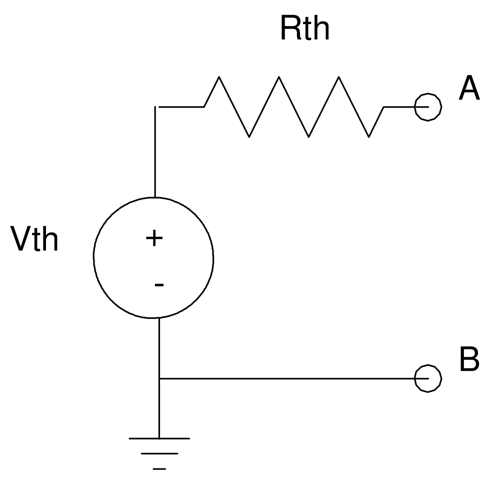
Finding Vth, Rth, In, Rn

Circuits are equivalent

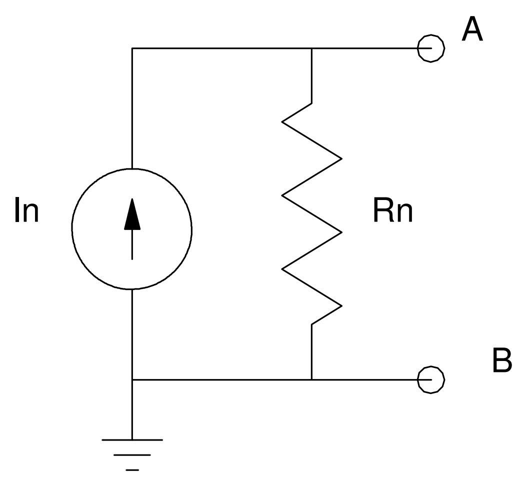
- Whatever works for your original circuit works for the Thevenin / Norton equivalent

Procedure:

- Vth: Measure the open-circuit voltage of your circuit.
- Rth = Rn: Turn off all sources (V = 0, I = 0). Measure the resulting resistance.
- In: Measure the short-circcuit current



Thevenin Equivalent

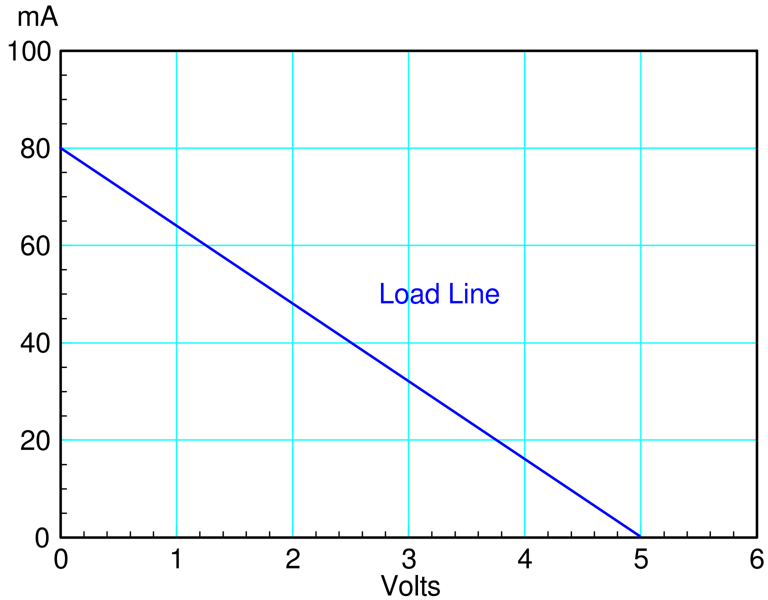


Norton Equivalent

Example: Given the load line, find the Thevenin equivalent

Vth:

- The the voltage at the output when the load is infinite
- Open circuit voltage
- Vth = 5.00V

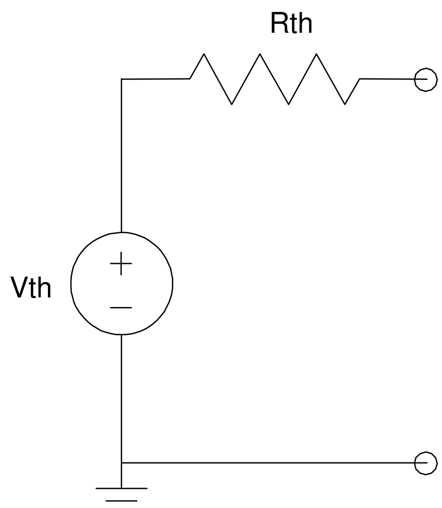


Rth:

- Rth is the slope of the load line

$$R_{th} = \left(\frac{5V}{80mA}\right) = 62.5\Omega$$

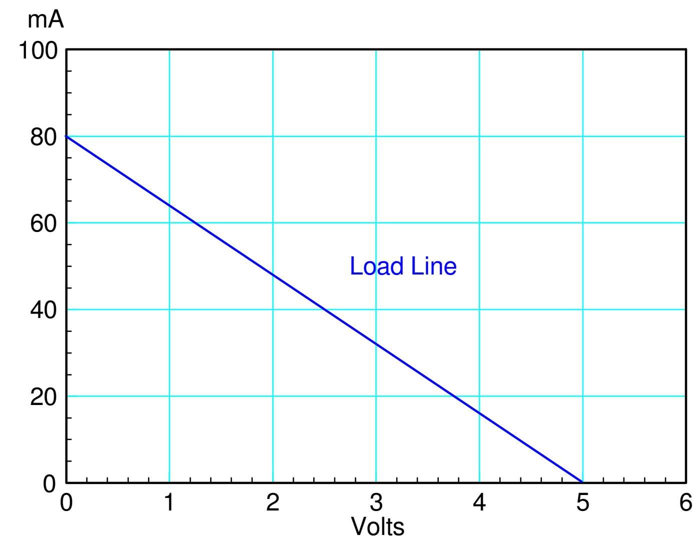
- Rth is the impedance looking in
  - Set the voltage sources to zero
  - Set the current sources to zero
  - Measure the impedance looking in



Example: Given the load line, find the Norton equivalent

In

- Short the output
- Current to the load is In
- In = 80mA

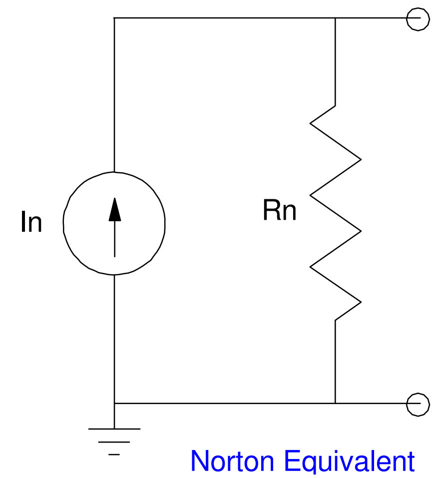


Rn

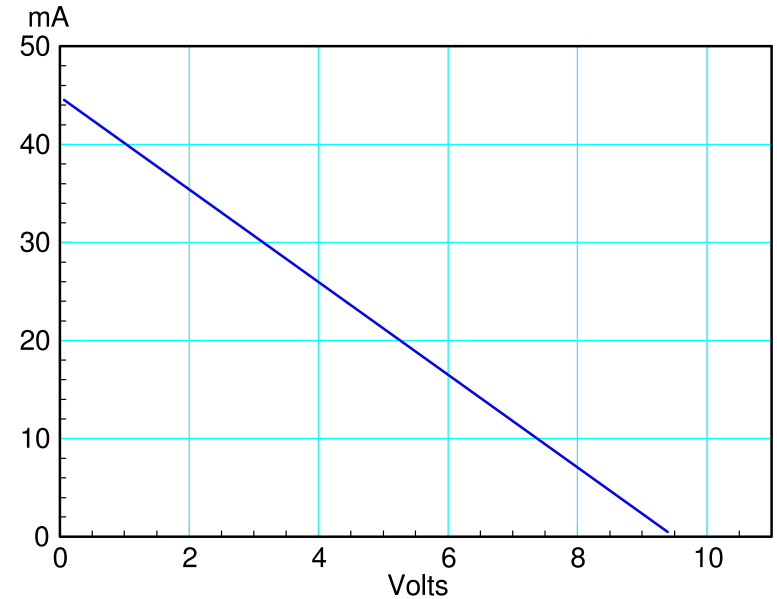
- Rn is the slope of the line

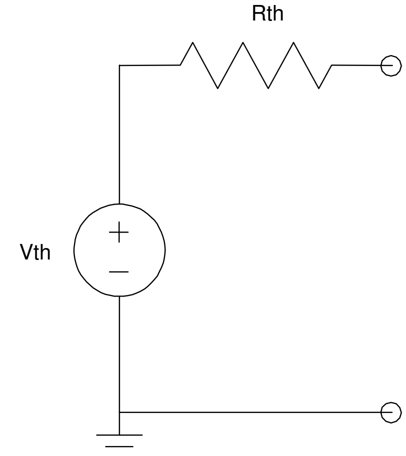
$$R_n = \left(\frac{5V}{80mA}\right) = 62.5\Omega$$

- Rn is the impedance looking in
  - Set the voltage sources to zero
  - Set the current sources to zero
  - Measure the impedance looking in

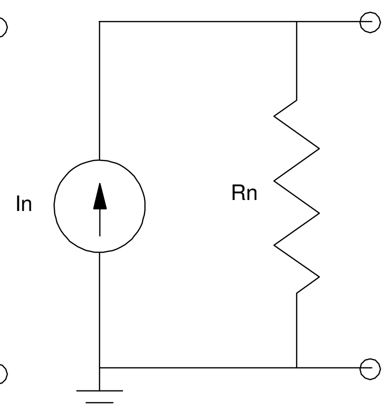


Problem: Determine the Thevenin and Norton equivalent





Thevenin Equivalent



Norton Equivalent

Theven & Norton Equivalents for Circuits

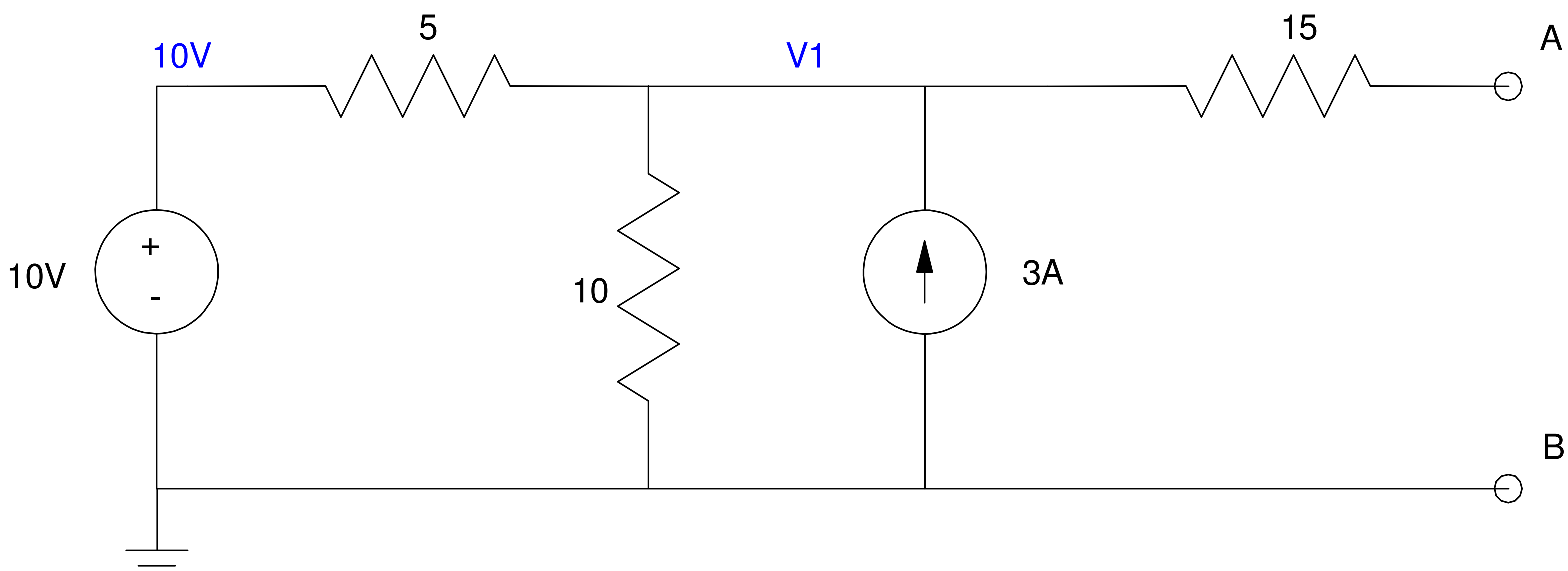
- Find two of three: V(open), I(short), Resistance looking in

Vth: Measure the open-circuit voltage.

$$\left(\frac{V_1-10}{5}\right)+\left(\frac{V_1}{10}\right)-3=0$$

$$V_{th}=V_1=16.67V$$

This is Vth

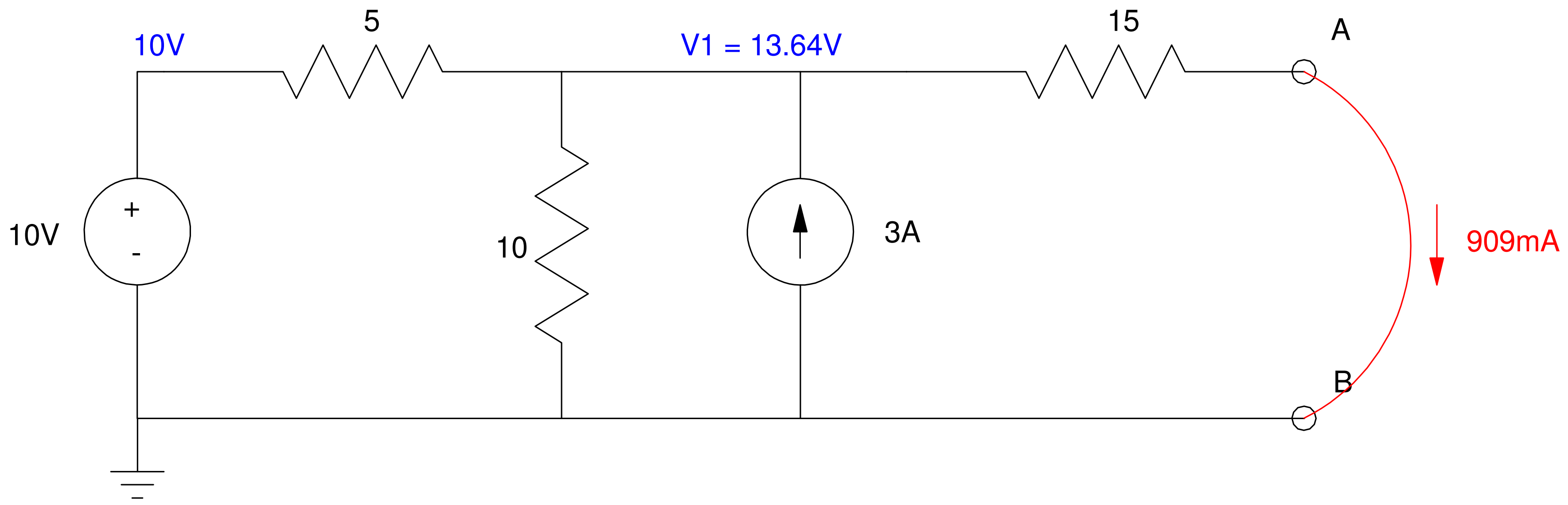


In: Short AB and measure the current. Again, this isn't obvious so write the node equation at V1

$$\left(\frac{V_1 - 10}{5}\right) + \left(\frac{V_1}{10}\right) - 3 + \left(\frac{V_1}{15}\right) = 0$$

$$V_1 = 13.64V$$

$$I_{short} = I_N = \frac{13.64V}{15\Omega} = 909.1mA$$



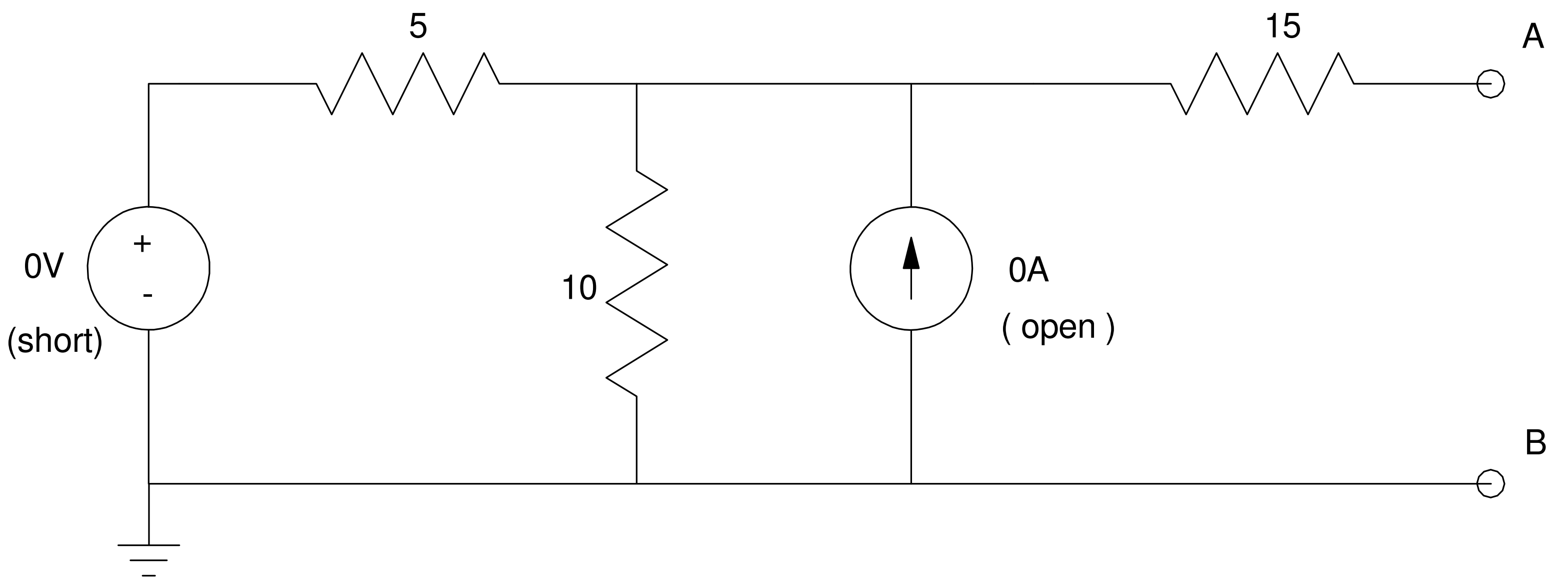
Rth: Turn off the sources (V = 0, I = 0). Measure the resistance between A and B

$$R_{AB} = 15 + 5||10$$

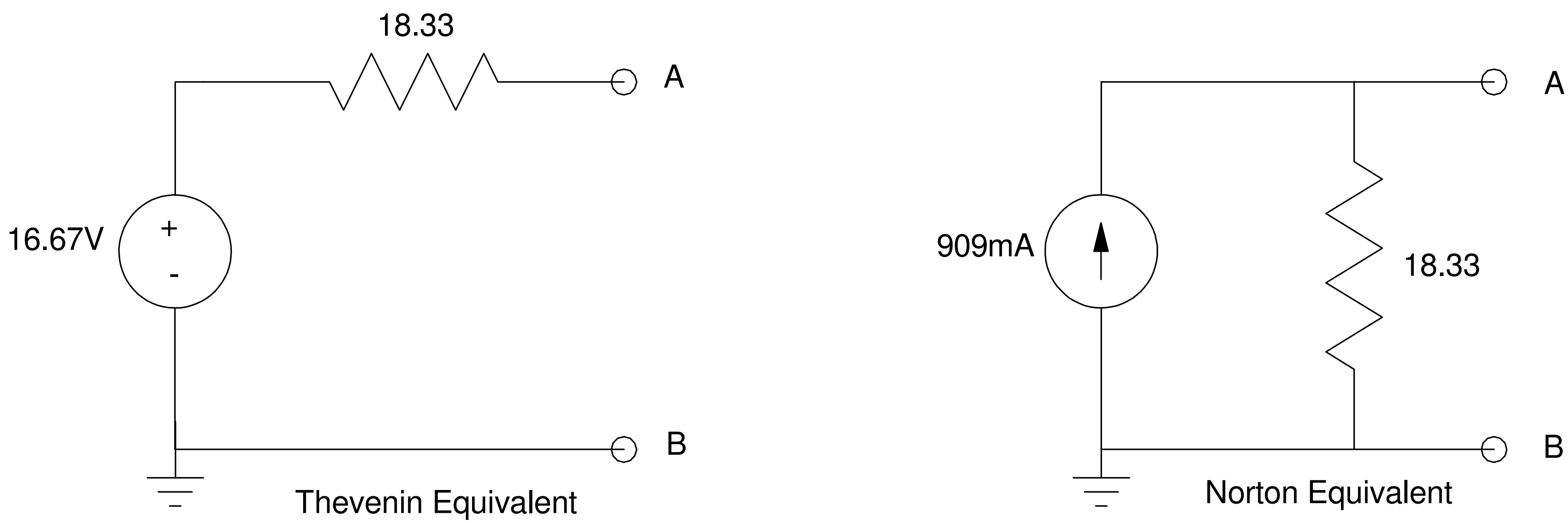
$$R_{AB} = 18.333\Omega$$

Note that you only need to compute two of these: the third redundant.

$$R_{AB} = \frac{V_{tn}}{I_n} = \frac{16.67V}{909mA} = 18.33\Omega$$



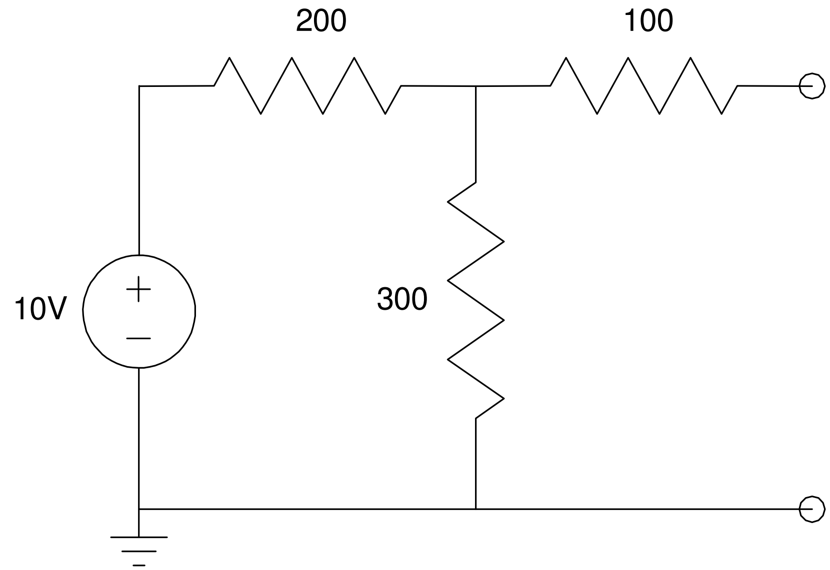
Result:

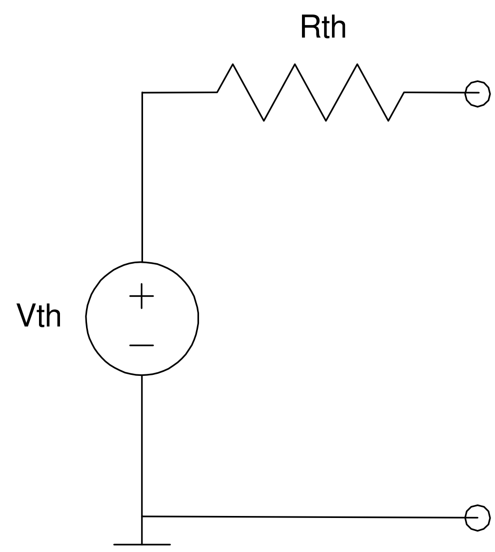


Resulting Thevenin and Norton equivalents of Example 1

Practice Problem

Find the Thevenin eqivalent circuit





Why Use Thevenin / Nortan Equivalents?

Sometimes, the Thevenin equivalent helps to explain what's happening

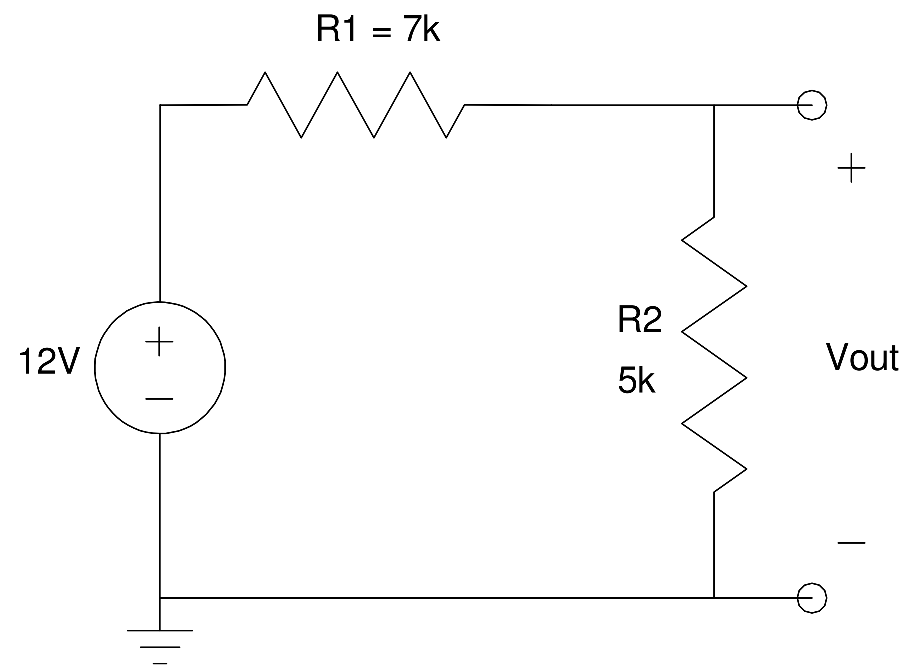
Example: Convert 12V to 5V

Solution: Use a votlage divider

$$V_2 = \left(\frac{R_2}{R_1+R_2}\right) V_1$$

$$V_2 = \left(\frac{5k}{5k+7k}\right) 12V$$

$$V_2 = 5V$$



Problem:

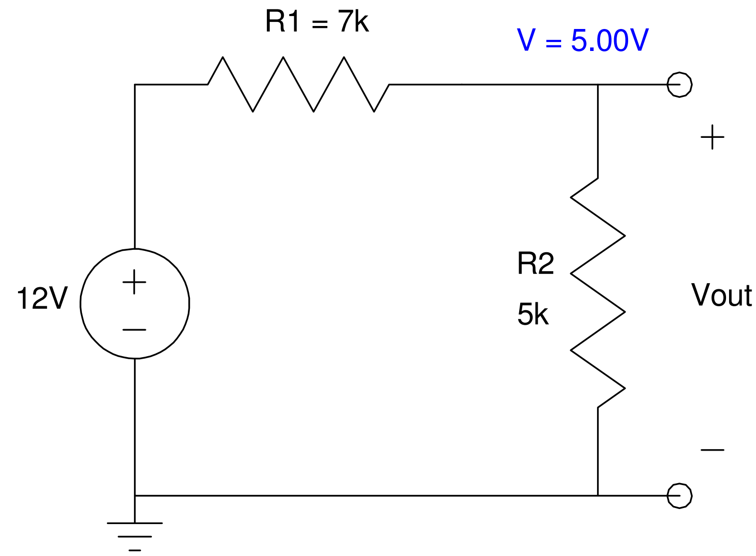
The voltage divider works when you build it

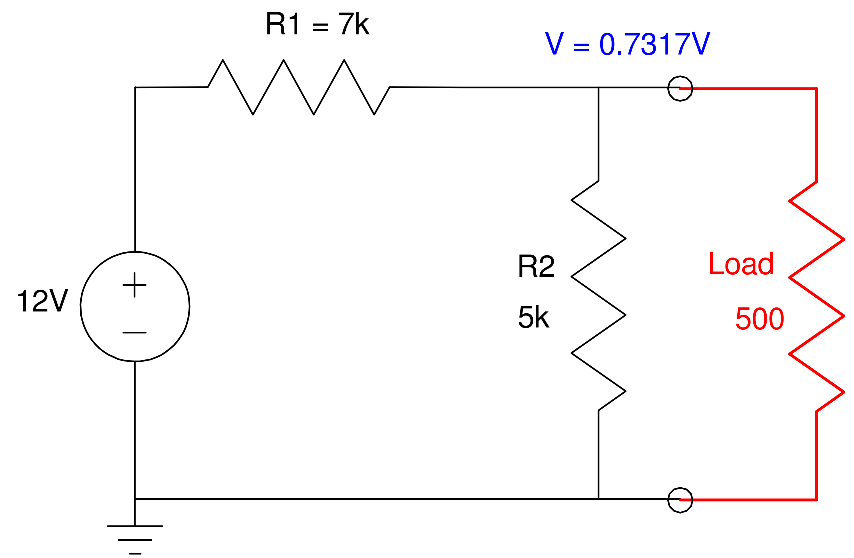
- The output is 5V

As soon as you connect a load to it, the voltage dissappears

- Add a load that draws 10mA @ 5V (500 Ohms)
- The output voltage drops to 0.73V

What's happening?





Solution:

Replaces the source with it's Thevenin equivalent

Open-Circuit Voltage (Vth)

$$V_{th} = \left(\frac{5k}{5k+7k}\right) 12V = 5V$$

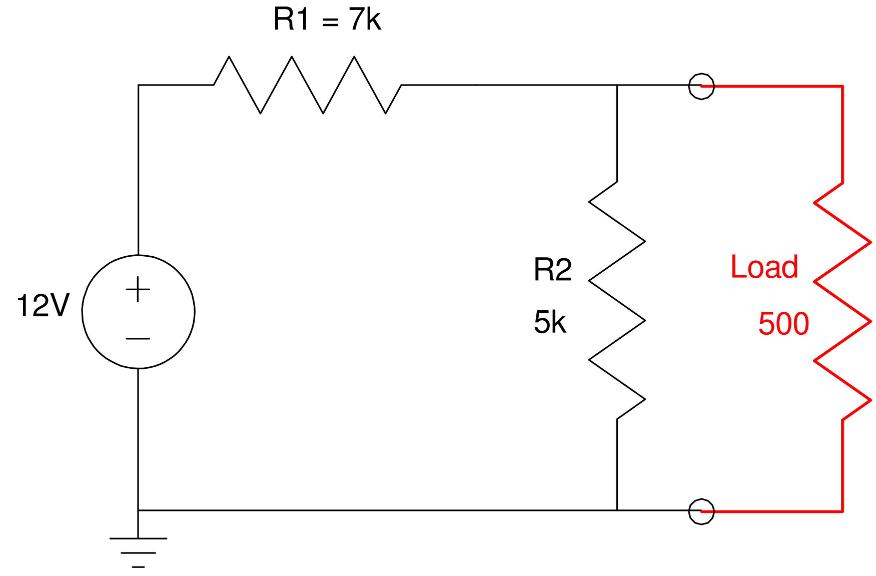
Thevenin Resistance

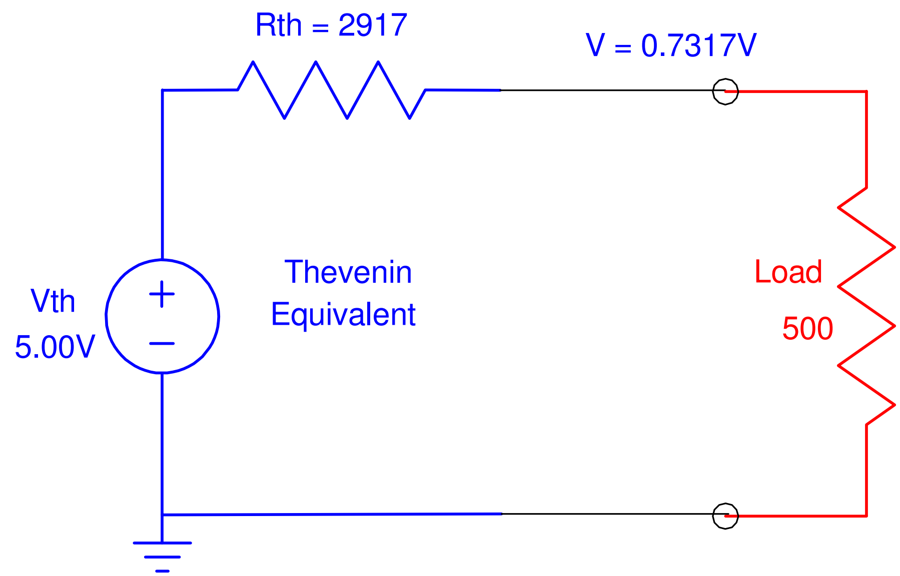
- Turn off the votlage source
- Compute Rin (Rth)

$$R_{th} = 5k || 7k = 2917\Omega$$

Add a 500 Ohm load, and the voltage drops by voltage division

$$V_{out} = \left(\frac{500}{500+2917}\right) 5V = 0.73V$$





Observation

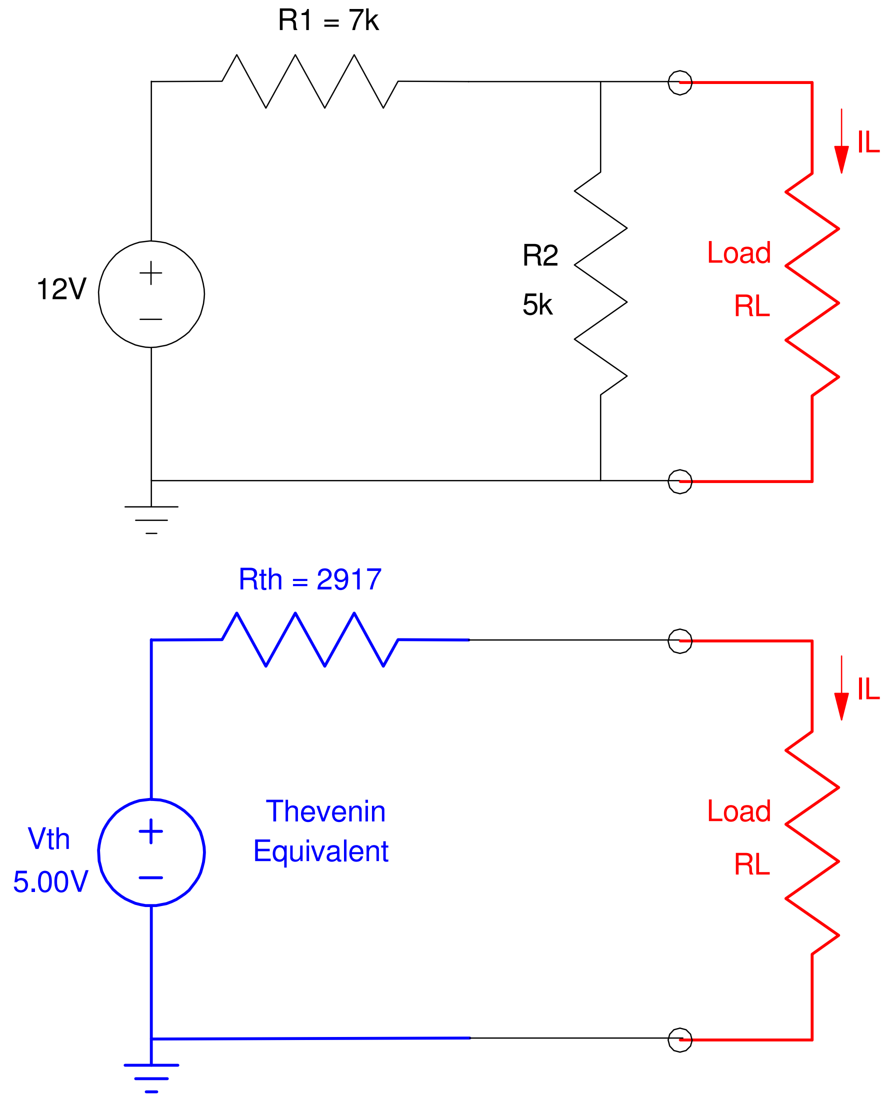
The Thevenin equivlent also tells you the limitations of this design

The 12V to 5V converter works

- As long as R_L >> Rth
- As long as R_L >> 2917

This is a 5V source that only works if you don't use it

- $R_L > 29k$
- $I_L < 200uA$



Circuit Simplification using Thevenin and Norton Equivalent

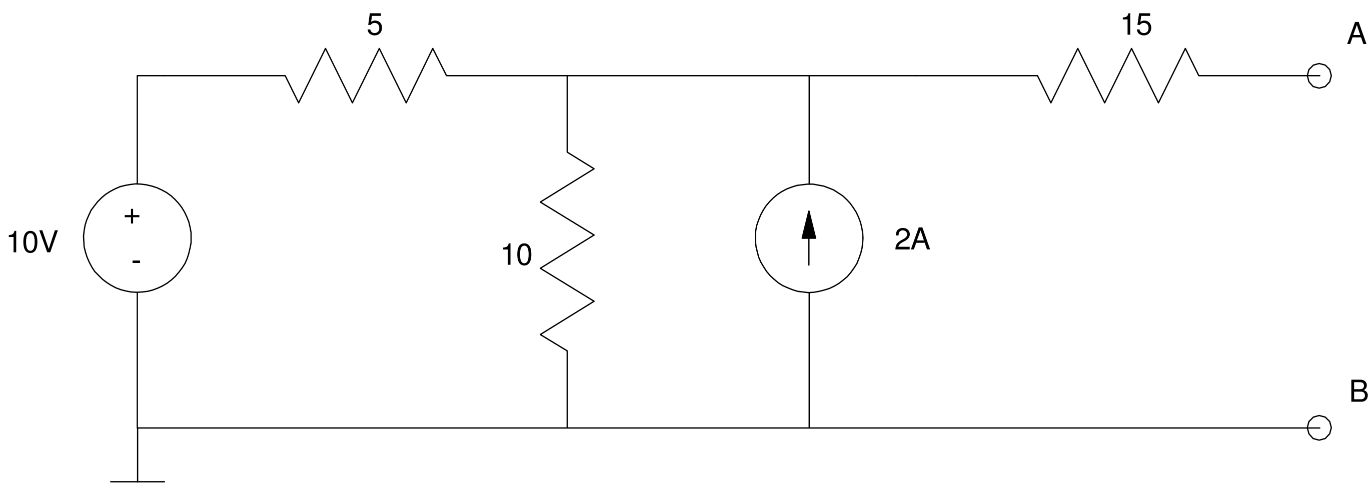
Sometimes you can simplify a circuit by flipping between Thevenin and Norton equivalents

$$R_{Thevenin} = R_{Norton}$$

$$V_{Thevenin} = I_{Norton} \cdot R$$

$$I_{Norton} = \frac{V_{Thevenin}}{R}$$

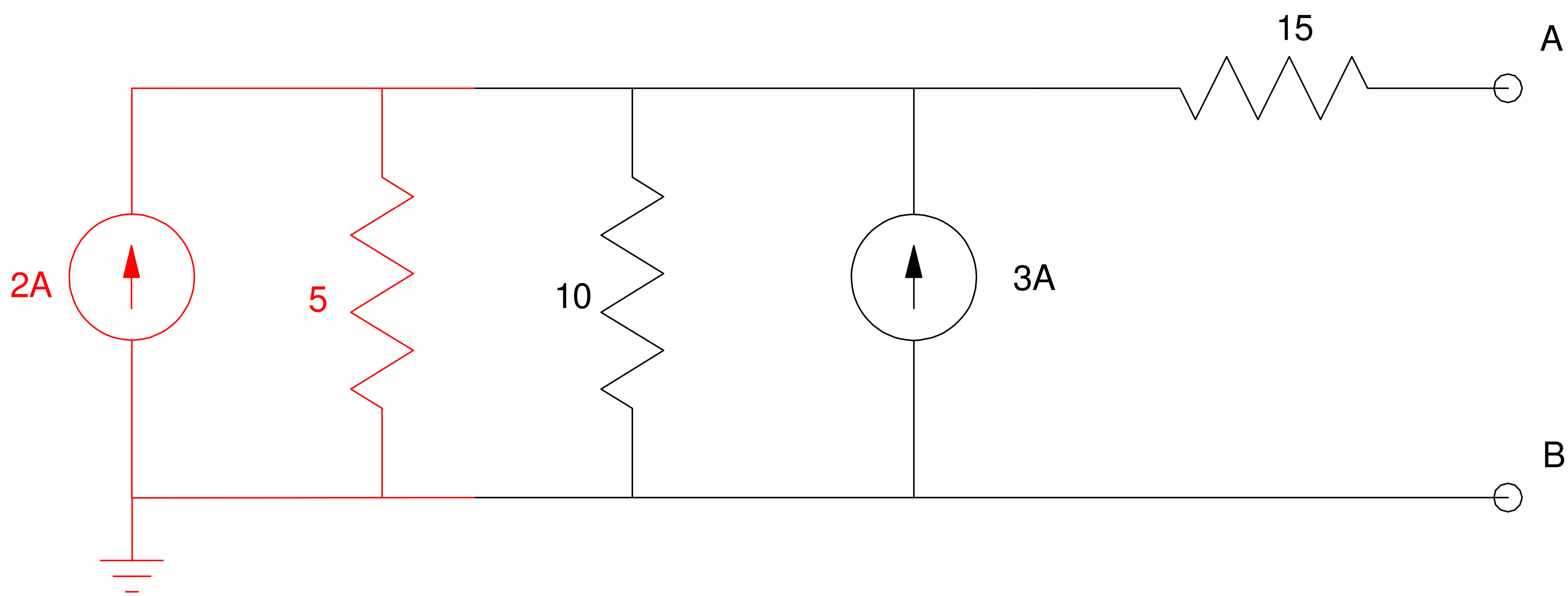
Eample: Find the Thevenin equivalent



Step 1: Convert the 10V / 5 Ohm resistor to its Norton equivalent

$$I_N = \frac{10V}{5\Omega} = 2A$$

$$R_N = R_{Th} = 5\Omega$$



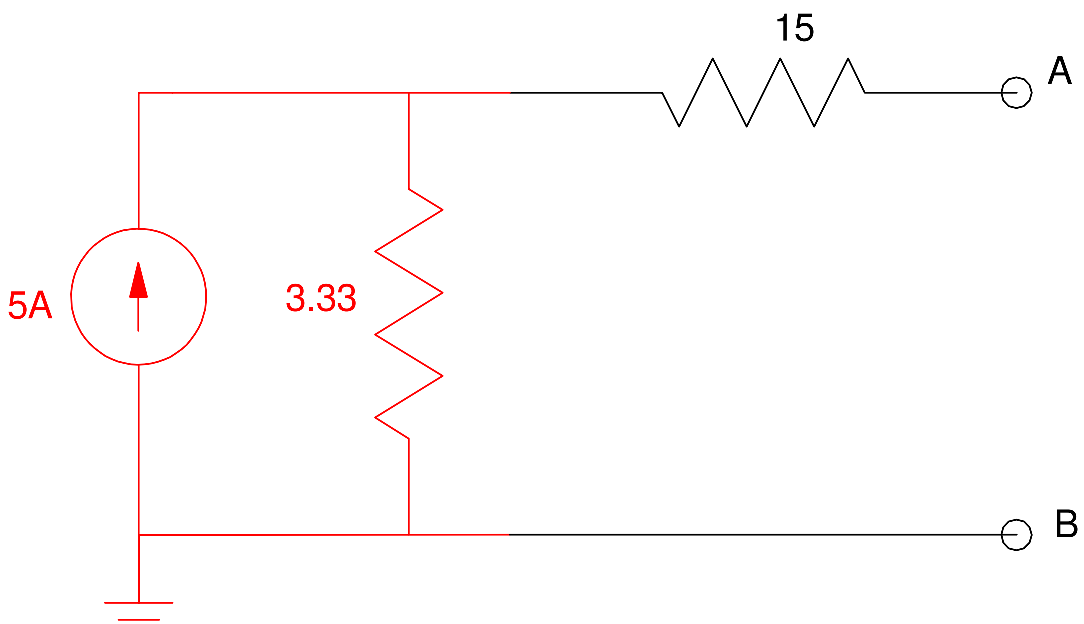
Convert the 10V : 5 Ohm source to its Norton equivalent (shown in red)

Add the resistors in parallel

$$R_{net} = 5||10 = \left(\frac{1}{5} + \frac{1}{10}\right)^{-1} = 3.333\Omega$$

Add the current soruces

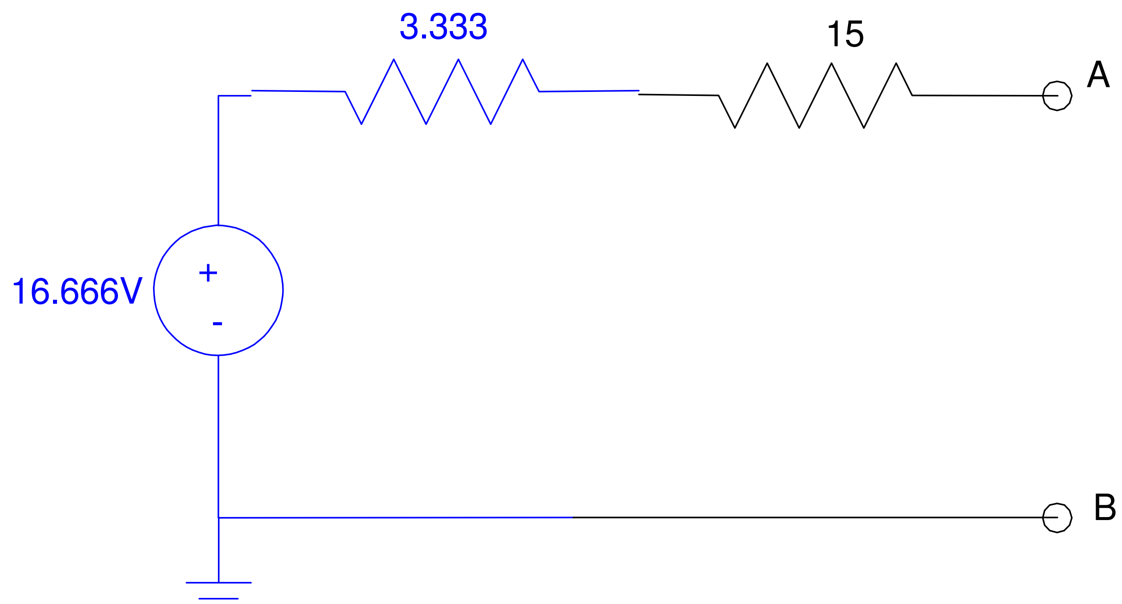
$$I_{net} = 2 + 3 = 5$$



Add the resistors and current sources in parallel:

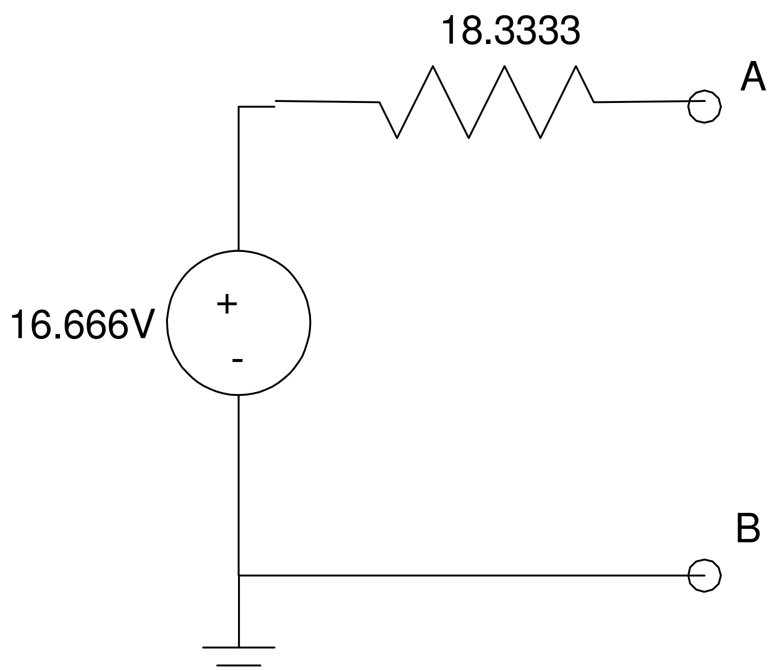
Convert back to a Thevenin equivalent

$$V_{th} = 5A \cdot 3.3333\Omega = 16.666V$$



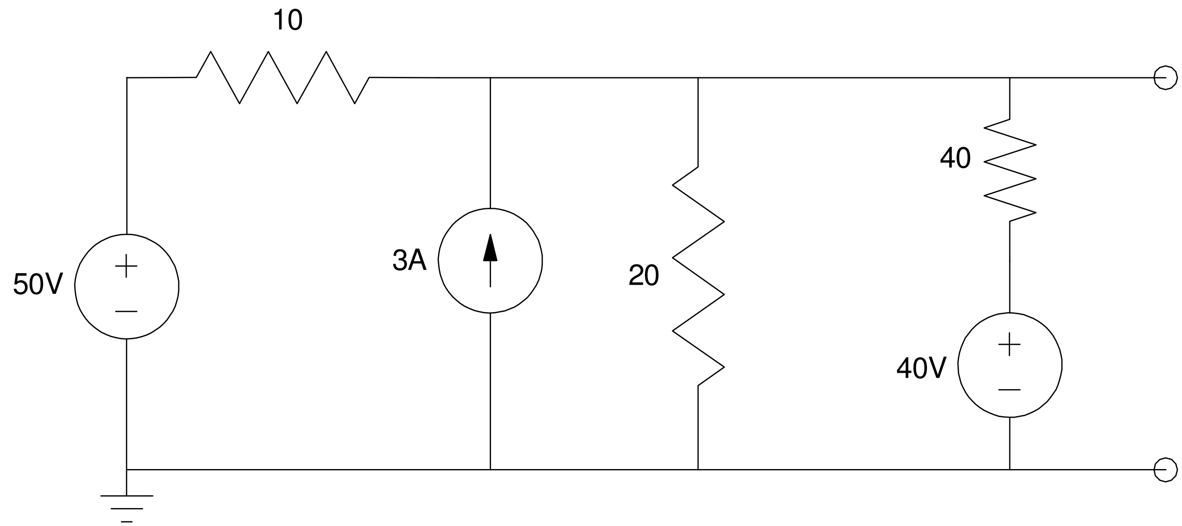
Convert to a Thevenin equivalent (shown in blue)

Add the resistors in series and you have the Thevenin equivalent looking in from terminals AB



Practice Problem

Find the Thevenin equivalent by using source transformations



Summary

For linear circuits (resistor circuits),

- If you plot the voltage vs. current at the load,
- You get a line

This is termed *the load line*

As far as the load is concerned

- A given circuit looks like
- Any other circuit with the same load line

This allows you to replace a circuit with a simpler model

- Thevenin equivalent or
- Norton equivalent