# Current Loops

## a.k.a. Current Mesh

## ECE 211 Circuits I
## Lecture #7

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

# Current Loops

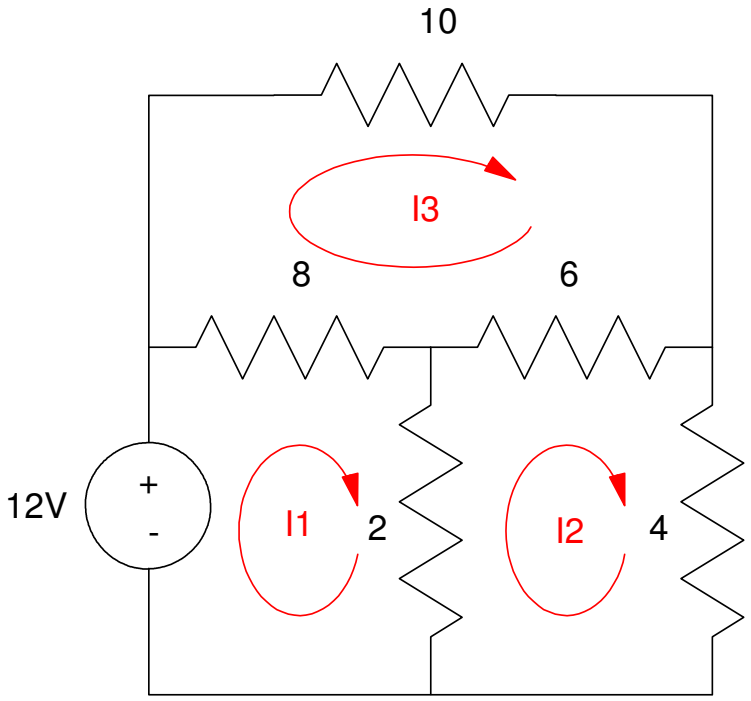
- Current Mesh
- Kirchoff's Current Loops
- Kirchoff's Voltage Law

Voltage-nodes are one way to come up with N equations for N unknowns

- Based upon conservation of current
- The sum of all currents from a node must be zero

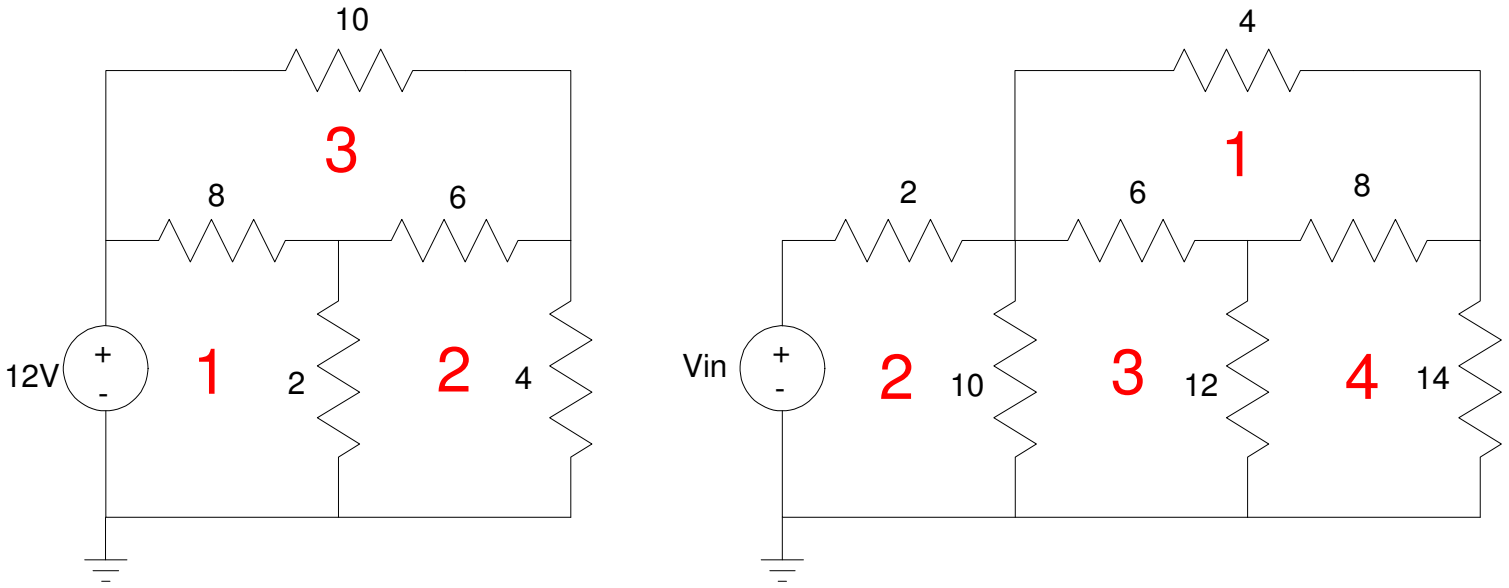
Current-loops are another technique

- Based upon conservation of voltage
- The sum of all voltages around a closed-path must be zero



# Step 1: Count Windows

- # of loop equations needed = # windows
- Not always possible



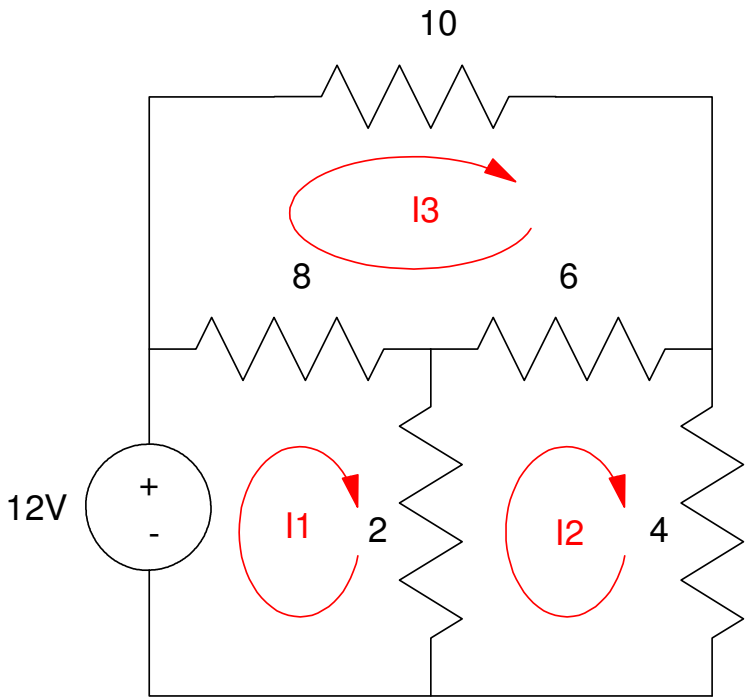

A circuit with 3 windows (left) and 4 windows (right).

# Step 2: Label Currents

- Make each loop go clockwise
- Labels are arbitrary

note: Direction is arbitrary

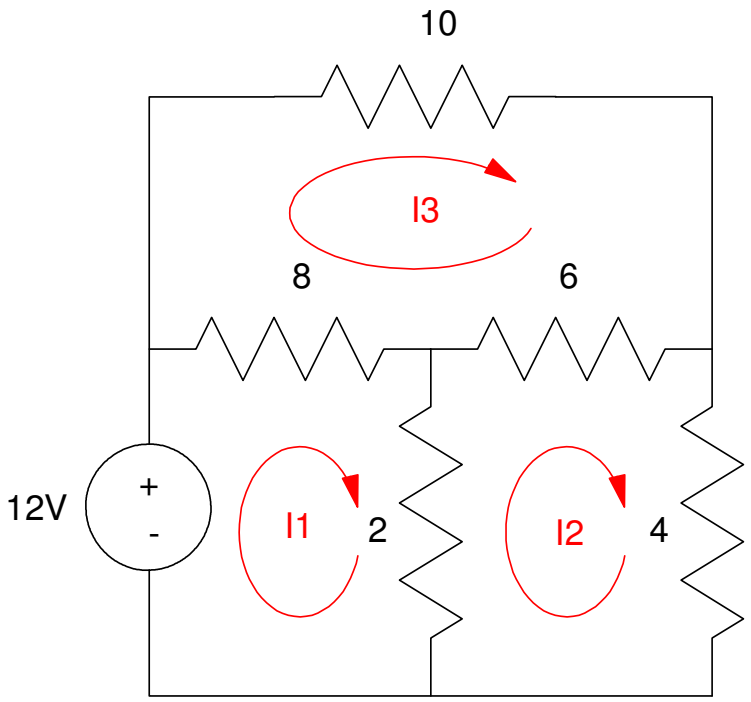
- You tend to get sign errors if you're inconsistent



# Step 3: Write equations

Write N equations for N unknowns

- Sum the voltage around each current loop
- Be consistent:
  - Add if you hit the plus sign first
  - Subtract if you hit the minus sign first
- Make sure you end up where you started
- The voltages must add up to zero




Loop I1:

$$-12 + (I_1 - I_3)8 + (I_1 - I_2)2 = 0$$

Loop I2:

$$(I_2 - I_1)2 + (I_2 - I_3)6 + (I_2)4 = 0$$

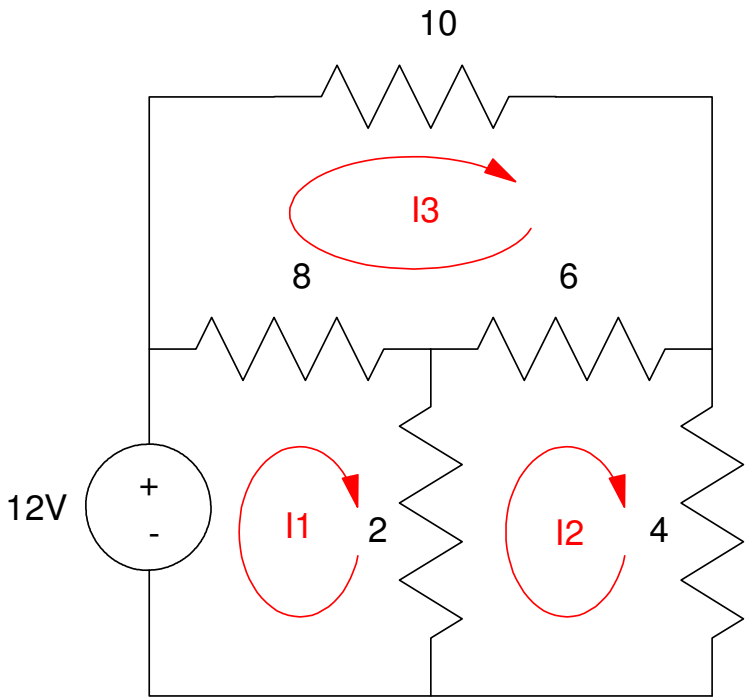
Loop I3:

$$(I_3 - I_1)8 + (I_3)10 + (I_3 - I_2)6 = 0$$

# Checking for Errors

Note the signs of each term

- When you go arount loop I1
  - All of the I1 terms are positive
  - All of the other terms are negative
- When you go around loop I2
  - All of the I2 terms are positive
  - All of the other terms are negative




Loop I1

$$-12 + (I_1 - I_3)8 + (I_1 - I_2)2 = 0$$

Loop I2

$$(I_2 - I_1)2 + (I_2 - I_3)6 + (I_2)4 = 0$$

Loop I3

$$(I_3 - I_1)8 + (I_3)10 + (I_3 - I_2)6 = 0$$

Also note the units match up

volts + volts + volts = 0

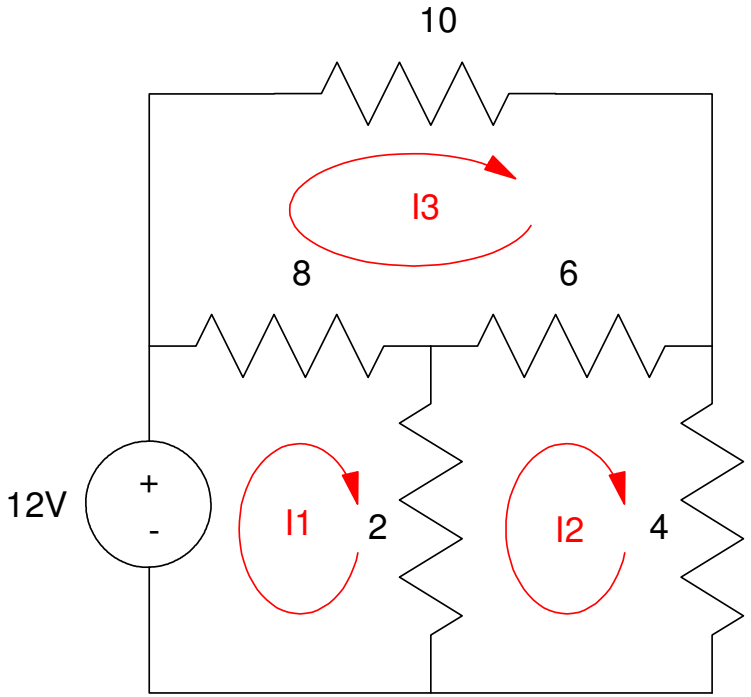
# Step 4: Solve

Group terms

$$10\,I_1 - 2\,I_2 - 8\,I_3 = 12$$
$$-2\,I_1 + 12\,I_2 - 6\,I_3 = 0$$
$$-8\,I_1 - 6\,I_2 + 24\,I_3 = 0$$

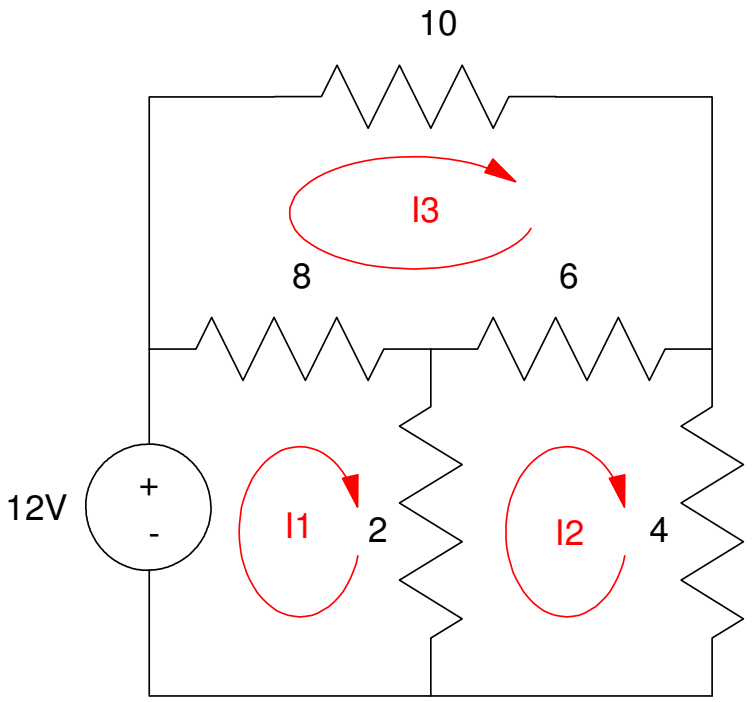
Place in matrix form

$$\begin{bmatrix} 10 & -2 & -8 \\ -2 & 12 & -6 \\ -8 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$



# Solve using Matlab:

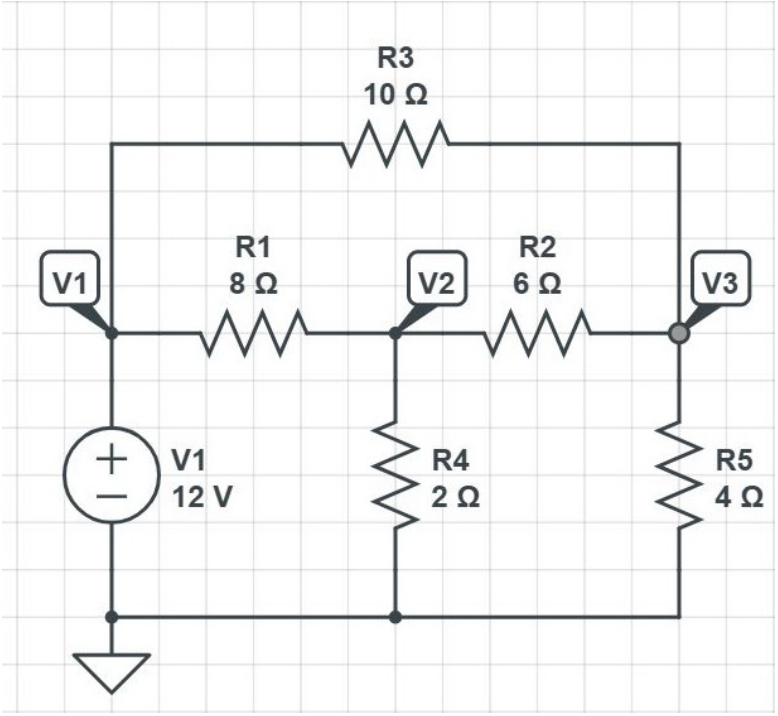
```
A = [10,-2,-8;-2,12,-6;-8,-6,24]

    10.   - 2.    - 8.
  - 2.      12.   - 6.
  - 8.    - 6.      24.

B = [12;0;0]

    12.
    0.
    0.

inv(A)*B

I1     2.0655738
I2     0.7868852
I3     0.8852459
```

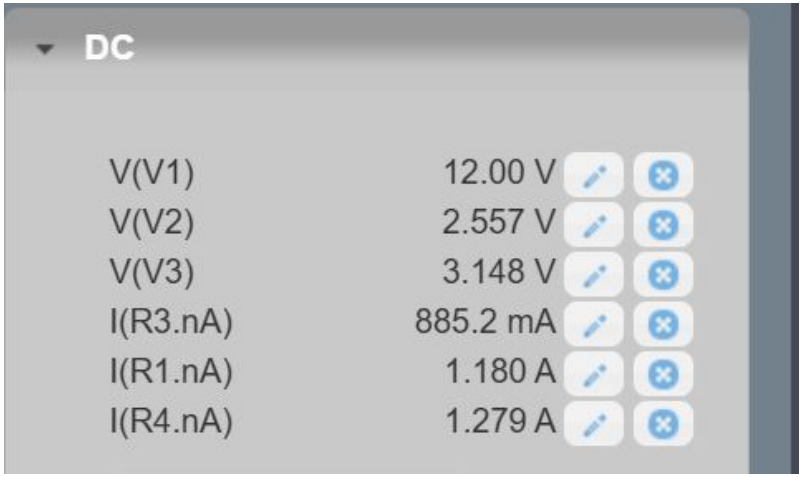


# Verification in CircuitLab

In CircuitLab, you can measure the currents through resistors by

- Measuring the voltage on both sides and computing current from V = IR, or
- Measure the current to a resistor by clicking on the side of the resistor
  - You might need to do some computations
  - I(8 Ohm) = I2 - I3
  - I(2 Ohm) = I1 - I2



# Verification in Lab

In lab, currents are hard to measure

- Fuses blow as soon as someone tries to use an ammeter as a voltmeter
- Also requires inserting an ammeter in series with an element

Easier to measure the voltages and compute the currents

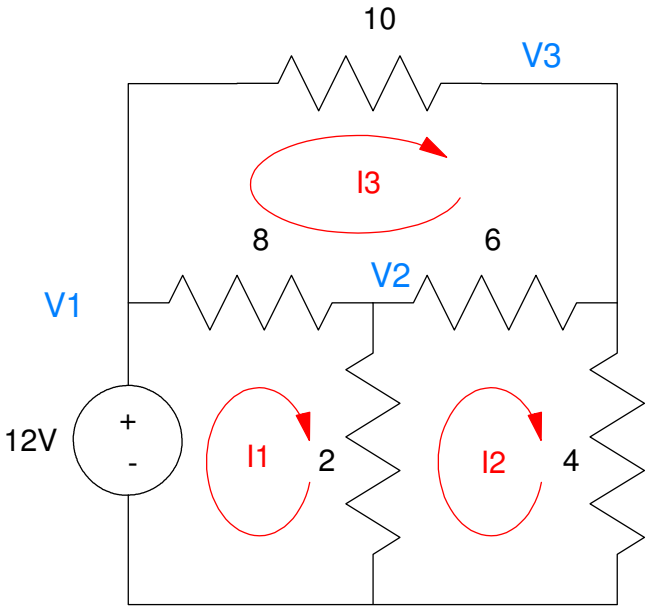
- V1 = 12.00V
- V2 = 2.557V
- V3 = 3.148V

so

$$I_3 = \left(\frac{V_1 - V_3}{10}\right) = 885.2mA$$

$$I_2 = \left(\frac{V_3}{4}\right) = 787.0mA$$

$$I_1 - I_3 = \left(\frac{V_1 - V_2}{8}\right) = 1.1804A$$



Example 2: Find the currents for the 4-window circuit. Assume Vin = +12V.

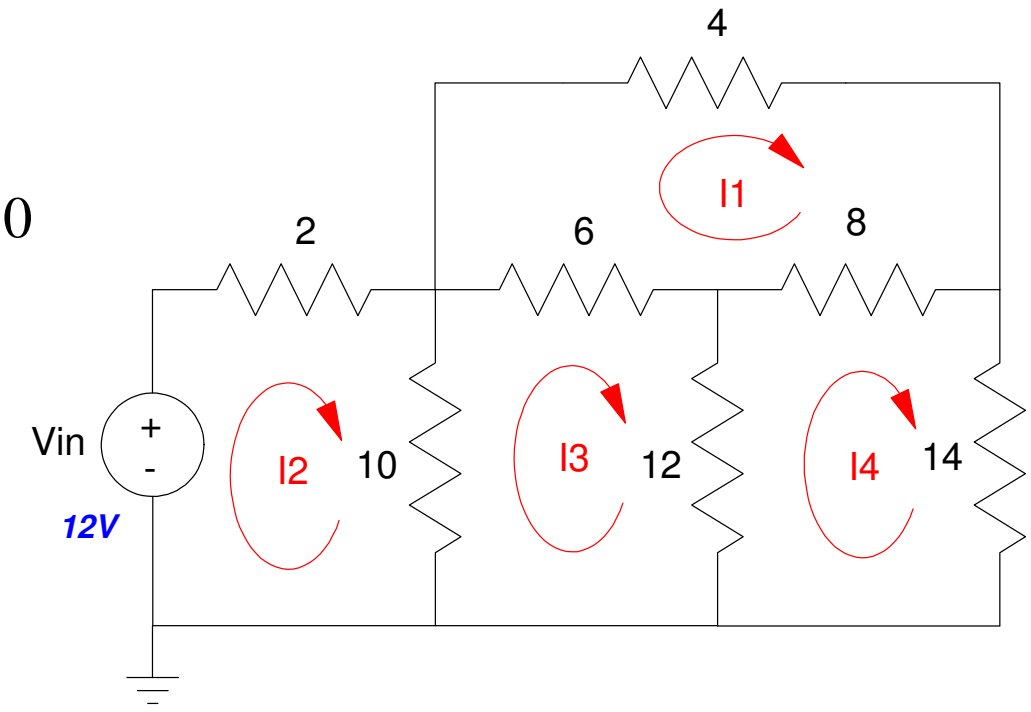
First, write 4 equations for 4 unknowns:

$$4I_1 + 8(I_1 - I_4) + 6(I_1 - I_3) = 0$$

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$10(I_3 - I_2) + 6(I_3 - I_1) + 12(I_3 - I_4) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$




Group terms:

$$18I_1 - 6I_3 - 8I_4 = 0$$

$$12I_2 - 10I_3 = 12$$

$$-6I_1 - 10I_2 + 28I_3 - 12I_4 = 0$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

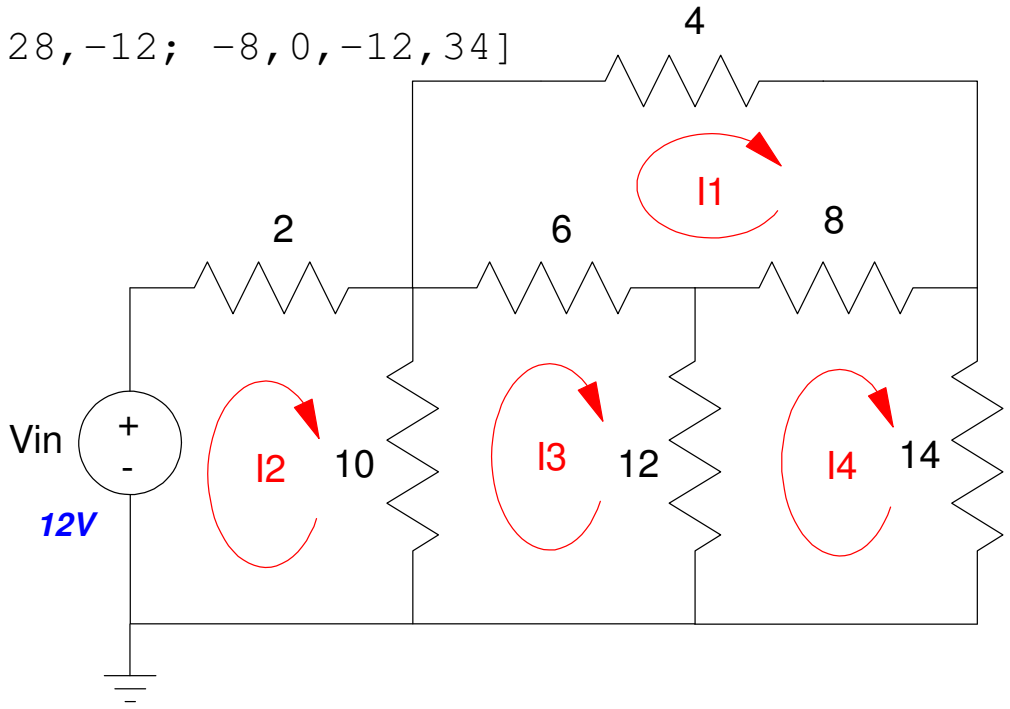
Place in matrix form:

$$\begin{bmatrix} 18 & 0 & -6 & -8 \\ 0 & 12 & -10 & 0 \\ -6 & -10 & 28 & -12 \\ -8 & 0 & -12 & 34 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solving in MATLAB:

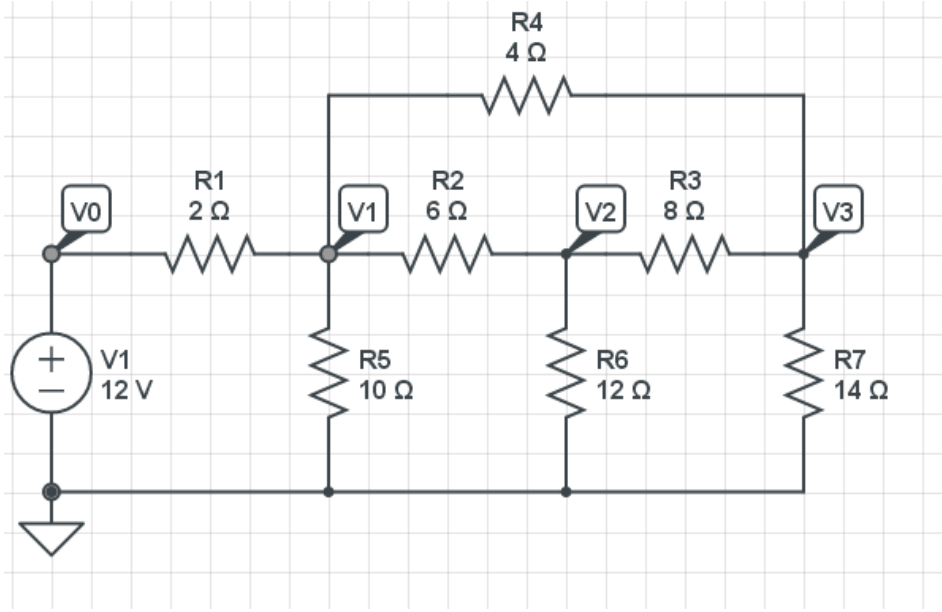
```
A = [18,0,-6,-8; 0,12,-10,0; -6,-10,28,-12; -8,0,-12,34]
B = [0;12;0;0]
inv(A)*B
```

```
I1:     0.5164104
I2:     1.7860913
I3:     0.9433096
I4:     0.4544411
```



## From Matlab

```
I1:     0.5164104
I2:     1.7860913
I3:     0.9433096
I4:     0.4544411
```



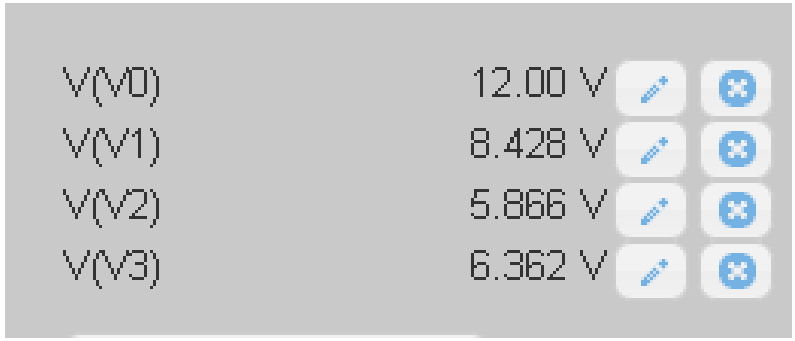

## From CircuitLab

$$I_1 = \left(\frac{V_1 - V_3}{R_4}\right) = 0.5165A$$

$$I_2 = \left(\frac{V_0 - V_1}{R_1}\right) = 1.7860A$$

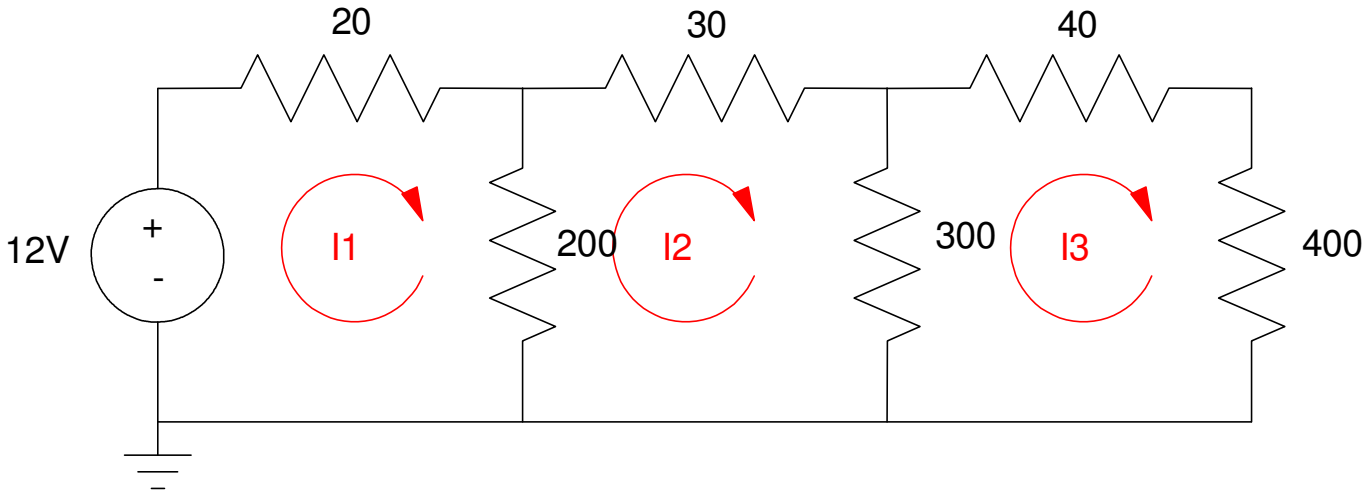
$$I_2 - I_3 = \left(\frac{V_1}{R_5}\right) = 0.8428A$$

$$I_4 = \left(\frac{V_3}{R_7}\right) = 0.4544A$$



# Practice Problem

Write 3 equations to solve for the 3 unknown currents



# Current Loops with Voltage Sources

Not a problem

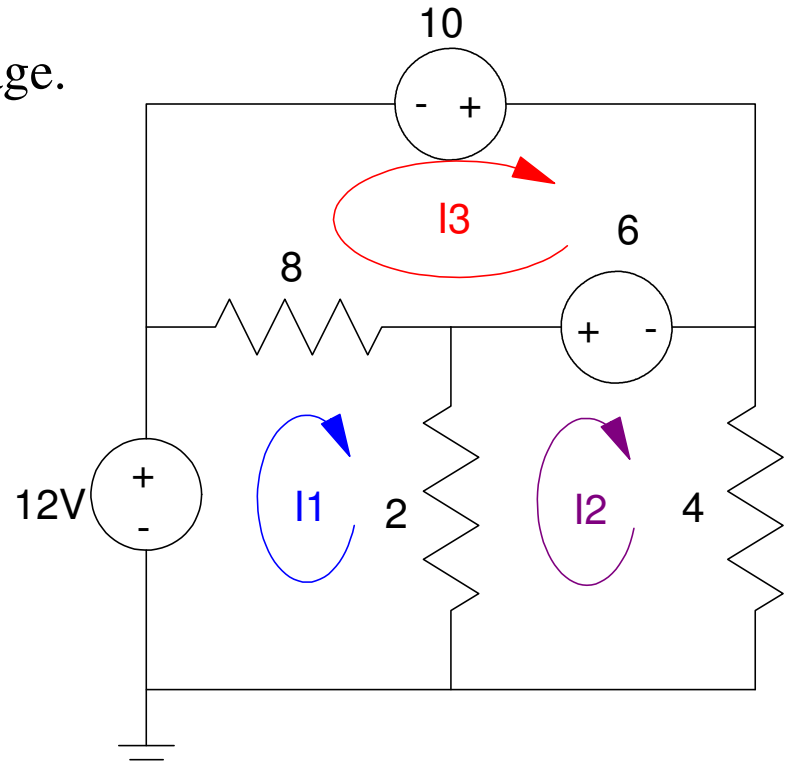
- If you encounter the + sign first, add the voltage.
- If you encounter the - sign first, subtract the voltage.

Example: Write 3 equations for 3 unknowns:

$$-12 + 8(I_1 - I_3) + 2(I_1 - I_2) = 0$$

$$2(I_2 - I_1) + 6 + 4(I_2) = 0$$

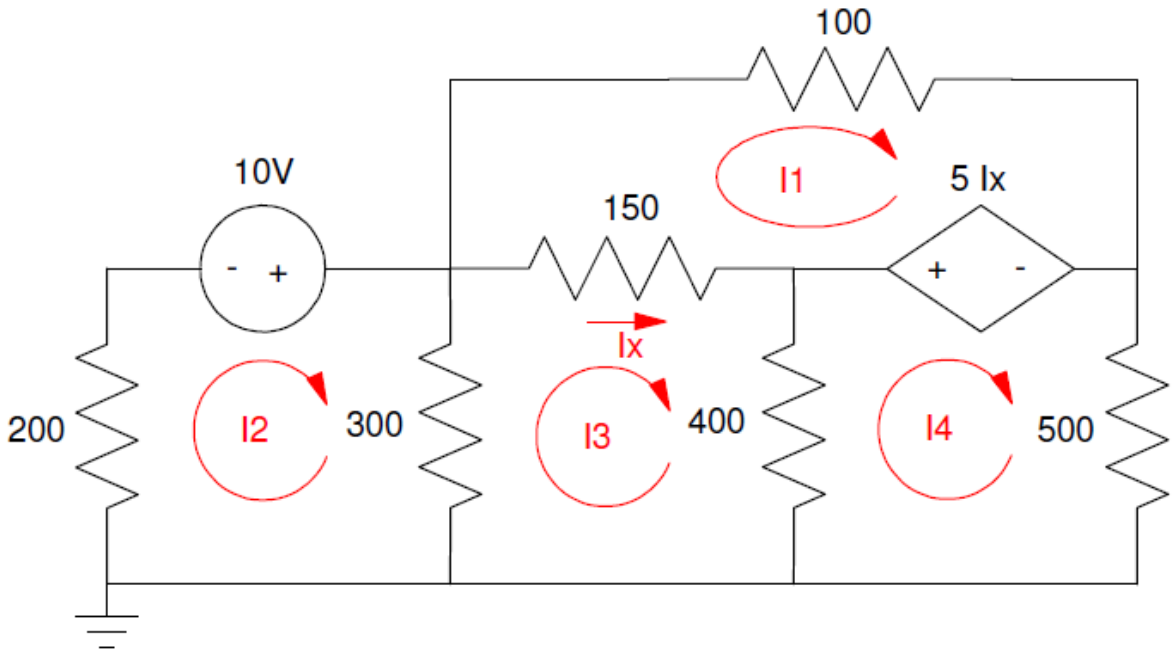
$$-10 + 6 + 8(I_3 - I_1) = 0$$



# Practice Problem

Write 5 equations to solve for the 5 unknowns

- I1, I2, I3, I4, Ix



# Summary

Current Loop Equations are based upon the conservation of voltage

- The sum of all voltages around a closed-path must be zero

To solve for N unknown currents, you write N current loop equations

You can then solve N equations for N unknowns using techniques from Math 129