Active Filters

Background

Up to now, we've been analyzing RLC circuits with constant inputs and with sinusoidal inputs at a known, fixed frequency. For such circuits, phasor analysis works well. Here's a problem though: what do you do if the frequency of the input varies?

This is where filters and Bode plots come into play.

- A filter is any circuit whose behavior changes with frequency. Essentially, that means any circuit which has inductors and/or capacitors.
- An active filter is a filter which uses one or more op-amps.
- A Bode plot is a graph which shows the gain of a given filter as the frequency varies.

Bode plots are a convenient way of describing how a filter behaves as the frequency changes. For example, the Bode plot for three different types of filters is shown in Figure 1.

- A low-pass filter is a filter which passes low frequencies and blocks or attenuated high frequencies.
- A band-pass filter is a filter which passes frequencies over a small band and rejects everything else
- A high-pass filter is a filter which blocks low frequencies and passes high frequencies.

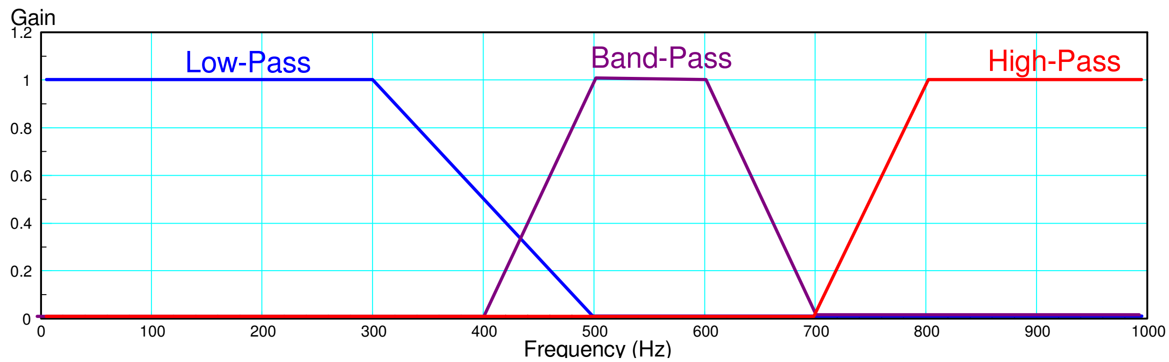

Figure 1: Examples of a Bode plot for a low-pass, band-pass, and high-pass filter

In this lecture, we'll look at analysis and design of four different types of filters:

- Bass boost.
- 1st-order active low-pass filter, and
- 2nd-order active low-pass filter

Many other types of filters exist. These are usually covered in Circuits II and Electronics.

Bass-Boost Filter

Assume you have the filter given in Figure 2. Determine the amplitude of Y assuming that

- X is a 1V peak sine wave,
- The frequency of X is in the range of 1 to 1000 rad/sec (0.15 to 159Hz).

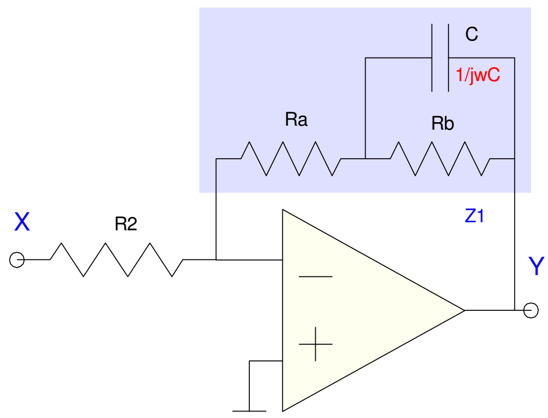

Figure 2: Bass-Boost filter

With analysis, you have a given filter and you want to know how its gain varies with frequency. With design, you find values of resistors and capacitors to give you some desired frequency response. Let's start with analysis.

The circuit of Figure 2 is that of an inverting amplifier. From before, the gain is

$$gain = -\left(\frac{Z_1}{R_2}\right)$$

Z1 includes a capacitor. Hence, its impedance varies with frequency.

- At low frequency, the capacitor has a high impedance. Z1 = Ra + Rb
- At high frequency, the capacitor has a low impedance. Z1 = Ra

This creates a bass-boost circuit:

$$gain = \begin{cases} -\left(\frac{R_a+R_b}{R_2}\right) & \omega \text{ small} \\ -\left(\frac{R_a}{R_2}\right) & \omega \text{ large} \end{cases}$$

Start with computing the impedance of Z1.

$$Z_1 = R_a + R_b \,||\, \frac{1}{j\omega C}$$

$$Z_1 = R_a + \left(\frac{R_b}{1+j\omega R_b C}\right)$$

The overall gain is thus

$$G = -\left(\frac{R_a+R_b+j\omega C R_a R_b}{R_2(1+j\omega R_b C)}\right)$$

If you know the values of R's and C's, you can compute the gain as a function of frequency. In Matlab, assuming all resistors are 100k and C is 0.1uF. the gain becomes

```
w = logspace(0,4,200)';
Ra = 1e5;
Rb = 1e5;
R2 = 1e5;
C = 0.1e-6;
num = Ra+Rb+j*w*C*Ra*Rb;
den = R2*(1+j*w*C*Rb);
G = -num ./ den;
semilogx(w,abs(G))
xlabel('rad/sec')
ylabel('Gain')
```

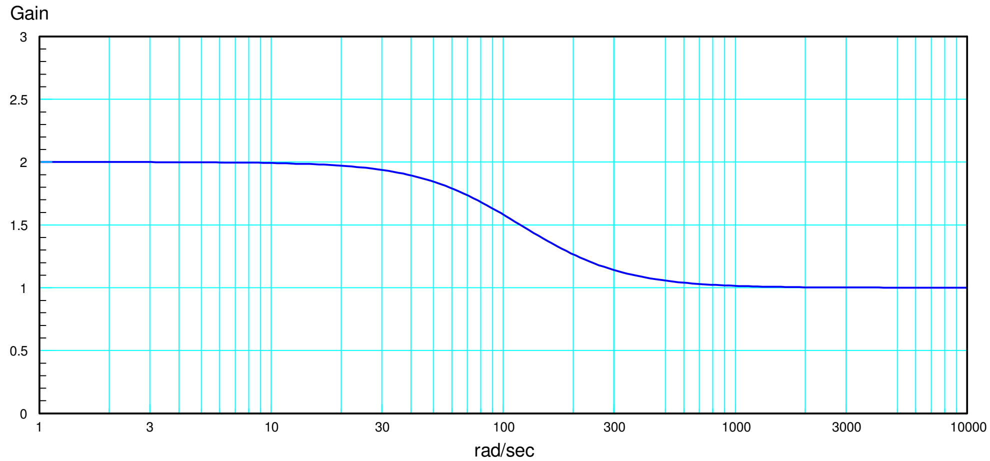

Figure 3: Bode plot for the bass-boost circuit with R's = 100k and C = 0.1uF

From the Bode plot, you can see how this filter behaves:

- Frequencies below 30 rad/sec are amplified by 2x
- Frequencies above 1000 rad/sec are unchanged (gain is one)

This circuit boosts the bass notes - hence the name *bass boost*.

One way to check your analysis is to throw the circuit into CircuitLab. Since the frequency of the input isn't specified, you can sweep the frequency from 1Hz to 1000Hz using a parameter sweep option as shown in Figure 3. For a frequency domain simulation

- Specify the input (V1 in this case)
- Specify the starting and ending frequency (1Hz to 1000Hz), and
- Specify the number of points per decade (20 to 100 usually).

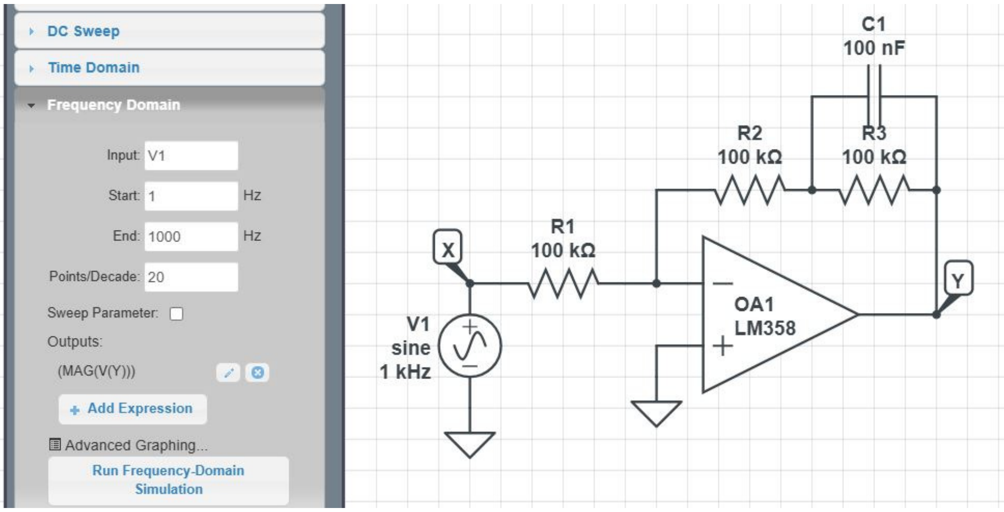

Figure 3: Run a Frequency Domain simulation in CircuitLab to find the gain vs frequency

The result should look just like what Matlab presented. It does.

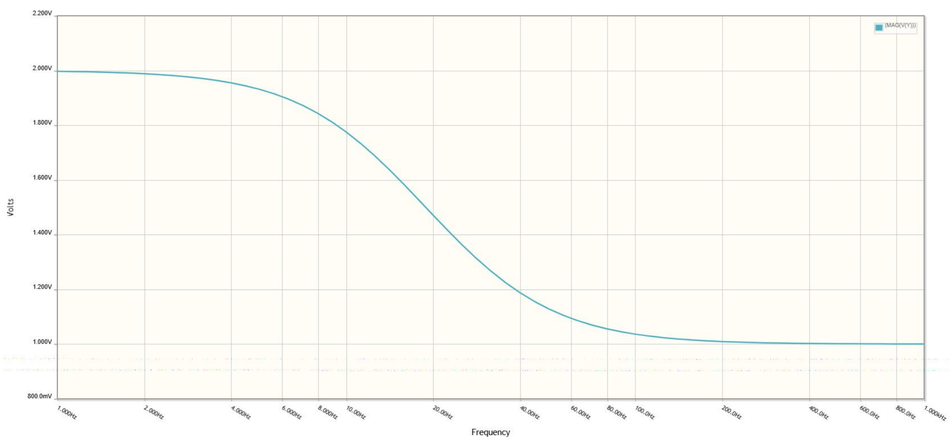

Figure 4: Result of a frequency domain simulation. Same results as Matlab verify previous calculations.

Design: The counter part to analysis is design. With design, you have some desired response. The problem is to determine the values of the resistors and capacitors to meet your desired response. For example, change this circuit so that it has

- A DC gain of 3x,
- A high-frequecny gain of 1x, and
- A corner frequency of 100Hz (where the gain starts to drop from 3x)

Solution: Let R2 be 100k. The high-frequency gain set Ra

$$hf = 1 = \left(\frac{R_a}{R_2}\right)$$

$$R_a = R_2 = 100k$$

The low-frequency gains sets Ra + Rb

$$lf = 3 = \left(\frac{R_a + R_b}{R_2}\right)$$

$$R_b = 200k$$

The corner frequency sets the value of C. At the corner frequency, Z1 switches from resistive to capacitive:

$$\left|\frac{1}{j\omega C}\right|_{\omega = 100Hz} = R_b$$

$$\frac{1}{RC} = \omega = 2\pi \cdot 100$$

$$C = 7.76nF$$

The resulting circuit is shown in Figure 5 along with it's Bode plot in Figure 6.

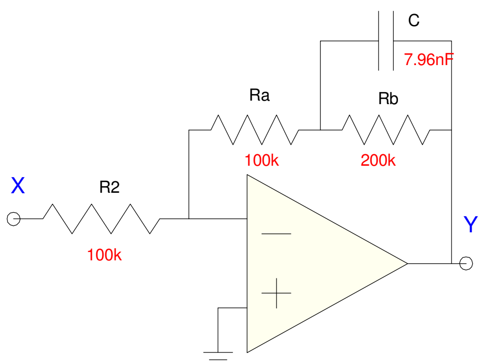

Figure 5: Final design for a bass-boost circuit

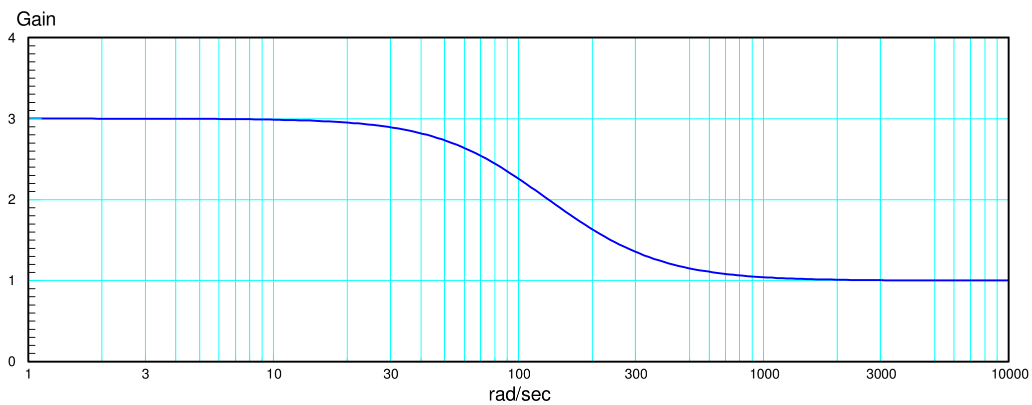

Figure 6: Bode plot for the circuit of Figure 5

First-Order Low-Pass Filter

Next, let's look at a 1st-order low pass filter as shown in Figure 7. The purpose of R and C is to

- Pass low frequencies (when the impedance of Zc >> R)
- Reject high frequencies (when Zc << R)

R1 and R2 provide a gain.

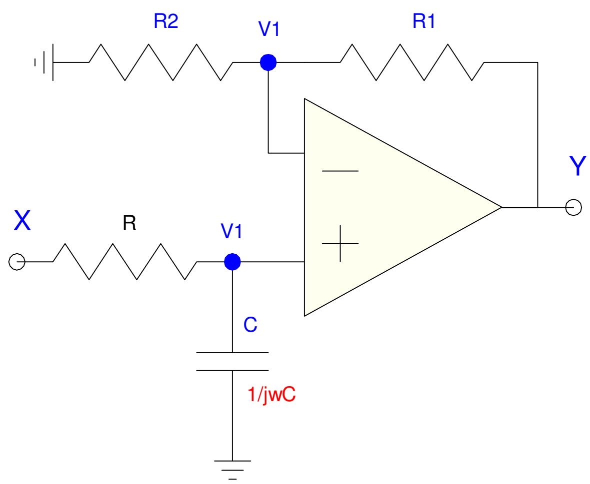

Figure 7: FIrst-order low-pass filter

The equations for this circuit are as follows. R1 and R2 create a non-inverting amplifier:

$$Y = \left(1 + \frac{R_1}{R_2}\right) V_1 = kV_1$$

The voltage node equation at V1 is

$$\left(\frac{V_1 - X}{R}\right) + \left(\frac{V_1}{1/j\omega C}\right) = 0$$

$$V_1 = \left(\frac{1}{j\omega RC + 1}\right) X$$

Put it together and you get

$$Y = \left(\frac{k}{j\omega RC + 1}\right) X$$

Analysis: Assume all R's are 100k and C = 0.1uF. Plot the gain vs. frequency (i.e. the Bode plot).

Solution in Matlab:

```
w = logspace(0,4,200)';
R1 = 1e5;
R2 = 1e5;
R = 1e5;
C = 0.1e-6;
k = 1 + R1/R2;
G = k ./ (1 + j*w*R*C);
semilogx(w,abs(G))
xlabel('rad/sec')
```

```
ylabel('Gain')
```

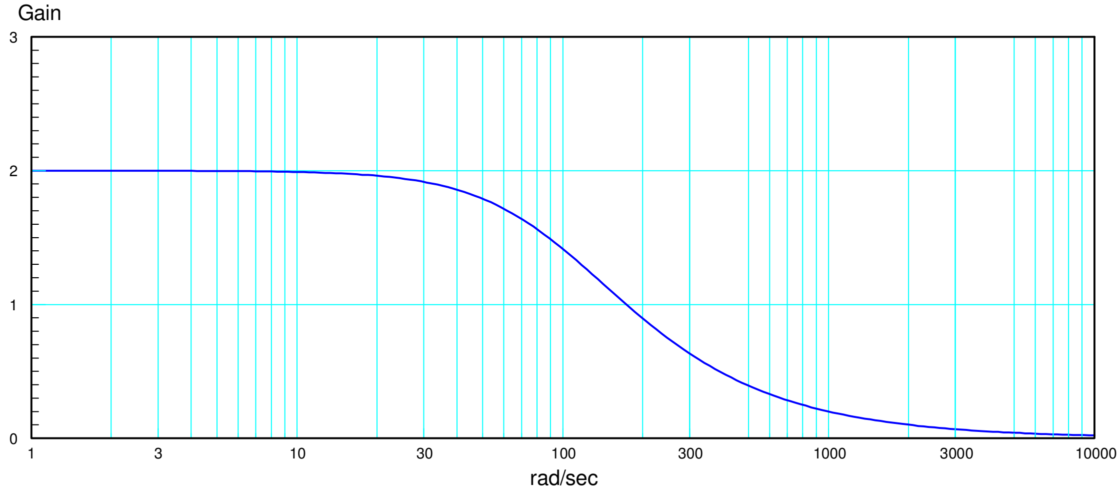

Figure 8: Gain vs. frequency when R's are 100k and C = 0.1uF

This checks in CircuitLab. For this circuit

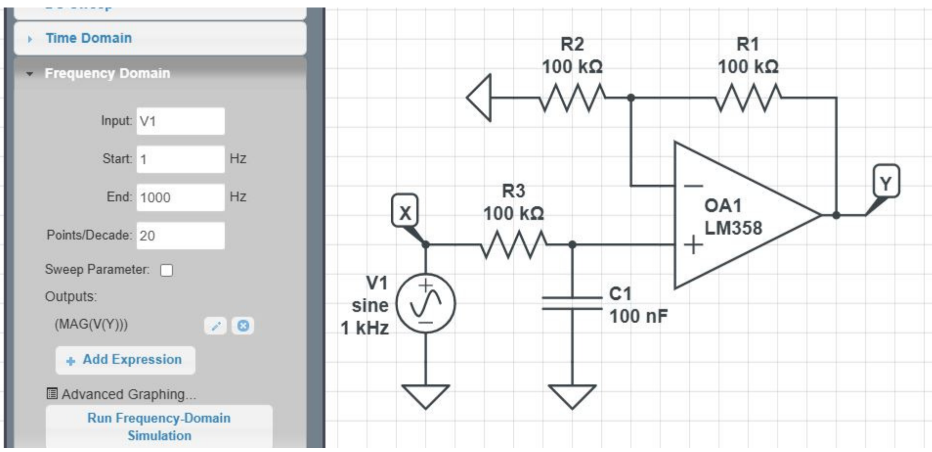

Figure 9: Input the circuit into CircuitLab and run a frequency-domain simulation

The result matches Matlab's result

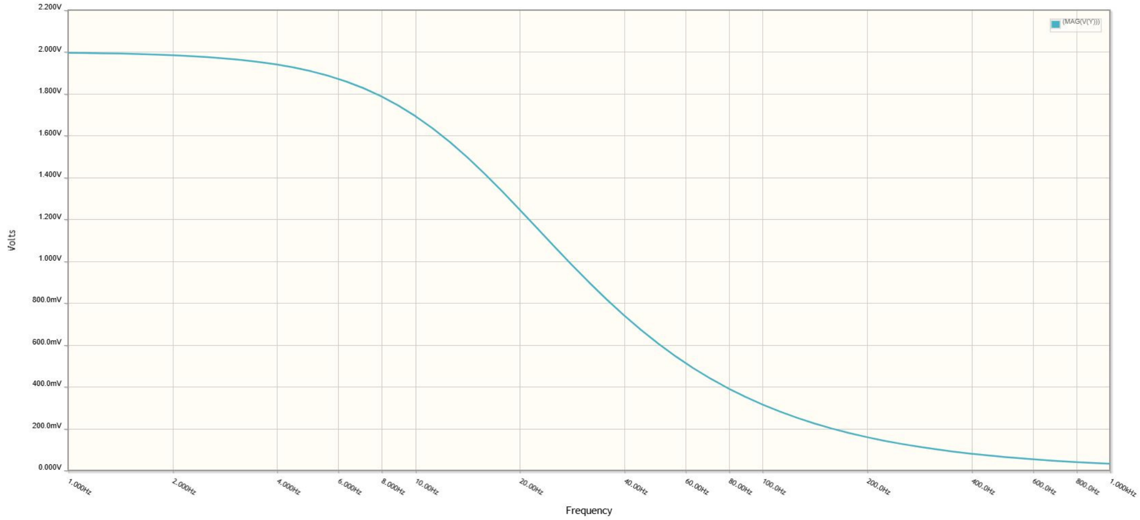

Figure 10: Bode plot from CircuitLab matches Matlab's results

Design: Change the filter so that it has

- A DC gain of 3, and
- A corner at 100Hz (628 rad/sec).

From before, the gain of the filter is

$$G = \left(\frac{k}{j\omega RC+1}\right)$$

A DC gain of 3 gives k=3, or

- R1 = 100k (arbitrary)
- R2 = 200k (gain = 3)

A corner at 100Hz gives

- $\omega RC = 1$ @ 628 rad/sec
- R = 100k (arbitrary)
- C = 15.9nF

The resulting circuit is shown in Figure 11 along with it's gain vs. frequency.

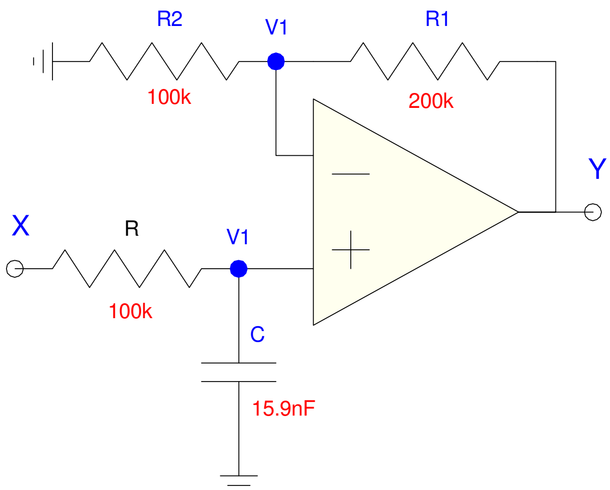

Figure 11: Resulting filter design

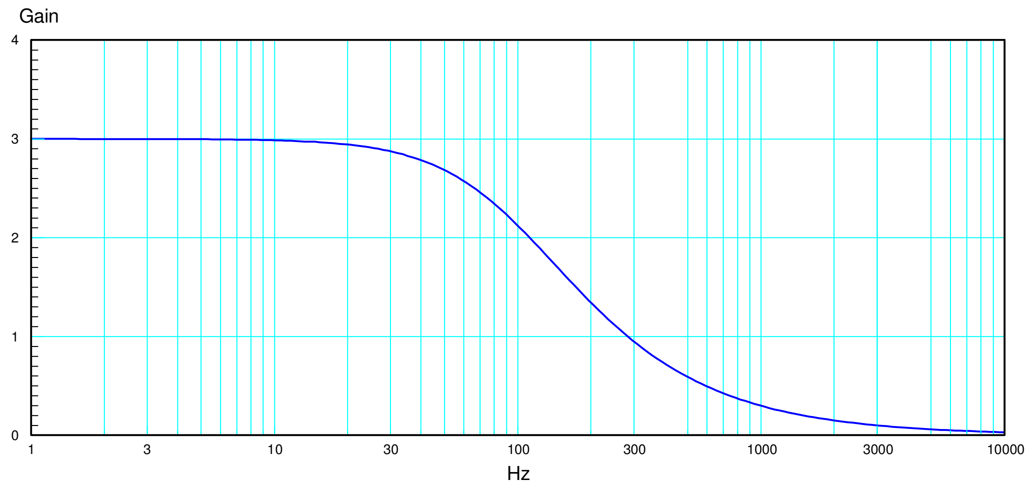

Figure 12: Resulting gain vs frequency for filter of Figure 11

Second-Order Low-Pass Filter

Finally, consider the filter of Figure 13: a 2nd-order low-pass filter. The topology of this filter isn't obvious. It's one that someone else came up with and works when you want to have complex poles.

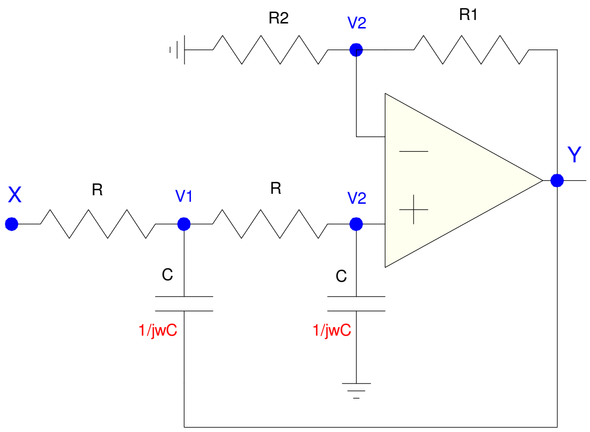

Figure 13: Second-order low-pass filter

The node equations for this filter are as follows. Note that R1 and R2 form a noninverting amplifier

$$Y = \left(1 + \frac{R_1}{R_2}\right) V_2 = kV_2$$

The voltage node equations are

$$\left(\frac{V_1 - X}{R}\right) + \left(\frac{V_1 - V_2}{R}\right) + \left(\frac{V_1 - Y}{1/j\omega C}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R}\right) + \left(\frac{V_2}{1/j\omega C}\right) = 0$$

After some effort, this simplifies to

$$Y = \left(\frac{k}{(j\omega RC)^2 + (3-k)(j\omega RC) + 1}\right) X$$

Analysis: Determine the gain vs. frequency when

- R1 = 90k
- R2 = 100k
- R = 100k
- C = 0.1uF

Solution: Throw the transfer function into Matlab and grind away

```
w = logspace(0,4,200)';
```

```
R1 = 90e3;
R2 = 100e3;
R = 100e3;
C = 0.1e-6;
k = 1 + R1/R2;
jwRC = j*w*R*C;
G = k./(jwRC.^2+(3-k)*jwRC+1);
semilogx(w,abs(G))
xlabel('rad/sec')
ylabel('Gain')
```

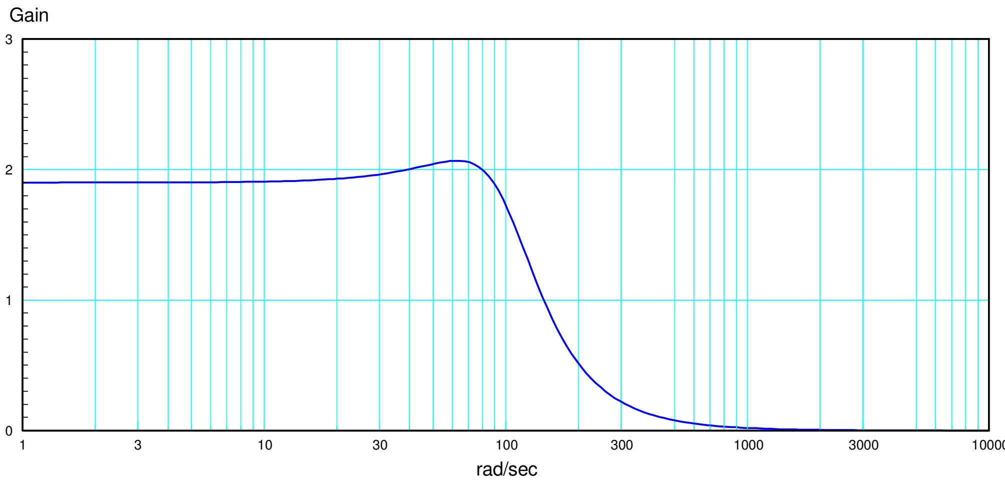

Figure 14: Gain vs. frequency

Design: Find R's and C's so that

- The corner frequency is 250Hz (crossover for a Cerwin Vega subwoofer), and
- The complex poles are at 45 degrees (gives a Butterworth filter - you'll cover this in electronics)

The corner frequency is 250Hz (500π rad/sec)

$$\frac{1}{RC} = 500\pi$$

R = 100k (arbitrary)

C = 6.37nF

k sets the angle of the pole

$$3 - k = 2\cos\theta$$

$$\theta = 45^0$$

$$k = 1.586 = 1 + \frac{R_1}{R_2}$$

R2 = 100k (arbitrary)

R1 = 58.6k

The resulting low-pass filer is thus

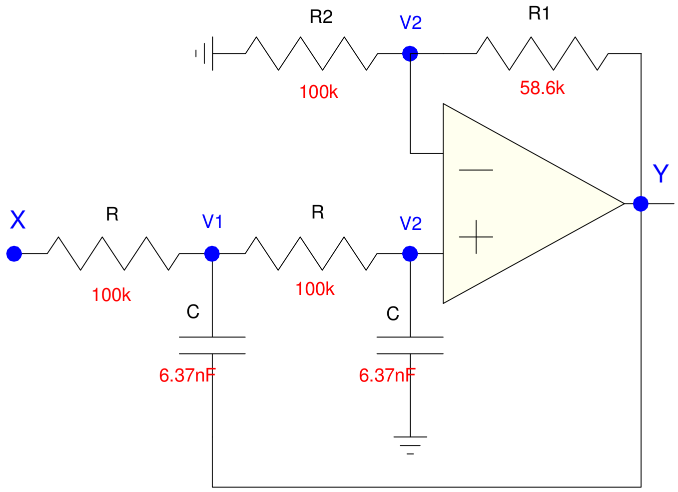

Figure 15: Low-pass filter with a corner at 250Hz

The gain vs. frequency is

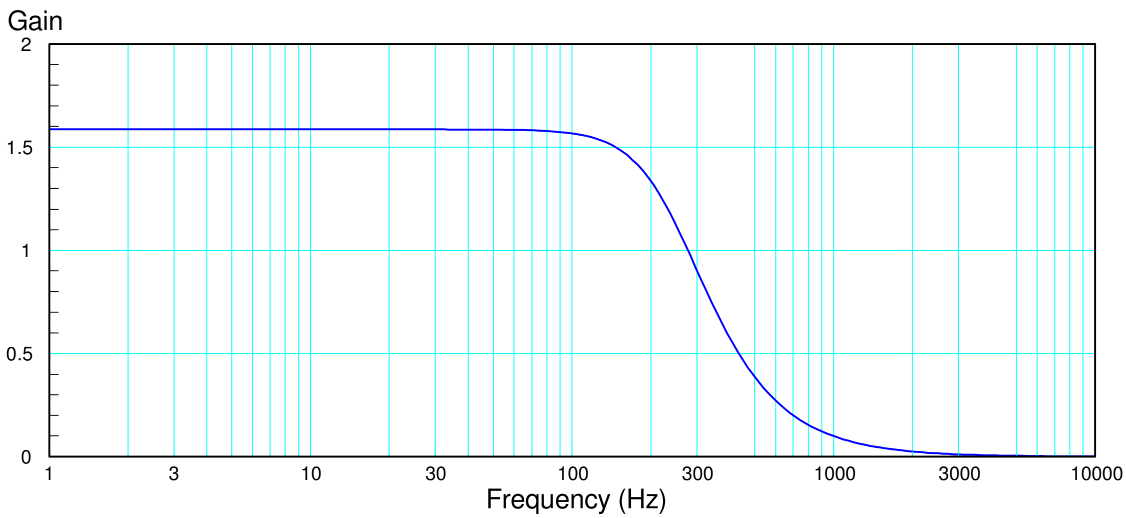

Figure 16: Resulting gain vs. frequency for 2nd-order filter design

When you take electronics, you'll cover other filters as well.

Summary

Filters are circuits where the gain varies with frequency

- Any circuit with inductors and/or capacitors

Active filters are op-amp circuits where the gain varies with frequency

- Op-amps allow gains > 1
- They also allow you to avoid using inductors

If the operating frequency is unknown, leave it as ω

- You can then use voltage nodes to determine the output as a function of frequency