Op-Amp Circuits with Phasors

Introduction

Previously, we looked at DC analysis of op-amp circuits with resistors. Three methods were used:

- Voltage Nodes: Write N equations for N unknown voltages then solve.
- Figure it Out: Determine what the voltages have to be for the currents to balance.
- CircuitLab: Build the circuit in CircuitLab then run a DC Solver to find the voltages and currents.

With both methods, one of the constraints was that Vp = Vm. This is usually the case when there's negative feedback.

In this lecture, we'll look at solving for the steady-state output for op-amp circuits with resistors, inductors, and capacitors. The methods used and the results will be very similar to what we did before - only now using complex numbers (i.e. phasors) when analyzing the circuit by hand, Time-Domain Simulation when using CircuitLab.

It's probably easiest to explain the use of phasors with op-amps through examples. This lecture will analyze three different op-amp circuits

- Circuit #1: An op-amp circuit with a single capacitor
- Circuit #2: An op-amp circuit with two capacitors, and
- Circuit #3: Another op-amp circuit with two capacitors

Note that none of these circuits use inductors. Inductors are large, expensive, lossy, and are prone to cross-talk. With op-amp circuits, they're not really needed. By using positive and negative feedback, any differential equation can be implemented using only capacitors. With these three circuits, you should get the idea for how to proceed with other op-amp circuits.

Single-Pole Low-Pass Filter

Starting out, consider the op-amp circuit in Figure 1 with

$$x(t) = 3\sin(50t)$$

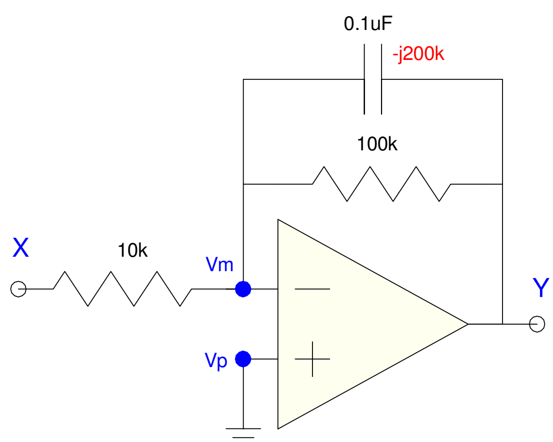

Figure 1: Single Op-Amp Circuit

Solving using Voltage Nodes: To start out, replace the components with their phasor equivalents. For the voltage source

$$\omega = 50 \frac{rad}{\sec}$$

$$X = 0 - j3$$

$$Z_c = \frac{1}{j\omega C} = -j200k\Omega$$

The voltage node equations are then

$$V_p = 0$$

$$V_m = V_p$$

$$\left(\frac{V_m - X}{10k}\right) + \left(\frac{V_m - Y}{100k}\right) + \left(\frac{V_m - Y}{-j200k}\right) = 0$$

Solving results in

$$\left(\frac{-(-j3)}{10k}\right) + \left(\frac{-Y}{100k}\right) + \left(\frac{-Y}{-j200k}\right) = 0$$

$$Y = 12 + j24$$

which is the phasor representation for y(t)

$$\boxed{y(t) = 12\cos(50t) - 24\sin(50t)}$$

Solving by Balancing Currents: Another way to find the voltages is to figure out what they need to be to make the currents balance. Start with noting that

Vp = Vm = 0

This results in the current through the 10k resistor being

$$I_x = \left(\frac{X}{10k}\right) = \left(\frac{(-j3)}{10k}\right)$$

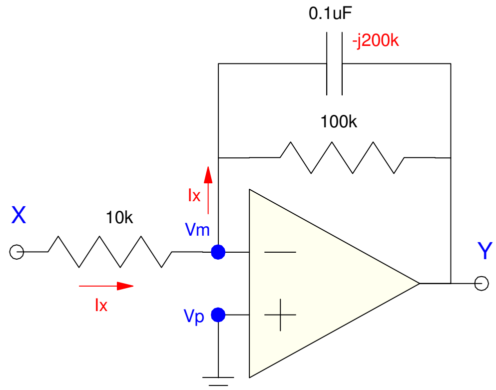

Figure 2: Figure out the voltages by making the currents balance

By conservation of current, Ix also flows through the parallel impedance of Z1

$$Z_1 = (100k)||(-j200k) = (80k - j40k)\Omega$$

This results in

$$Y = -Z_1 I_x$$

$$Y = -(80k - j40k)\left(\frac{-j3}{10k}\right)$$

$$Y = 12 + j24$$

which is the same answer from before.

Solving using CircuitLab: Another way to find the output is to simulate the circuit in CircuitLab. Start with inputting the circuit as shown in Figure 3. Since the output is time-varying (i.e. an AC signal), run a Time-Domain Simulation. Start the simulation at 1 second to give the transients time to settle out. Run for 200ms to see a few cycles of a sine wave.

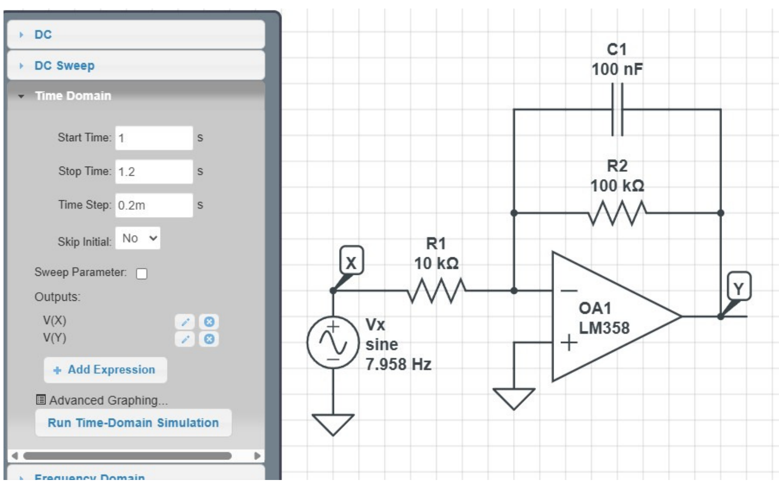

Figure 3: Input the circuit in CircuitLab and run a Time-Domain Simulation.

The resulting voltages at X and Y are shown in Figure 4.

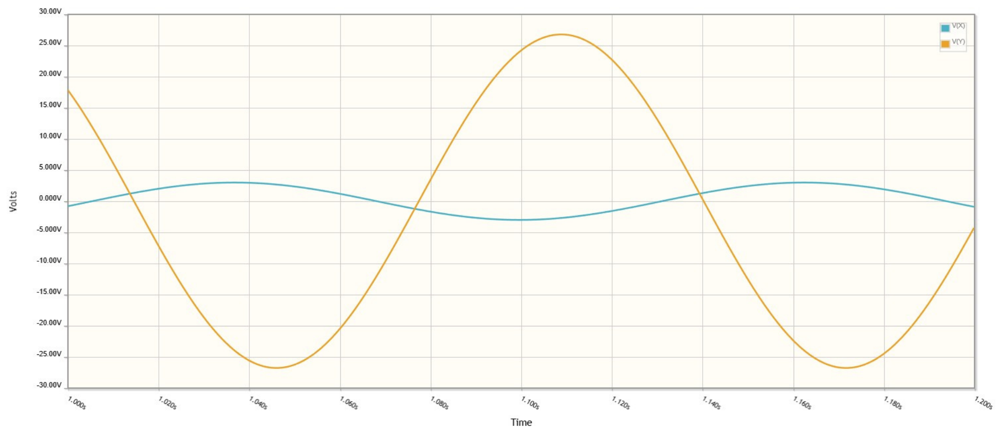

Figure 4: Voltage Vx (blue) and Vy (orange)

This matches up with Matlab calculations for both Vx and Vy as shown in Table 2.

	Vx	Vy
CircuitLab	3.000V	26.78V
Matlab	3.000V	26.83V

Table 1: Magnitude of voltages from CircuitLab and Matlab

Circuit #2: Two Capacitor Circuit

Next, consider an op-amp circuit with two capacitors, such as the one presented in Figure 5 when

$$x(t) = 3\sin(50t)$$

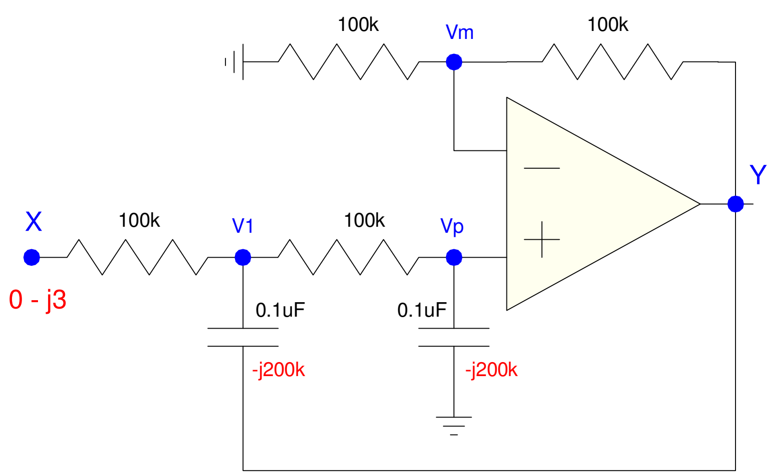

Figure 5: Op-Amp Circuit with Two Capacitors

Start with converting the sources and capacitors to their phasor equivalents

$$\omega = 50$$

$$X = 0 - j3$$

$$\frac{1}{j\omega C} = -j200k\Omega$$

Now solve. With voltage nodes, there are five voltage nodes. Hence, we need five equations for five unknowns. Start with the easy ones

$$X = 0 - j3$$

$$V_p = V_m$$

Add in the voltage node equations to get the last three equations

$$\left(\frac{V_1 - X}{100k}\right) + \left(\frac{V_1 - V_p}{100k}\right) + \left(\frac{V_1 - Y}{-j200k}\right) = 0$$

$$\left(\frac{V_p - V_1}{100k}\right) + \left(\frac{V_p}{-j200k}\right) = 0$$

$$\left(\frac{V_m}{100k}\right) + \left(\frac{V_m - Y}{100k}\right) = 0$$

Group terms

$$X = -j3$$

$$V_p - V_m = 0$$

$$\left(\frac{-1}{100k}\right)X + \left(\frac{-1}{-j200}\right)Y + \left(\frac{1}{100k} + \frac{1}{100k} + \frac{1}{-j200k}\right)V_1 + \left(\frac{1}{100k}\right)V_p = 0$$

$$\left(\frac{-1}{100k}\right)V_1 + \left(\frac{1}{100k} + \frac{1}{-j200k}\right)V_p = 0$$

$$\left(\frac{-1}{100k}\right)Y + \left(\frac{2}{100k}\right)V_m = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ \left(\frac{-1}{100k}\right) & \left(\frac{-1}{-j200}\right) & \left(\frac{2}{100k} + \frac{1}{-j200k}\right) & \left(\frac{1}{100k}\right) & 0 \\ 0 & 0 & \left(\frac{-1}{100k}\right) & \left(\frac{1}{100k} + \frac{1}{-j200k}\right) & 0 \\ 0 & \left(\frac{-1}{100k}\right) & 0 & 0 & \left(\frac{2}{100k}\right) \end{bmatrix} \begin{bmatrix} X \\ Y \\ V_1 \\ V_p \\ V_m \end{bmatrix} = \begin{bmatrix} -j3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
% X   Y   V1   Vp   Vm
a1 = [1 0 0 0 0];
a2 = [0 0 0 1 -1];
a3 = [-1, 1/(2j), 1+1-1/(2j), -1, 0];
a4 = [0, 0, -1, 1-1/(2j), 0];
a5 = [0, -1, 0, 0, 2];
A = [a1;a2;a3;a4;a5];
B = [-3j;0;0;0;0];
V = inv(A)*B


Vx         0 - 3.0000i
Vy   -3.6923 - 5.5385i
V1   -0.4615 - 3.6923i
Vp   -1.8462 - 2.7692i
Vm   -1.8462 - 2.7692i
```

Solving by figuring it out doesn't quite work for me for this circuit. It's too complex. I'd just use voltage nodes.

Solving using CircuitLab does work though. First, input the circuit

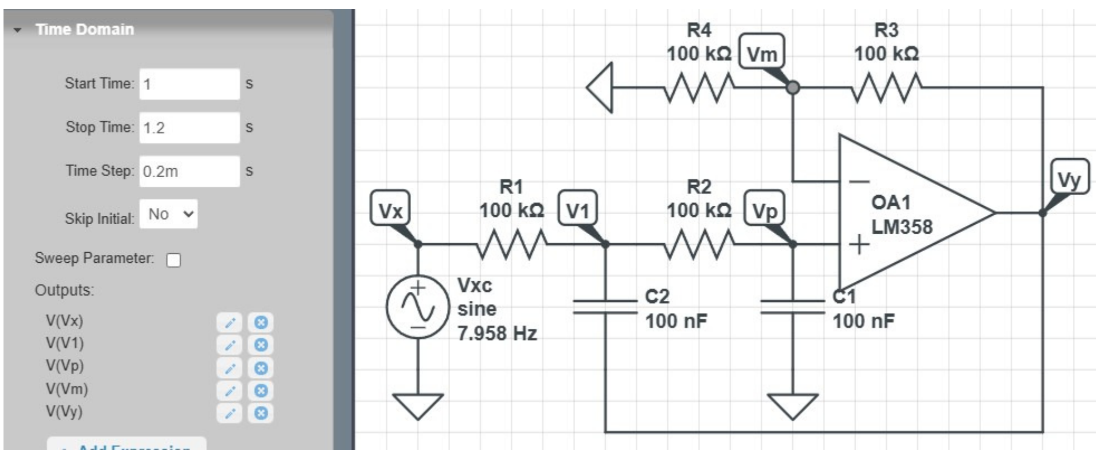

Figure 6: Two capacitor circuit in CircuitLab

Run a time-domain simulation. Delay the start time to let the transients settle out (phasors give the steady-state solution)

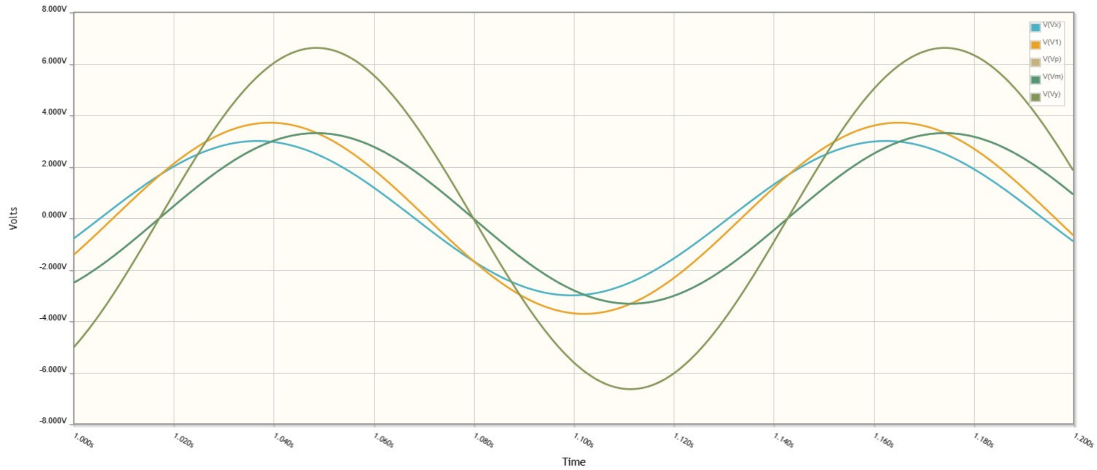

Figure 7: Steady-state solution from CircuitLab

The magnitude of the voltages should match calculations. From Table 2, they do.

	Vx	V1	Vp	Vy
CircuitLab	3.000V	3.690V	3.298V	6.616V
Matlab	3.000V	3.721V	3.328V	6.656V

Table 2: CircuitLab and Matlab results match - verifying calculations

Circuit #3:

Finally, consider the circuit given in Figure 8. Find the node voltages when

$$X = 3\sin(50t)$$

Using phasors

$$\omega = 50$$

$$X = 0 - j3$$

$$0.2\mu F \rightarrow \frac{1}{j\omega C} = -j100k\Omega$$

$$0.1\mu F \rightarrow \frac{1}{j\omega C} = -j200k\Omega$$

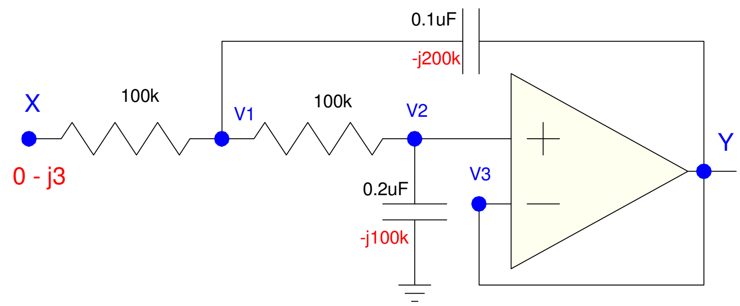

Figure 8: Find the phasor voltages

For this circuit, there are five voltages labeled. To solve, five equations are needed. Start with the easy ones:

$$X = 0 - j3$$

$$V_3 = V_2$$

$$V_3 = Y$$

For the remaining two equations, sum the currents to zero at V1 and V2

$$\left(\frac{V_1 - X}{100k}\right) + \left(\frac{V_1 - Y}{-j200k}\right) + \left(\frac{V_1 - V_2}{100k}\right) = 0$$

$$\left(\frac{V_2 - V_1}{100k}\right) + \left(\frac{V_2}{-j100k}\right) = 0$$

This gives five equations and five unknowns. Skipping a few steps and writing these equations in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ \left(\frac{-1}{100k}\right) & \left(\frac{1}{j200k}\right) & \left(\frac{2}{100k} - \frac{1}{j200k}\right) & \left(\frac{-1}{100k}\right) & 0 \\ 0 & 0 & \left(\frac{-1}{100k}\right) & \left(\frac{1}{100k} - \frac{1}{j100k}\right) & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Place in Matlab and solve

```
% X   Y   V1   V2   V3
a1 = [1 0 0 0 0];
a2 = [0 0 0 1 -1];
a3 = [0, 1, 0, 0, -1]
a4 = [-1/100000, 1/(200000j), 1-1/(200000j)+1/100000,-1/100000,0];
a5 = [0, 0, -1/100000, 1/100000-1/100000j, 0];
A = [a1;a2;a3;a4;a5];
B = [-3j;0;0;0;0];
V = inv(A)*B
```

```
Vx         0 - 3.0000i
Vy   -1.4118 - 0.3529i
V1   -1.0588 - 1.7647i
V2   -1.4118 - 0.3529i
V3   -1.4118 - 0.3529i
```

You can check these results with CircuitLab. First input the circuit

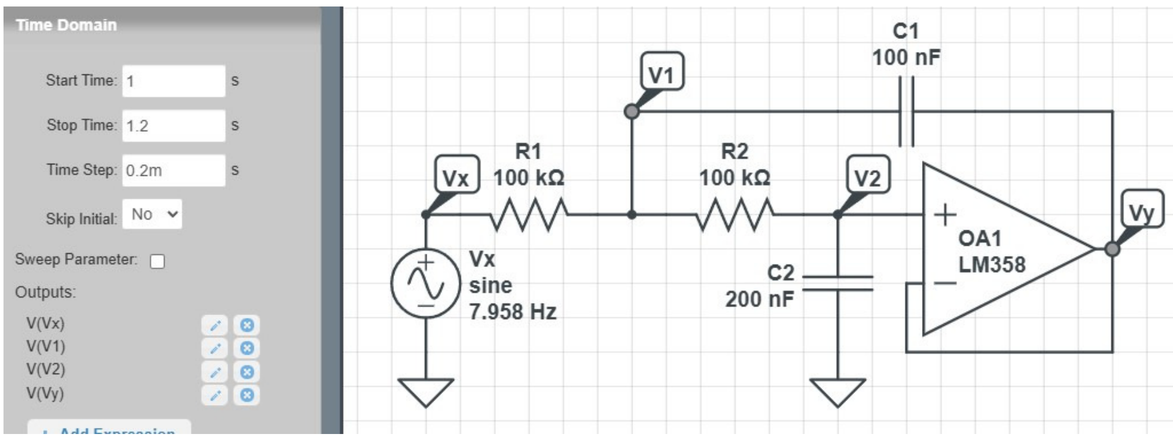

Figure 9: Input the circuit i CircuitLab and run a Time Domain Simulation

Then run a time-domain simulation

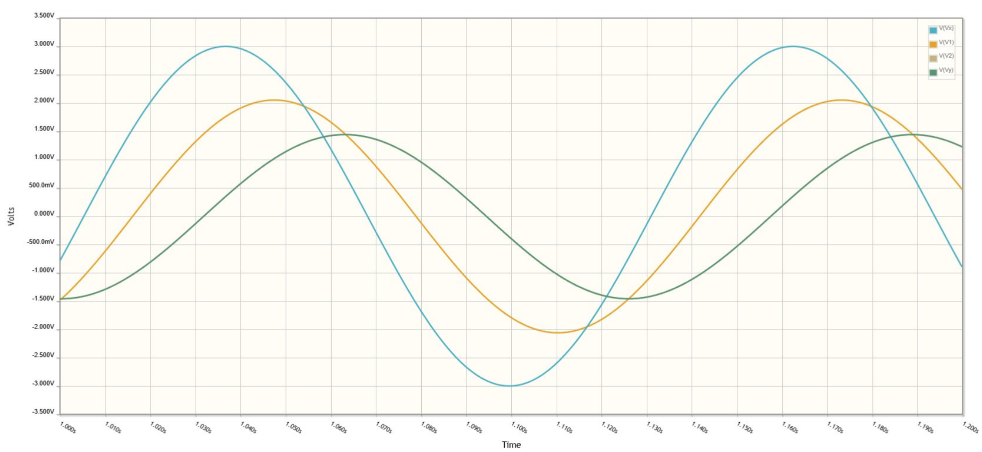

Figure 10: Voltages from a time-domain simulation

The resulting peak voltage from CircuitLab matches up with the magnitudes computed with Matlab

	Vx	V1	Vy
CircuitLab	3.000V	2.051V	1.440V
Matlab	3.000V	2.058V	1.455V

Table 3: Simulated and calculate voltages for Circuit 3

Note that the simulation results match up with calculated values from Matlab. This confirms our calculations.

Summary

Nor surprisingly, phasors work with op-amp circuits. The same procedures are used to solve for the node voltage as we did with our previous DC analysis. The only difference is not we're using complex numbers.

CircuitLab is also a useful tool for checking your calculations. Since the waveforms are time-varying sine waves. a time-domain simulation needs to be run. When done, the peak voltages should match the magnitude of the voltages you computed (i.e. polar form is more useful in lab).