
Voltage Nodes

ECE 211 Circuits I

Lecture #5

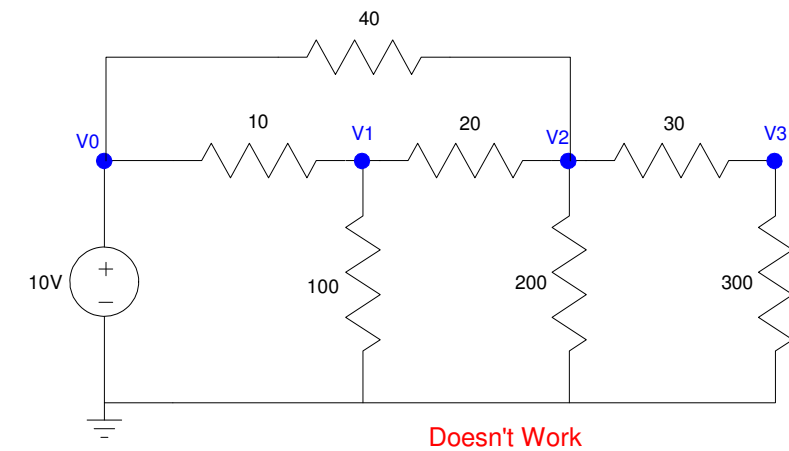
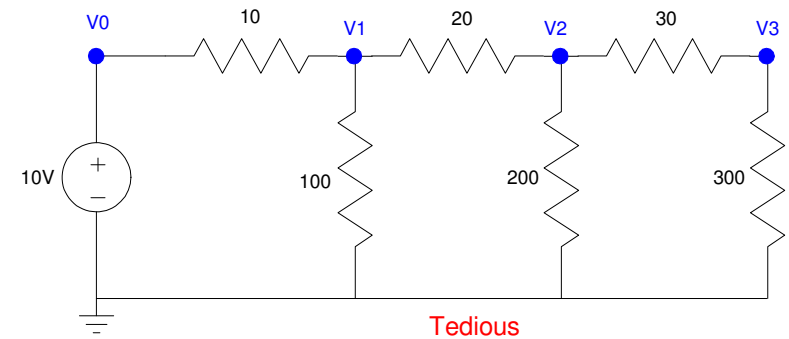
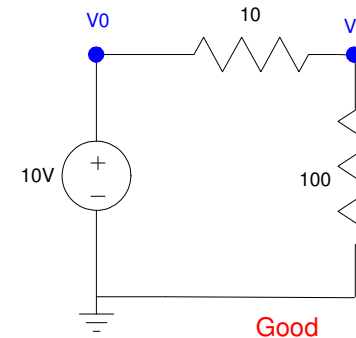
Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Background

In the previous lecture, voltage division was used to find the voltages in a circuit

- For some circuits, this works really well
- For other, it's really tedious
- For still others, voltage division just doesn't work

We need a better tool



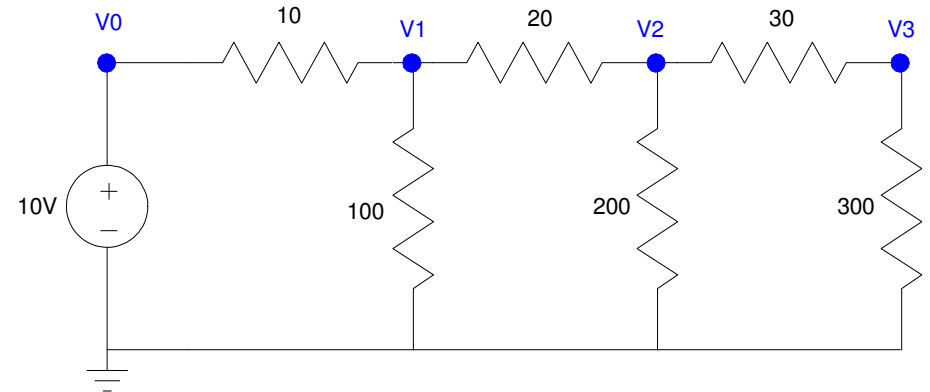
Main Tools for Circuit Analysis

To solve a circuit, we primarily use three tools:

- Voltage Nodes (Kirchoff's current law)
- Current Loops (Kirchoff's voltage law)
- Thevenin Equivalents (stay tuned for these...)

The goal of the first two is to obtain N equations so solve for N unknowns.

- Topic for Math 129: Linear Algebra
- How to solve N equations for N unknowns



$$a_0 V_0 + a_1 V_1 + a_2 V_2 + a_3 V_3 = k_1$$

$$b_0 V_0 + b_1 V_1 + b_2 V_2 + b_3 V_3 = k_2$$

$$c_0 V_0 + c_1 V_1 + c_2 V_2 + c_3 V_3 = k_3$$

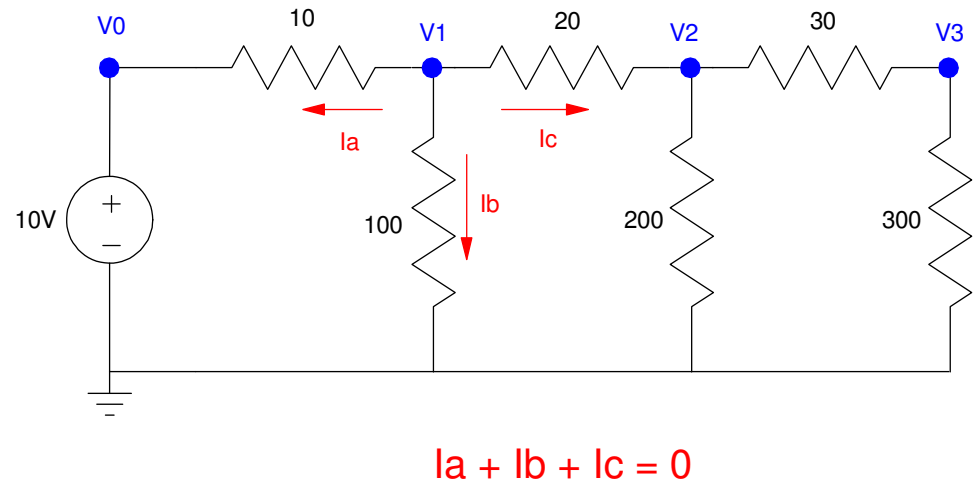
$$d_0 V_0 + d_1 V_1 + d_2 V_2 + d_3 V_3 = k_4$$

Voltage Nodes

- The voltages at each node are such that the current from each node sums to zero
- This holds at every node

Idea:

- I know how to solve N equations for N unknowns
 - Math 129 Linear Algebra
- If I can reduce the problem to this form, I can solve the problem



Note:

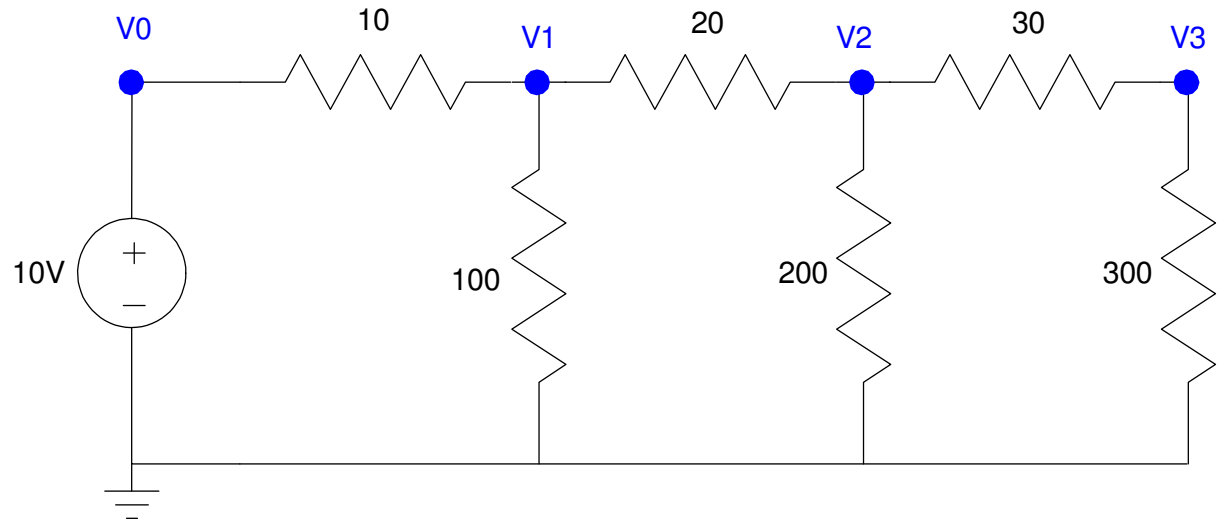
- This is a common theme throughout ECE
- What's important is getting the equations right
 - Matlab can solve 50 equations for 50 unknowns just as easily as 2 equations for 2 unknowns
 - Focus on getting the equations right.

Example #1

- Find the voltages V_0 , V_1 , V_2 , and V_3
- Solved previously
 - CircuitLab
 - Voltage division

Result:

- $V_0 = 10.000\text{V}$
- $V_1 = 8.553\text{V}$
- $V_2 = 7.369\text{V}$
- $V_3 = 6.699\text{V}$



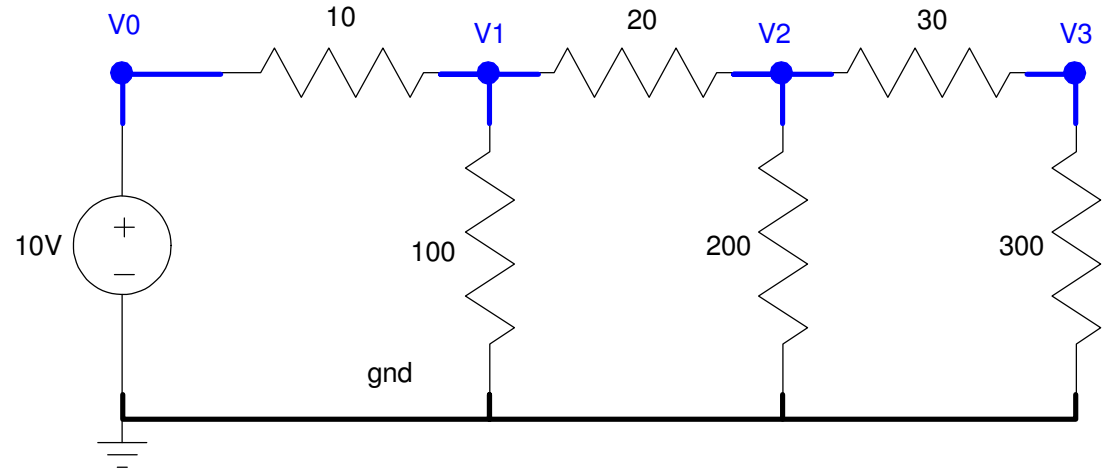
Now solve using Voltage Nodes

Step #1

- Define circuit ground
- Label each voltage node
- Count the number of voltage nodes

The number of voltage nodes is the number of equations you need

- 4 voltage nodes
- We need 4 equations



Step #2

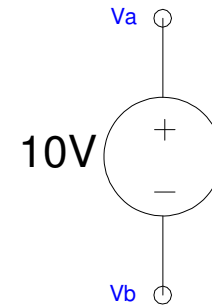
Start writing equations

Start with the easy equations

- Voltage sources

These define the voltage across two nodes

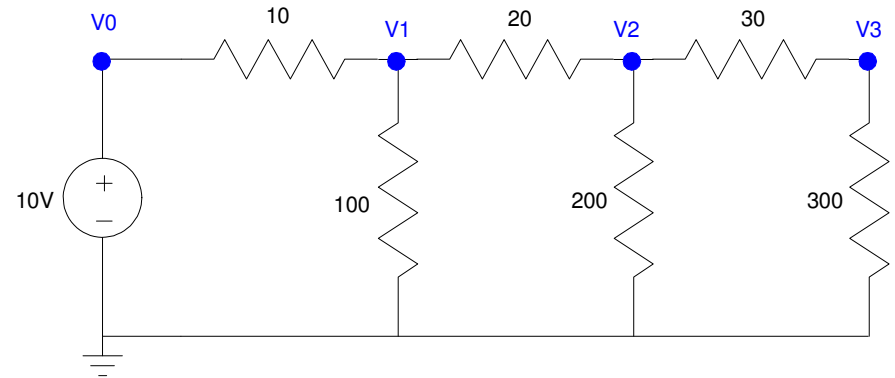
$$V_a - V_b = 10V$$



For our circuit

- Equation #1 of 4

$$V_0 = 10V$$



Step #3

Write the voltage node equations

- Need 3 more equations
- At each voltage node, the currents flowing *from* the node must sum to zero

Node V1:

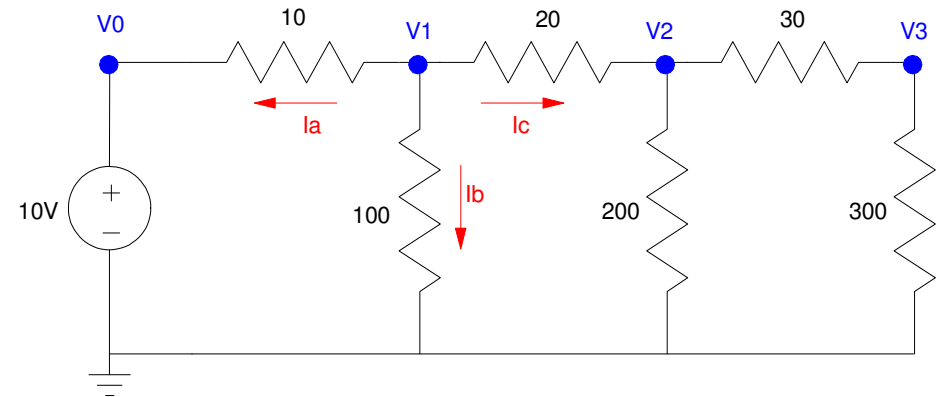
- Equation #2 or 4

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_1 - V_2}{10} \right) + \left(\frac{V_1}{100} \right) + \left(\frac{V_1 - V_2}{20} \right) = 0$$

Note:

- All the V1 terms are positive
- All the other terms are negative
- All units match (amps + amps + amps)
- Quick way to check for sign errors



Node V2:

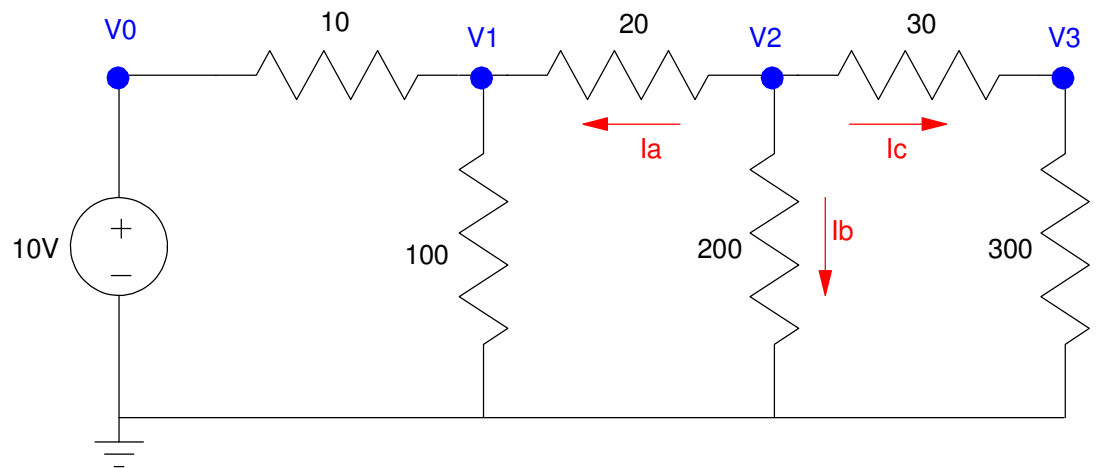
- Equation #3 of 4

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Note:

- All the V2 terms are positive
- All the other terms are negative
- All units match (amps + amps + amps)



Node V3

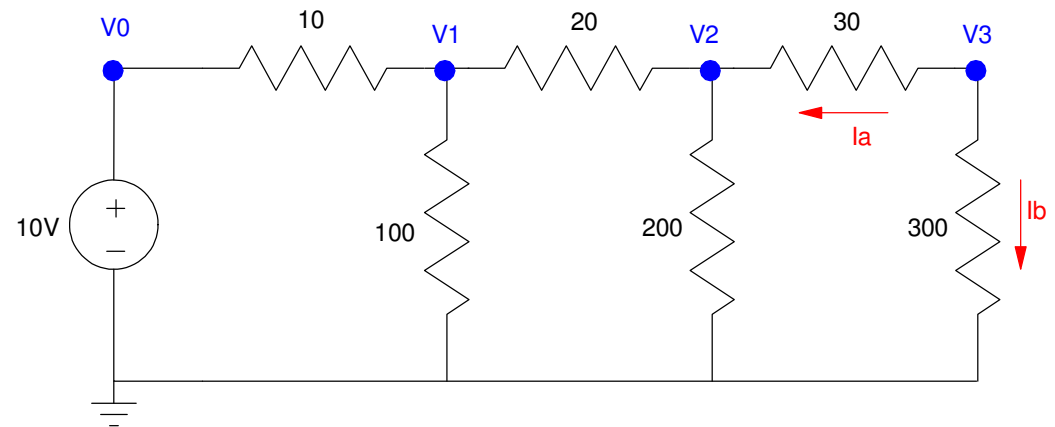
- Equation #4 of 4

$$I_a + I_b = 0$$

$$\left(\frac{V_3 - V_2}{30} \right) + \left(\frac{V_3}{300} \right) = 0$$

Note:

- All the V3 terms are positive
- All the other terms are negative
- All units match (amps + amps)



Net Result

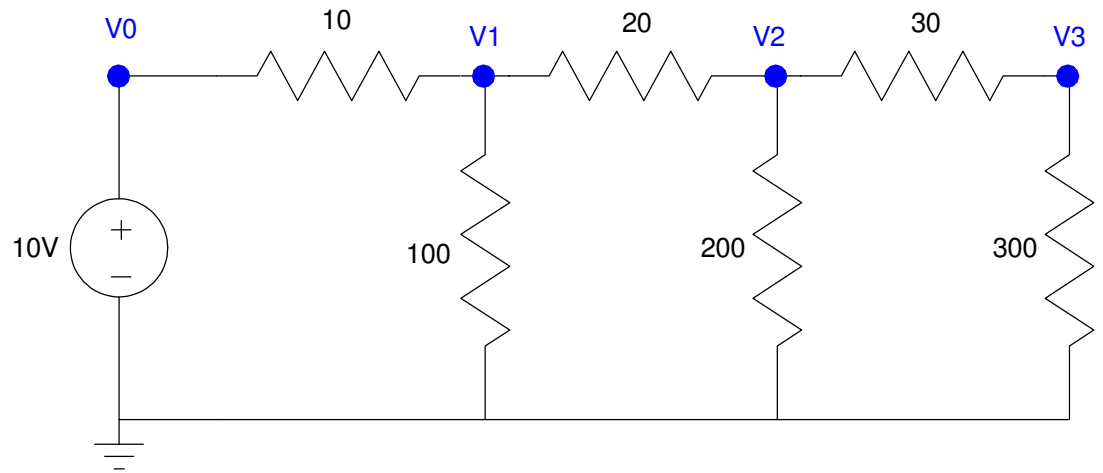
4 equations for 4 unknowns

$$V_0 = 10$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



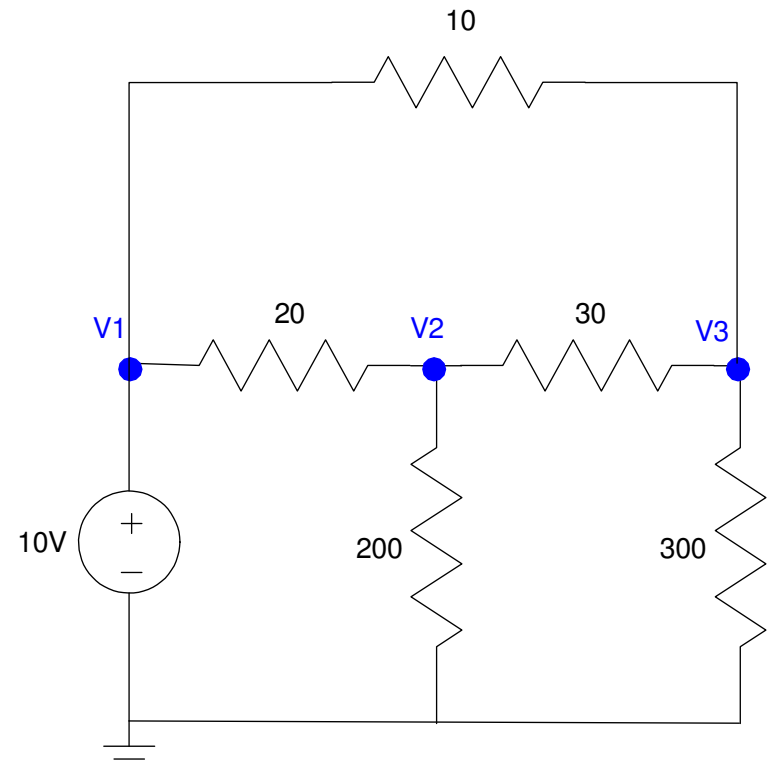
Note:

- This is typically where you stop on tests
- Solving requires Matlab (homework only)

Practice Problem

Write the voltage node equations for the following circuit

- 3 equations for the 3 unknown voltages



Step #4: Solve

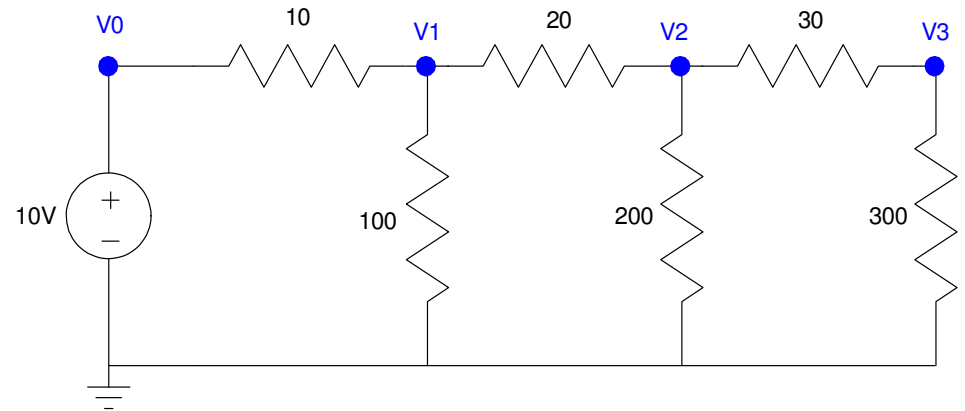
Previous equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



Group terms

$$V_0 = 10$$

$$\left(\frac{-1}{10}\right) V_0 + \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{20}\right) V_1 + \left(\frac{-1}{20}\right) V_2 = 0$$

$$\left(\frac{-1}{20}\right) V_1 + \left(\frac{1}{20} + \frac{1}{200} + \frac{1}{30}\right) V_2 + \left(\frac{-1}{30}\right) V_3 = 0$$

$$\left(\frac{-1}{30}\right) V_2 + \left(\frac{1}{30} + \frac{1}{300}\right) V_3 = 0$$

Place in matrix form

$$V_0 = 10$$

$$\left(\frac{-1}{10}\right)V_0 + \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{20}\right)V_1 + \left(\frac{-1}{20}\right)V_2 = 0$$

$$\left(\frac{-1}{20}\right)V_1 + \left(\frac{1}{20} + \frac{1}{200} + \frac{1}{30}\right)V_2 + \left(\frac{-1}{30}\right)V_3 = 0$$

$$\left(\frac{-1}{30}\right)V_2 + \left(\frac{1}{30} + \frac{1}{300}\right)V_3 = 0$$

becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{20}\right) & \left(\frac{-1}{20}\right) & 0 \\ 0 & \left(\frac{-1}{20}\right) & \left(\frac{1}{20} + \frac{1}{200} + \frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30} + \frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
a1 = [1,0,0,0];  
a2 = [-1/10, 1/10+1/100+1/20, -1/20, 0];  
a3 = [0, -1/20, 1/20+1/200+1/30, -1/30];  
a4 = [0, 0, -1/30, 1/30+1/300];  
A = [a1;a2;a3;a4]
```

```
    1.0000         0         0         0  
   -0.1000    0.1600   -0.0500         0  
         0   -0.0500    0.0883   -0.0333  
         0         0   -0.0333    0.0367
```

```
B = [10 ; 0 ; 0 ; 0]
```

```
10  
0  
0  
0
```

```
V = inv(A) * B
```

```
V0    10.0000  
V1     8.5529  
V2     7.3694  
V3     6.6994
```

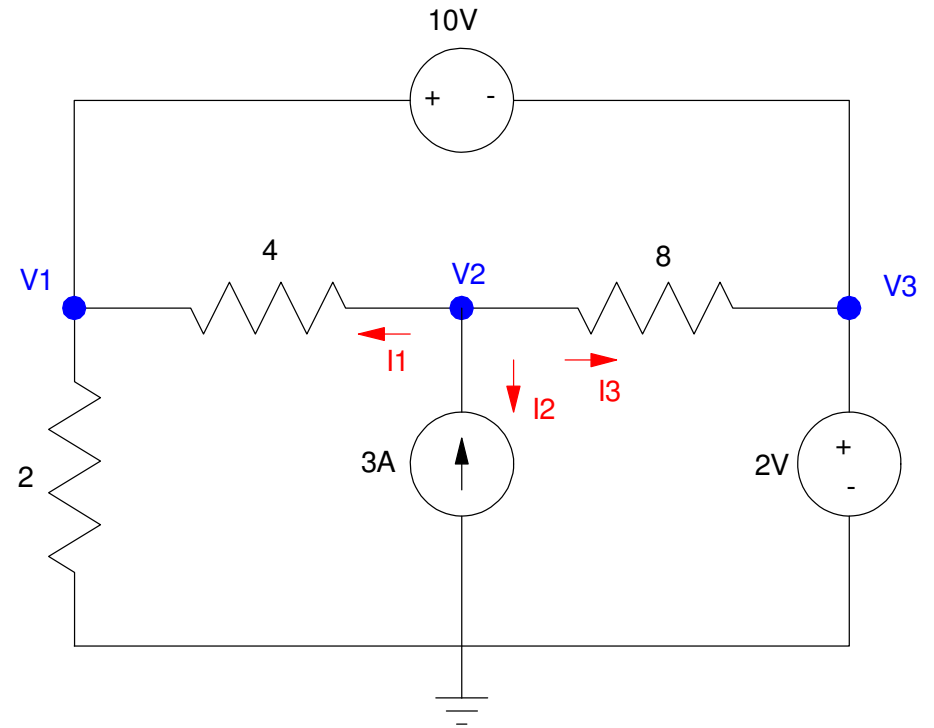
Current Sources

Current sources aren't an issue with voltage nodes

- They just define the current flowing

Example: Find $\{V1, V2, V3\}$

- 3 unknowns voltages
- Write 3 equations for 3 unknowns



Start with the easy ones

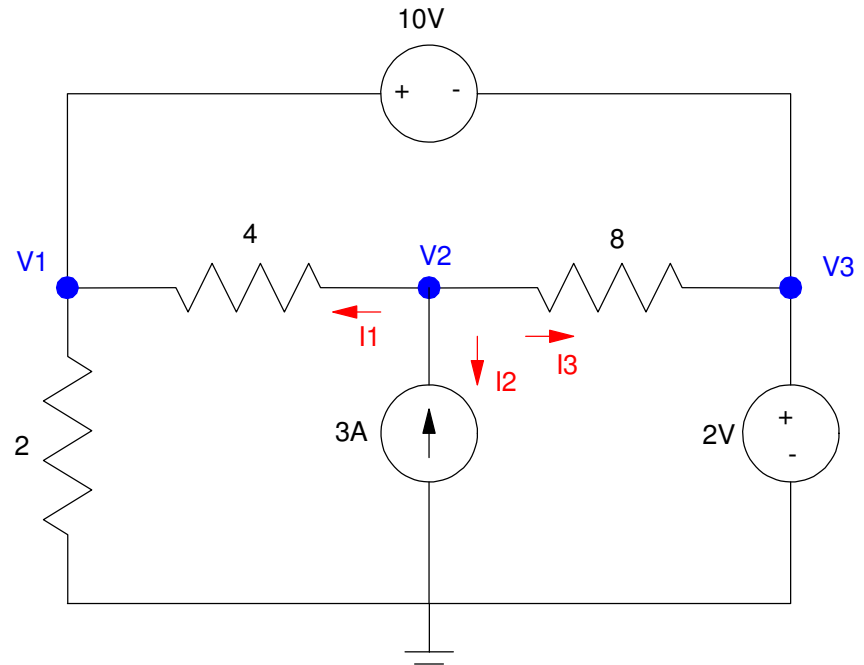
$$V_3 - 0 = 2$$

$$V_1 - V_3 = 10$$

Now we need one more equation.
Write the voltage node equation at V2

$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_2 - V_1}{4}\right) + (-3) + \left(\frac{V_2 - V_3}{8}\right) = 0$$



Group terms

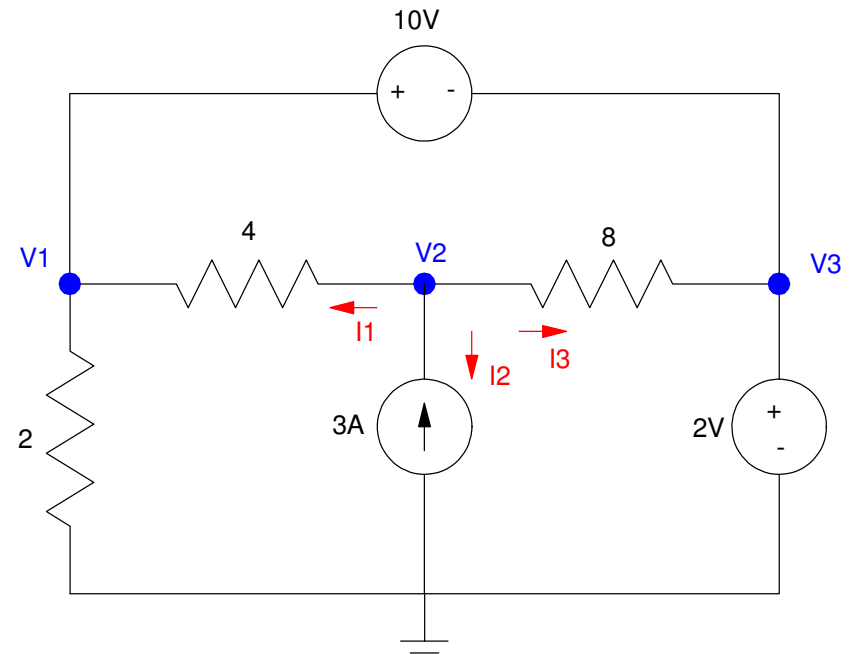
$$V_3 = 2$$

$$V_1 - V_3 = 10$$

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 3$$

Place in matrix form:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ \left(\frac{-1}{4}\right) & \left(\frac{1}{4} + \frac{1}{8}\right) & \left(\frac{-1}{8}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix}$$



Solving in Matlab:

```
A = [0,0,1 ; 1,0,-1 ; -0.25,0.375,-0.125]
```

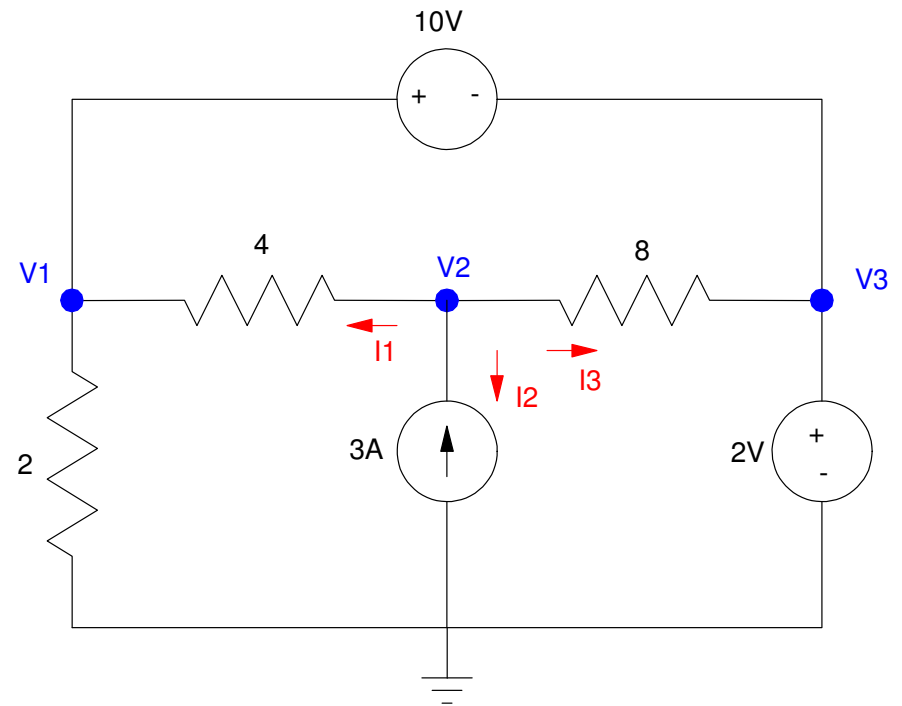
```
      0      0      1.0000
1.0000      0     -1.0000
-0.2500  0.3750  -0.1250
```

```
B = [2;10;3]
```

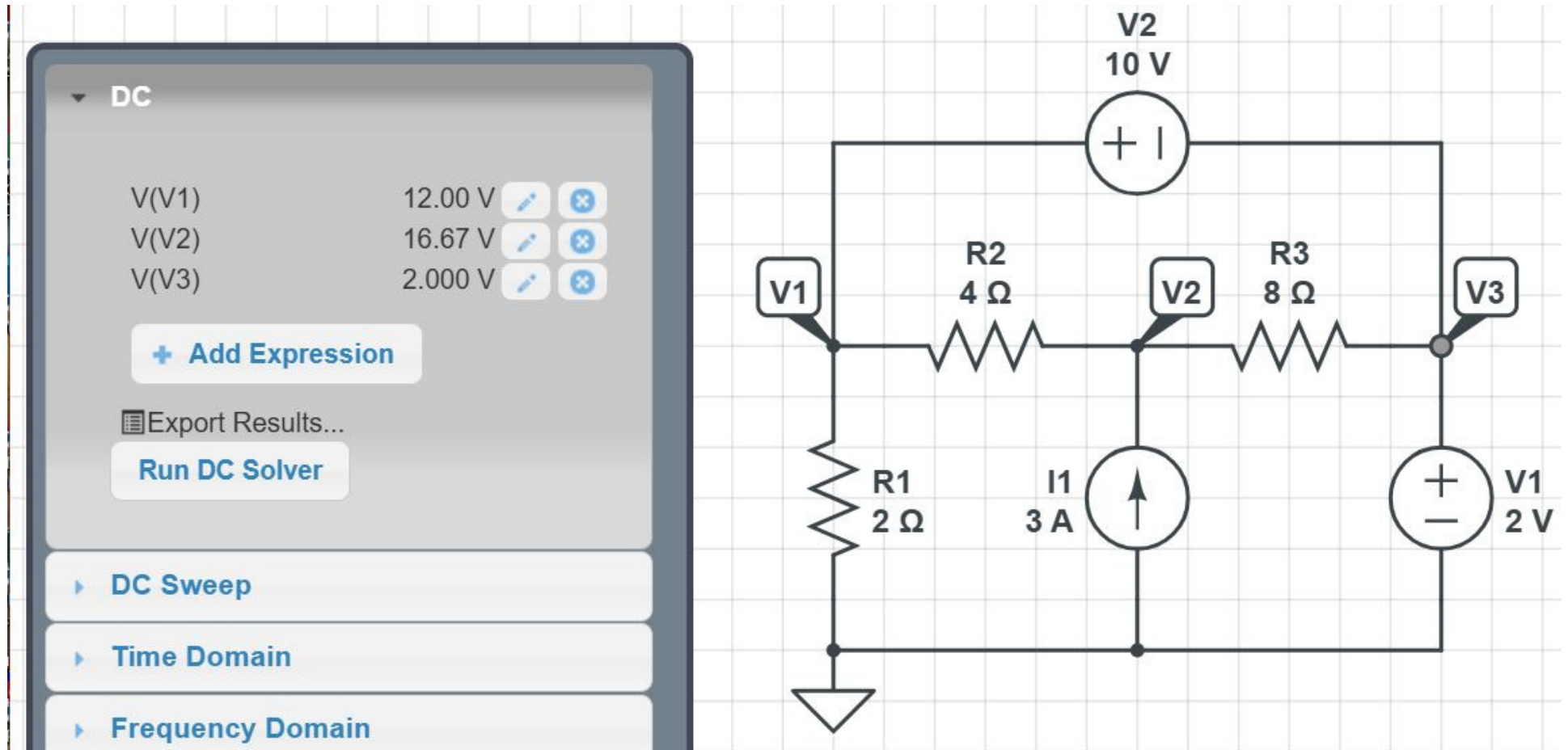
```
      2
     10
      3
```

```
V = inv(A)*B
```

```
V1    12.0000
V2    16.6667
V3     2.0000
```



Check with CircuitLab



Dependent Sources

Dependent sources are

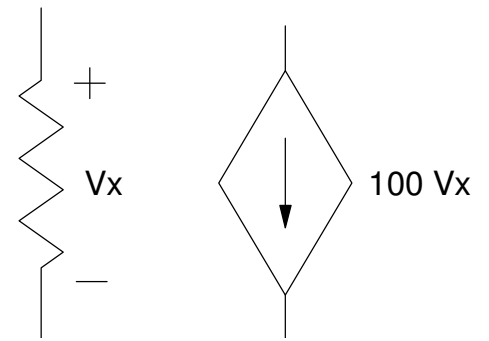
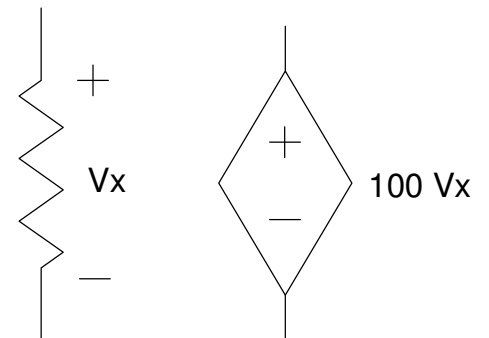
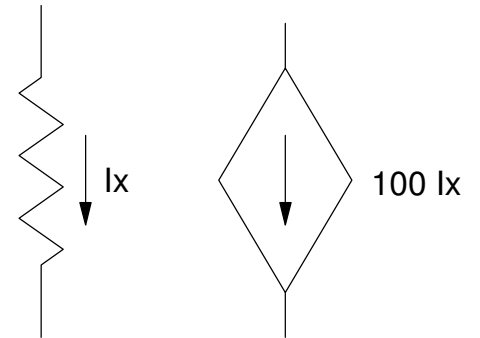
- Sources (voltage or current)
- Which depend upon another variable

Example:

- Current-controlled current source
 - BJT transistor
- Voltage-controlled voltage source
 - Amplifier
- Voltage-controlled current source
 - MOSFET

Devices which do this are covered in Electronics

- For this class, just treat these as sources



Voltage Node Equations with Dependent Sources

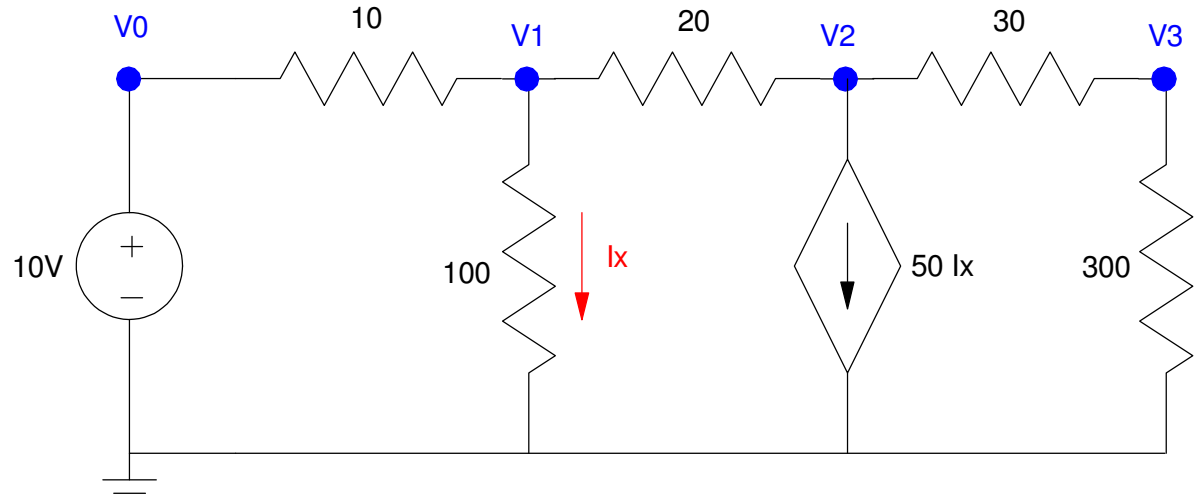
Each dependent source adds one more equation

- Define V_x or I_x

Example: Write the voltage node equations

- 4 voltages
- 1 dependent source
- = 5 unknowns

Write 5 equations for 5 unknowns



5 equations for 5 unknowns

10V Source:

$$V_0 = 10$$

I_x :

$$I_x = \left(\frac{V_1}{100} \right)$$

Node V1:

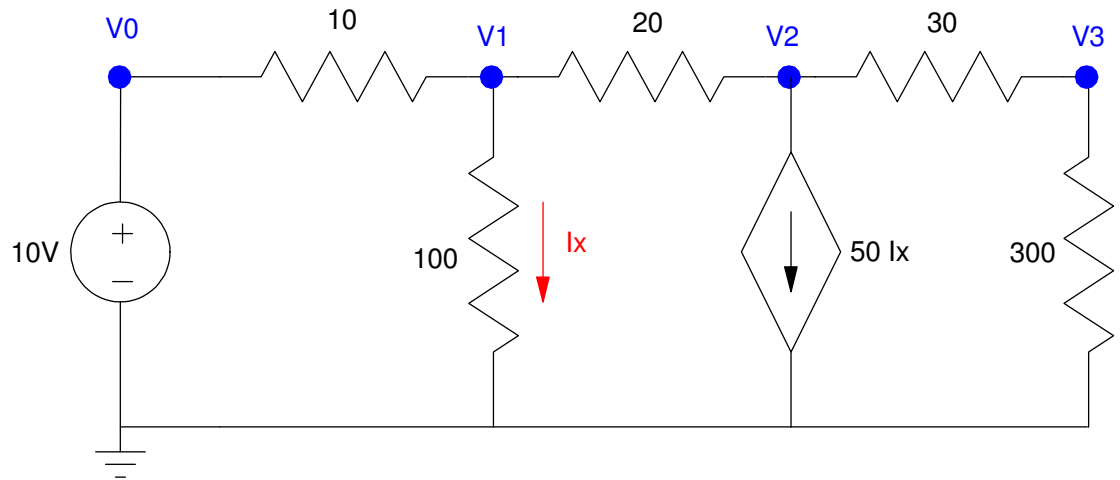
$$\left(\frac{V_1 - V_0}{10} \right) + \left(\frac{V_1}{100} \right) + \left(\frac{V_1 - V_2}{20} \right) = 0$$

Node V2:

$$\left(\frac{V_2 - V_1}{20} \right) + 50I_x + \left(\frac{V_2 - V_3}{30} \right) = 0$$

Node V3:

$$\left(\frac{V_3 - V_2}{30} \right) + \left(\frac{V_3}{300} \right) = 0$$



Another Variation

Independent Source

$$V_0 = 10$$

V_x

$$V_x = V_1 - V_2$$

Node V1

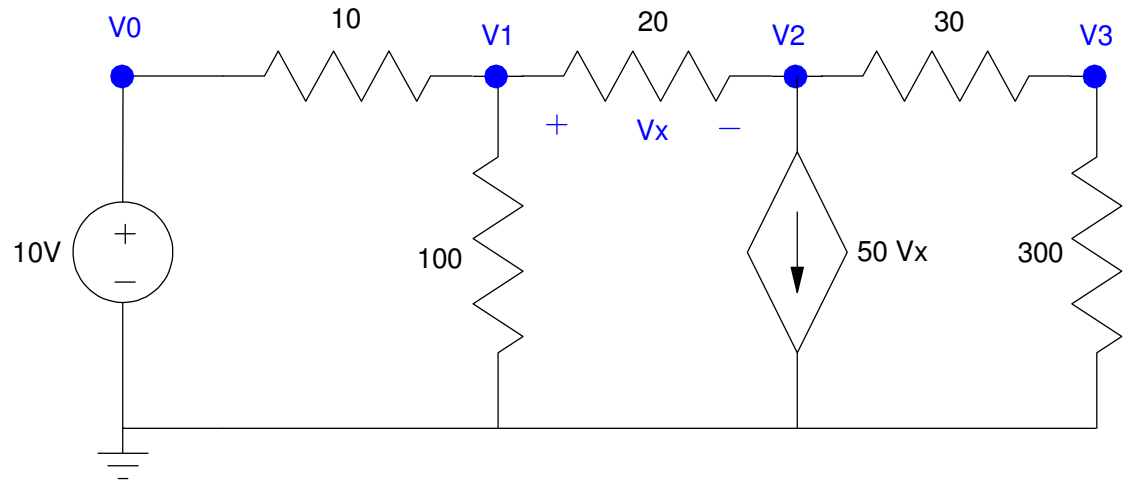
$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

Node V2

$$\left(\frac{V_2 - V_1}{20}\right) + 50V_x + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Node V3

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



Summary

Voltage Nodes is one way to compute the voltages in a circuit

- Define N voltage nodes
- Write N equations for N unknowns
- Solve using Matlab

This is based on conservation of current

- The sum of the currents from a node must be zero
- $I_a + I_b + I_c = 0$

