Super Loops

a.k.a. Super Meshes

ECE 211 Circuits I

Lecture #8

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Super-Loops

Current sources cause problems with current loops

- You don't know the voltage across the current source
- You can't sum the voltages to zero

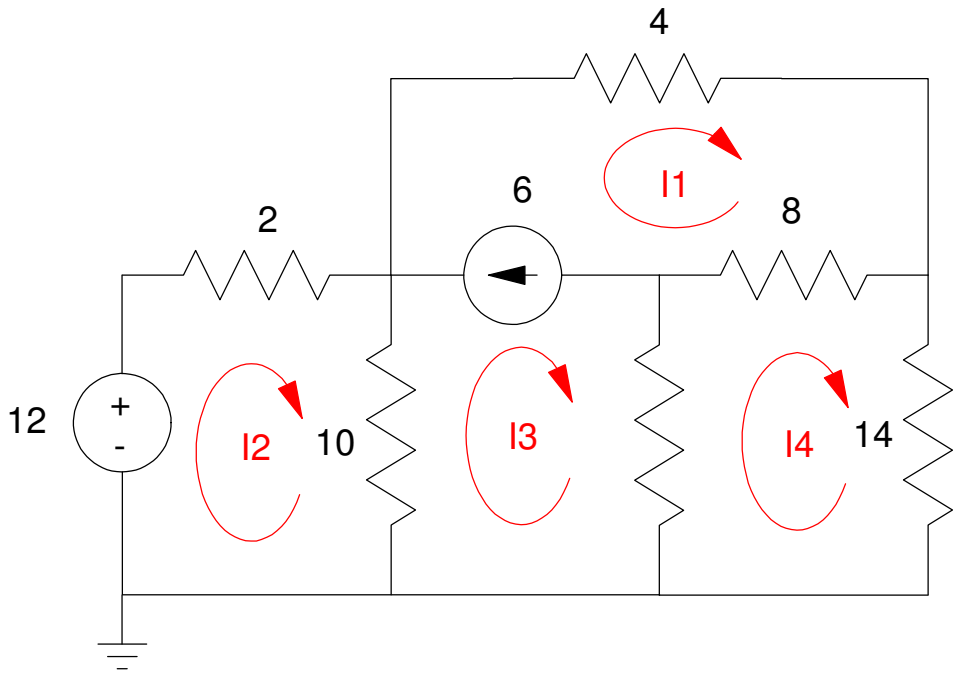
Kirchoff's Voltage Law: The sum of the voltages around any closed path must sum to zero.

- Pick a different closed path to complete N equations for N unknowns
- If this path includes several current loops, the path is called a *SuperLoop*

note: There are usually multiple ways to do this

Example: Find I1 .. I4

- 4 windows
- Write 4 equations for 4 unknowns



Easy one: (current source)

$$I_1 - I_3 = 6$$

Loop I2 and I4

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

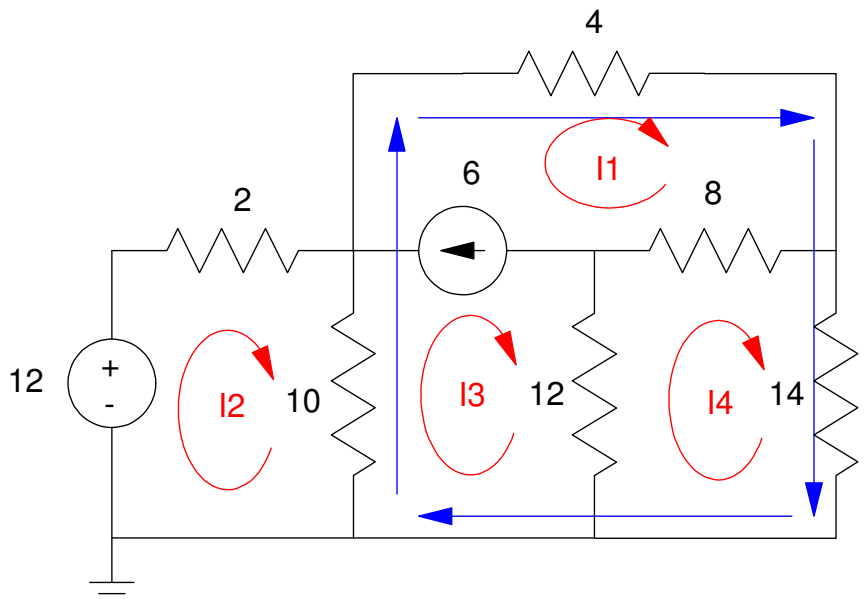
$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

4th equation???

SuperLoop:

Option 1:

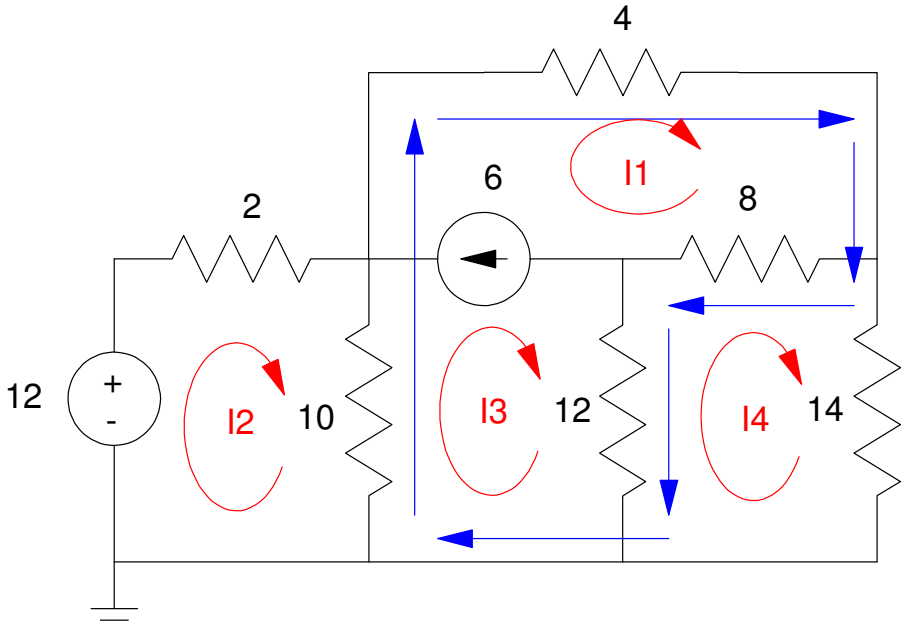
$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$



Option 2:

$$10(I_3 - I_2) + 4(I_1) + 8(I_1 - I_4) + 12(I_3 - I_4$$

Both are valid



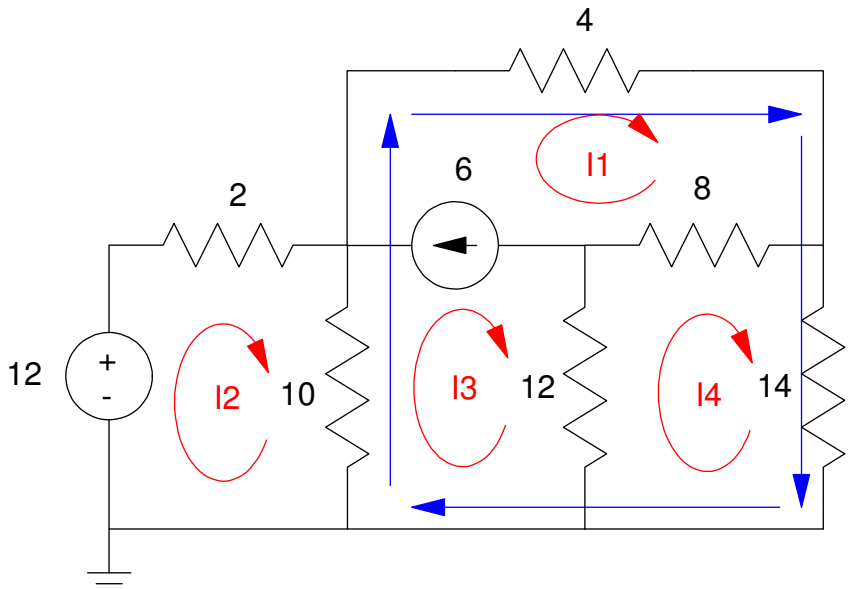
Solving: Group terms

$$12I_2 - 10I_3 = 12$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

$$I_1 - I_3 = 6$$

$$4I_1 - 10I_2 + 10I_3 + 14I_4 = 0$$

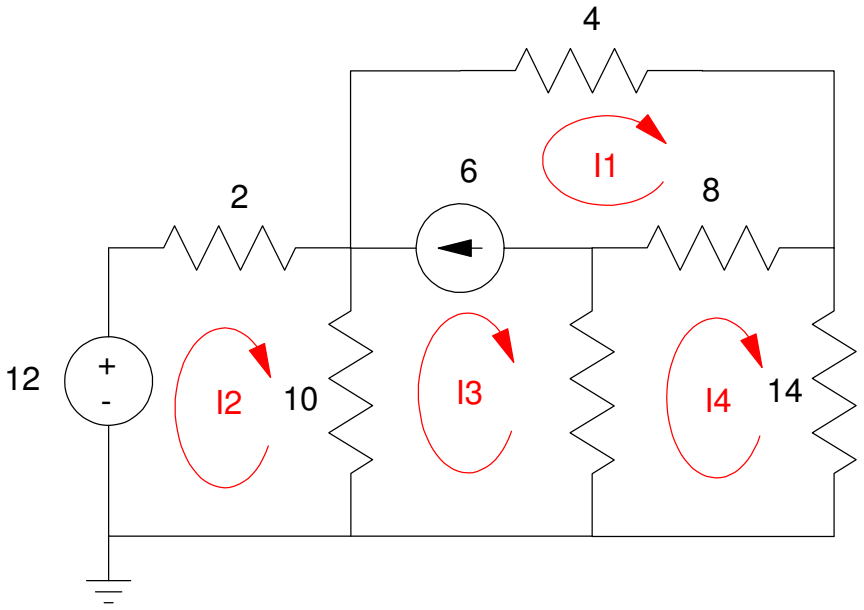


Put in matrix form:

$$\begin{bmatrix} 0 & 12 & -10 & 0 \\ -8 & 0 & -12 & 34 \\ 1 & 0 & -1 & 0 \\ 4 & -10 & 10 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 6 \\ 0 \end{bmatrix}$$

Throw into MATLAB and solve

```
A = [0,12,-10,0; -8,0,-12,34; 1,0,-1,0; 4,-10,10,14]

     0.    12.   - 10.    0.
   - 8.    0.    - 12.    34.
     1.    0.    - 1.     0.
     4.   - 10.    10.    14.

B = [12;0;6;0]

    12.
     0.
     6.
     0.

inv(A)*B

I1     3.5712271
I2    -1.0239774
I3    -2.4287729
I4    -0.0169252
```



Current can be negative. It just means the direction we assumed for I1..I4 was wrong.

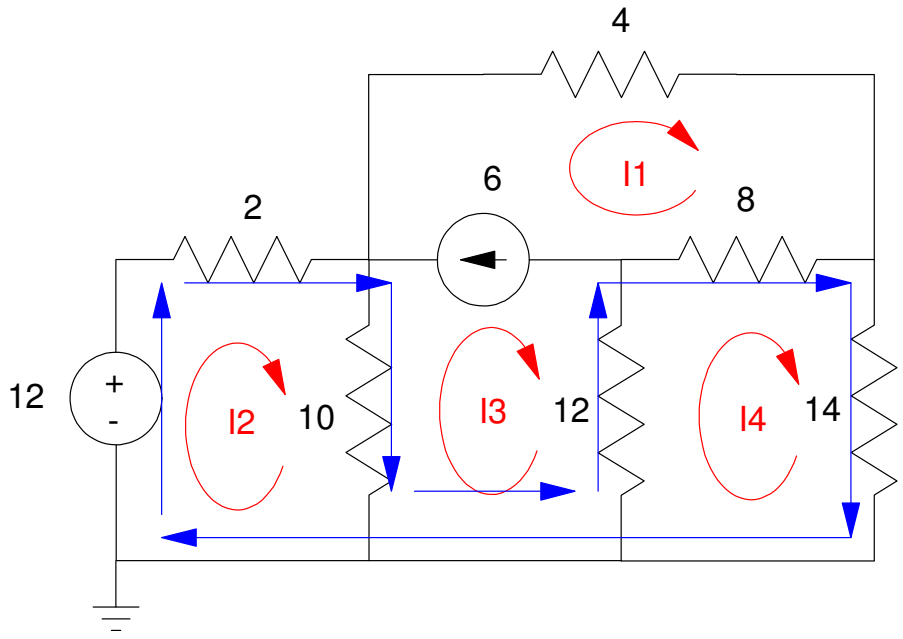
Note: Some superloops don't work that some paths for the 4th equation are not valid. For example, suppose you use this loop:

$$-12+2(I_2)+10(I_2-I_3)+12(I_4-I_3)+8(I_4-I_1)+14(I_4)=0$$

This shows up in Matlab as an error

- You do not have 4 linearly independent equations
- Your equations miss the 4 Ohm resistor

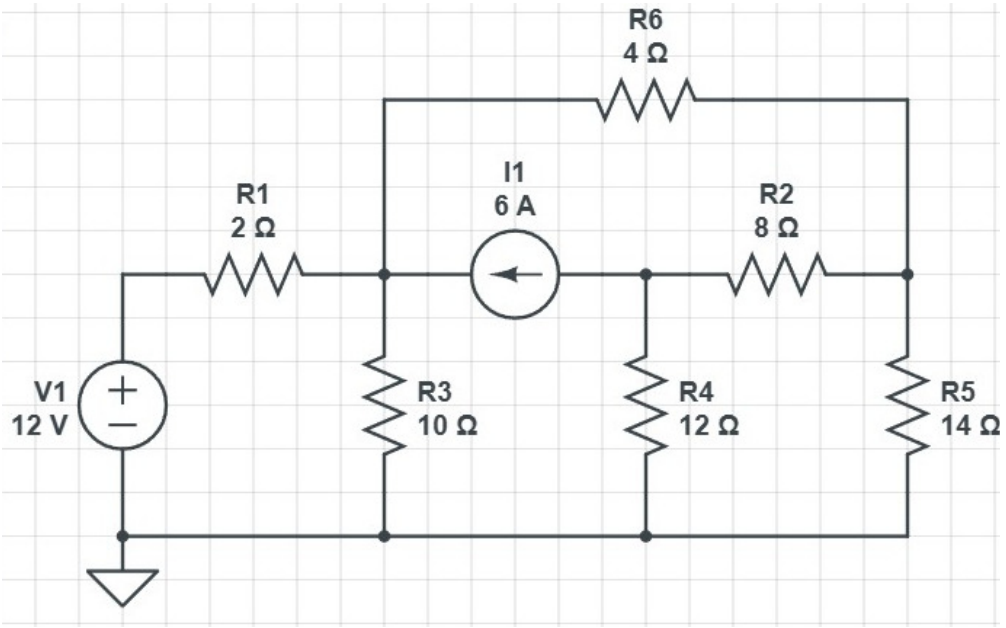
```
A = [0,12,-10,0; -8,0,-12,34;
1,0,-1,0; -8,12,-22,34];
B = [12;0;6;12];

I = inv(A)*B
        !--error 19
Problem is singular.
```

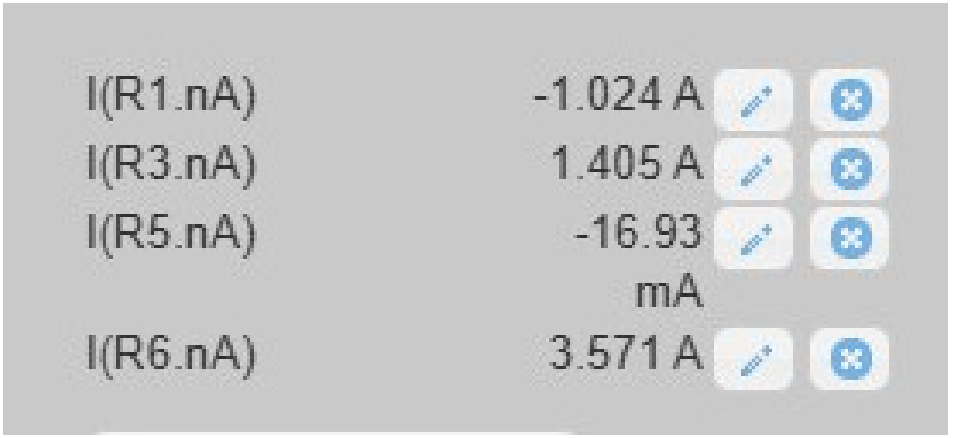


Checking with CircuitLab

Results match up with calculations

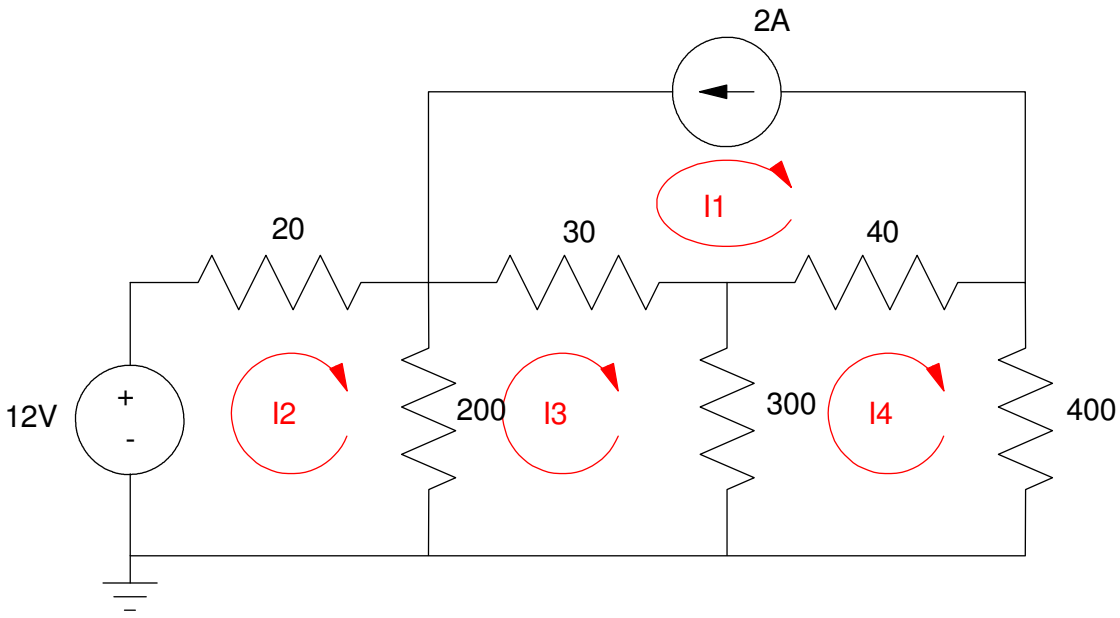
Calculation		Simulation	
I1	**3.5712271**	I(R6)	3.571 A
I2	**-1.0239774**	I(R1)	-1.024 A
I3	**-2.4287729**		
I4	**-0.0169252**	I(R5)	-16.93 mA





Practice Problem

Write 4 equations to solve for the 4 unknown currents



Super Loops with Dependent Sources

Dependent sources mean you need extra equations

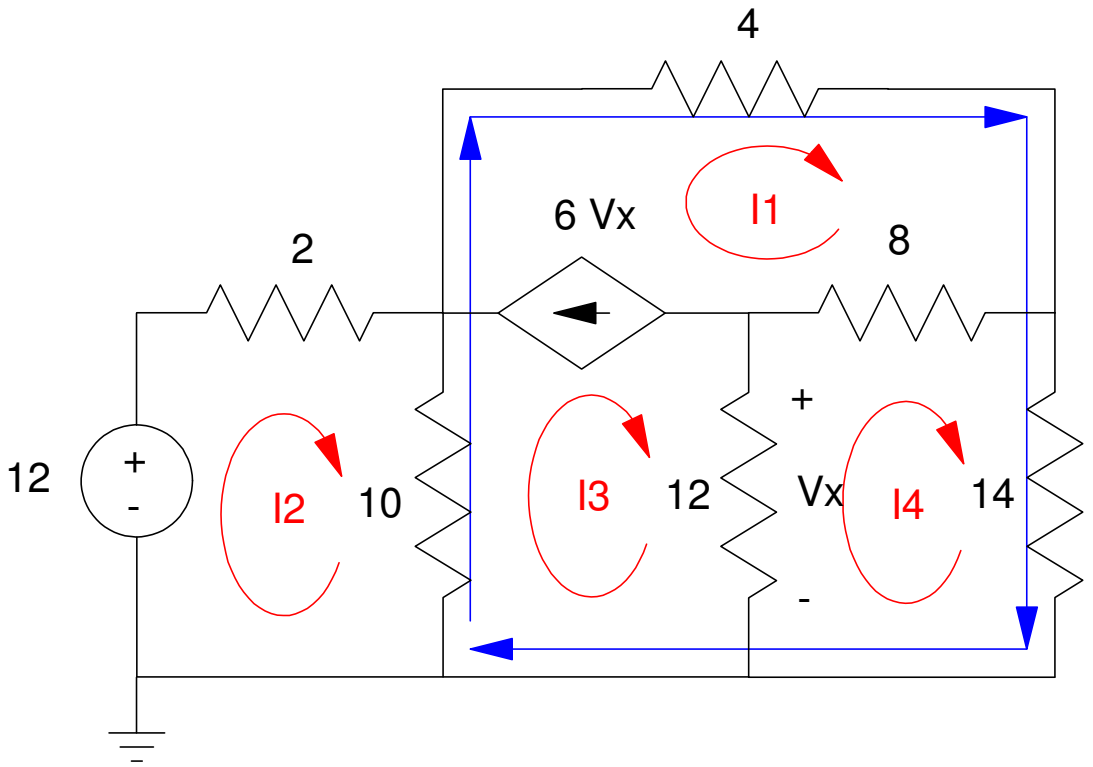
- One more equation for each dependent source

Example: Find I1..I4

Step 1: Define the currents (shown in red)

Step 2: Determine how many equations you need. There are five unknowns (I1, I2, I3, I4, Vx).

- 5 equations for 5 unknowns.



Step 3: Write the equations

Easy ones:

$$V_x = 12(I_3 - I_4)$$

$$6V_x = I_1 - I_3$$

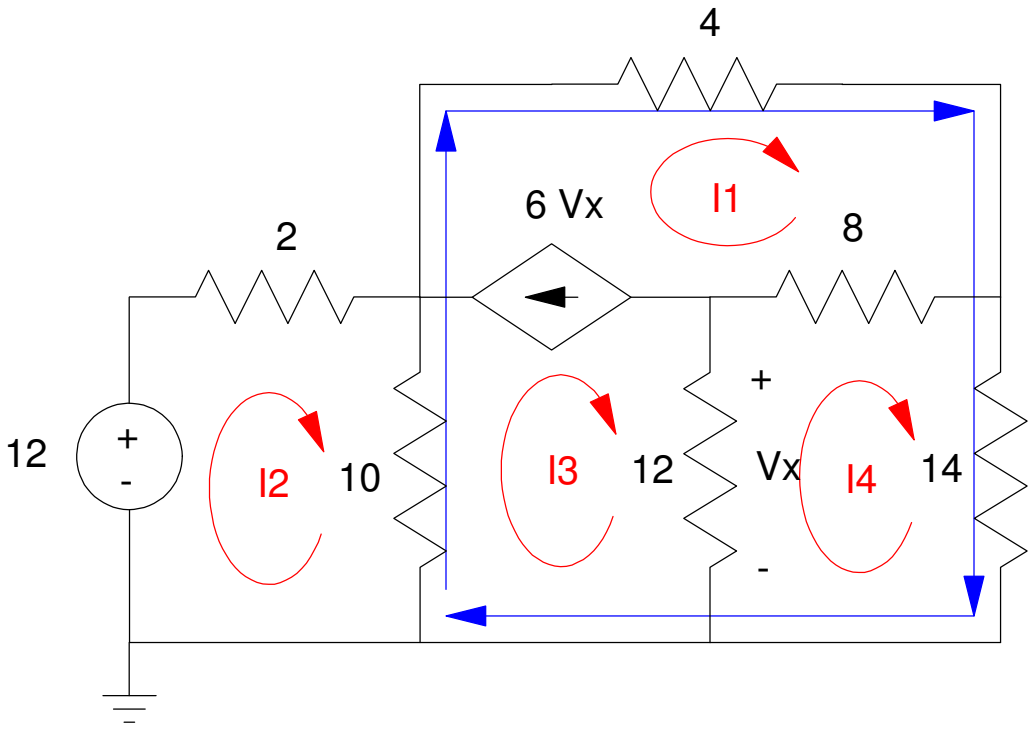
Loop I2 and I4

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

SuperLoop (shown in blue)

$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$



Step 4: Solve. Group terms:

$$12I_3 - 12I_4 - V_x = 0$$

$$I_1 - I_3 - 6V_x = 0$$

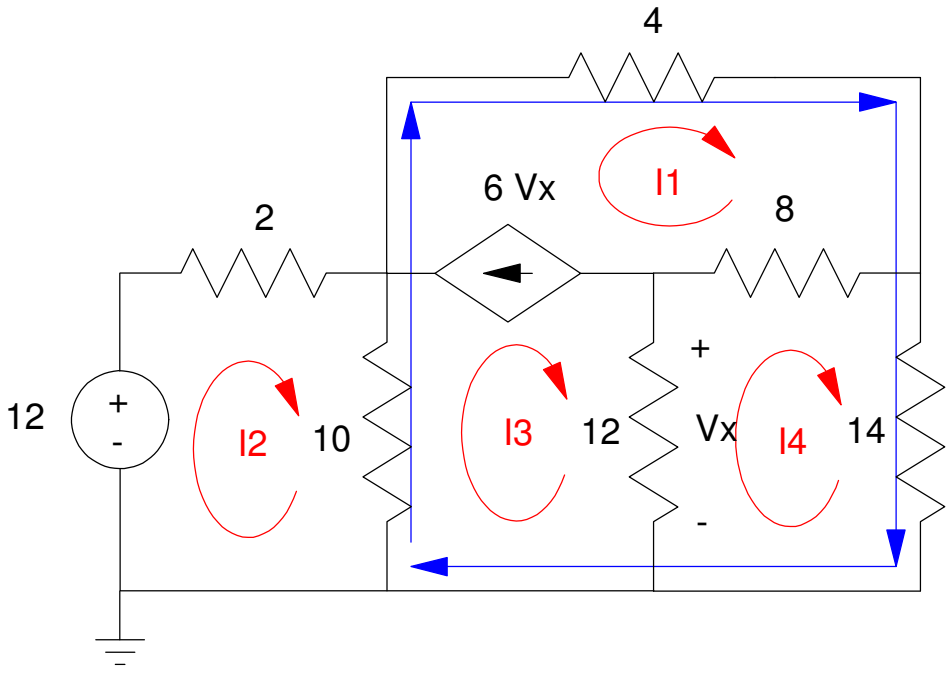
$$12I_2 - 10I_3 = 12$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

$$4I_1 - 10I_2 + 10I_3 + 14I_4 = 0$$

Write these in matrix form

$$\begin{bmatrix} 0 & 0 & 12 & -12 & -1 \\ 1 & 0 & -1 & 0 & -6 \\ 0 & 12 & -10 & 0 & 0 \\ -8 & 0 & -12 & 34 & 0 \\ 4 & -10 & 10 & 14 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$



Throwing these equations into MATLAB and solve:

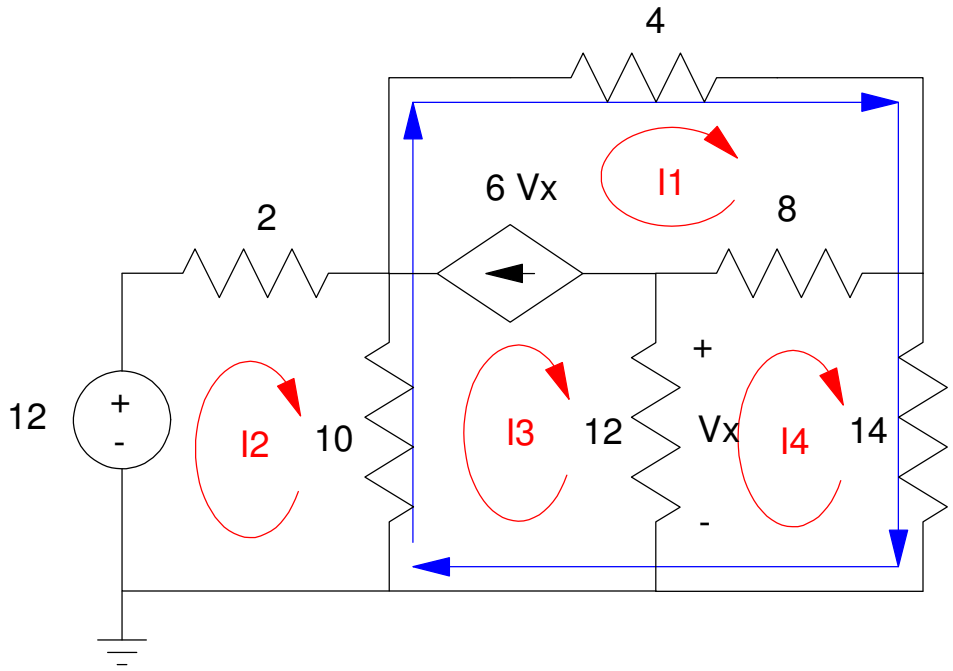
```
A = [0,0,12,-12,-1 ; 1,0,-1,0,-6 ; 0,12,-10,0,0];
A = [A ; -8,0,-12,34,0 ; 4,-10,10,14,0]
```

```
     0.     0.     12.   - 12.   - 1.
     1.     0.    - 1.     0.    - 6.
     0.     12.   - 10.    0.      0.
   - 8.     0.    - 12.    34.     0.
     4.   - 10.     10.    14.     0.
```

```
B = [0;0;12;0;0];
```

```
I = inv(A)*B
```

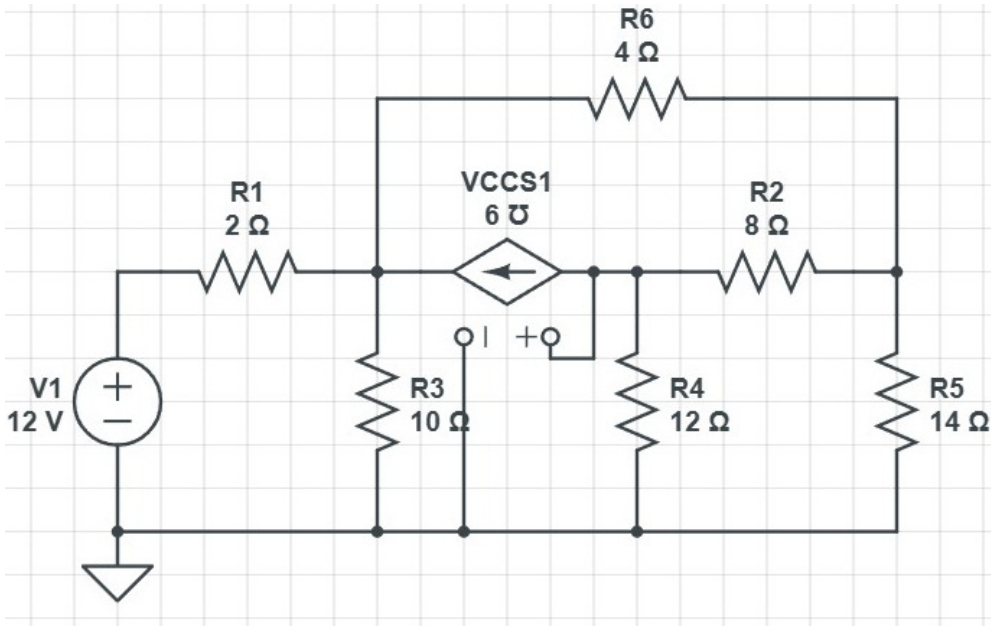
```
I1     1.0219378
I2     1.3210662
I3     0.3852794
I4     0.3764369
Vx     0.1061097
```

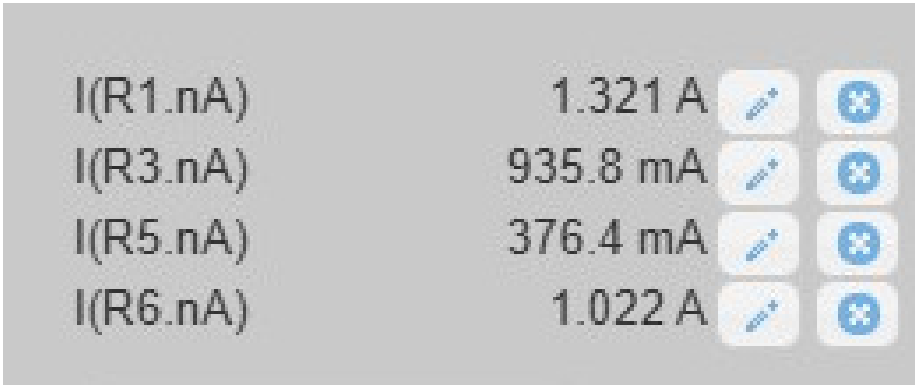


CircuitLab

Results match calculations

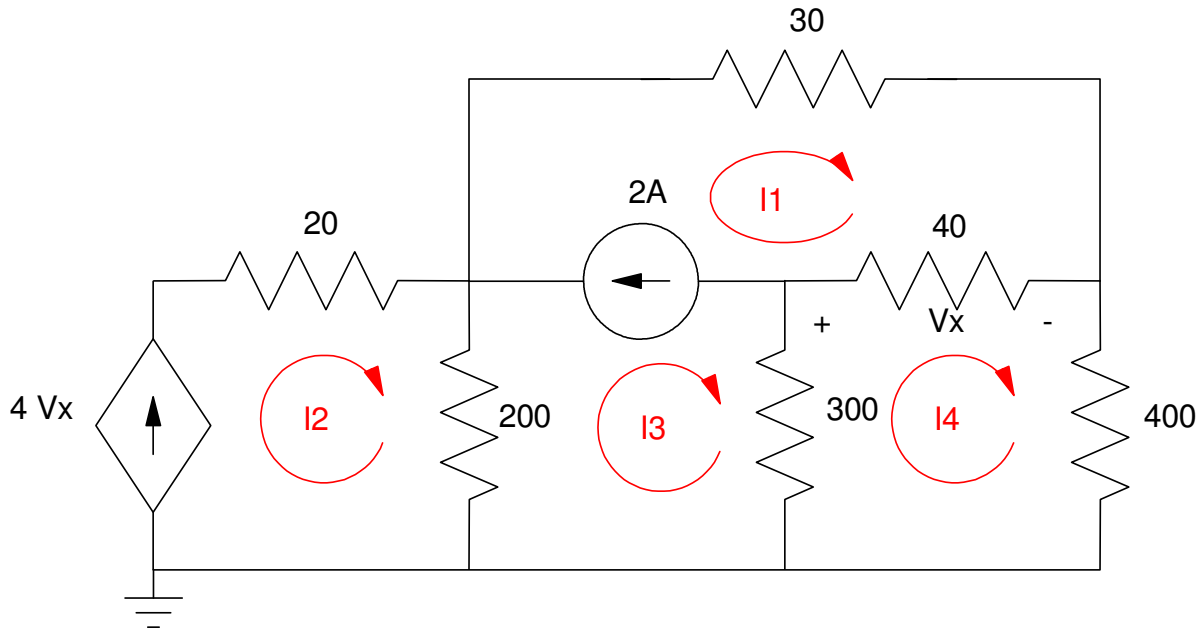
Calculation		Simulation	
I1	**1.0219378**	I(R6)	1.022 A
I2	**1.3210662**	I(R1)	1.321 A
I3	**0.3852794**		
I4	**0.3764369**	I(R5)	376.4 mA
Vx	**0.1061097**		





Practice Problem:

Write 4 equations to solve for the four unknown currents



Summary

With current loops, the goal is to write N loop equations to solve for N unknown currents

Sometimes, you can't write a given loop equation

- Typically when there's a current source
- Voltage is unknown
- You likewise can't sum the voltages to zero

Super-Loops allow you to write the remaining equations

- The sum of the voltages around *any* closed path must be zero
- Often times, there are several super-loop equations you could use.