Phasors & Current Loops

ECE 211 Circuits I

Lecture #26

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

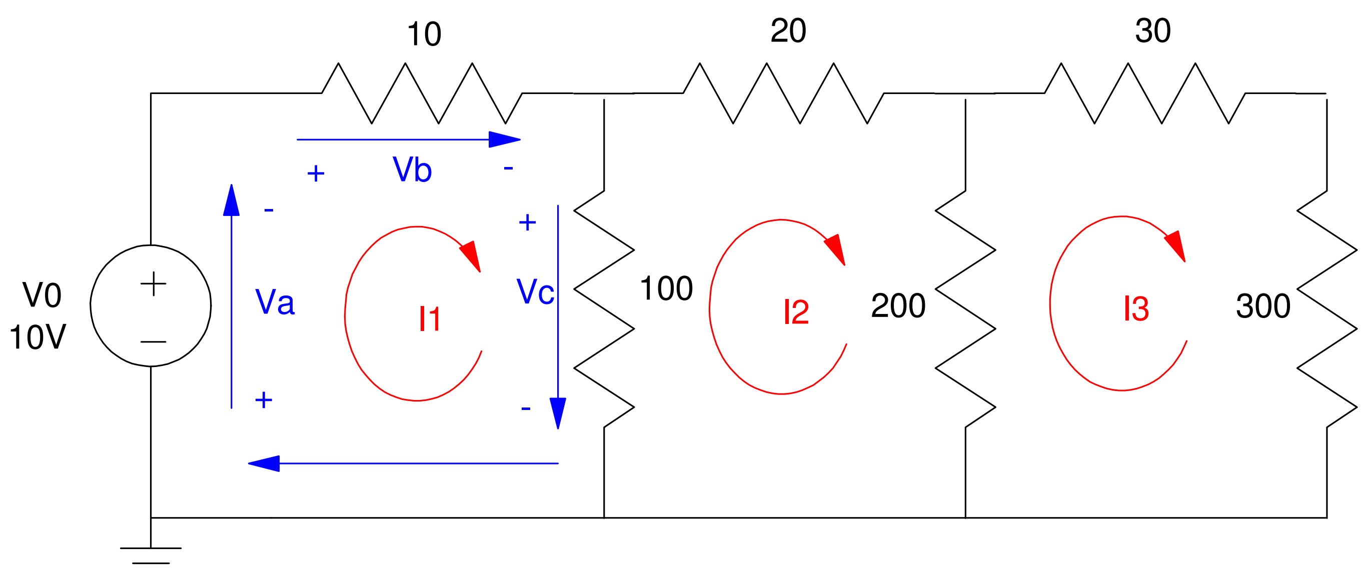
Current Loops (review)

Another way to solve for voltages and currents in a circuit

- Write N equations for N unknowns

Based on Conservation of Voltage

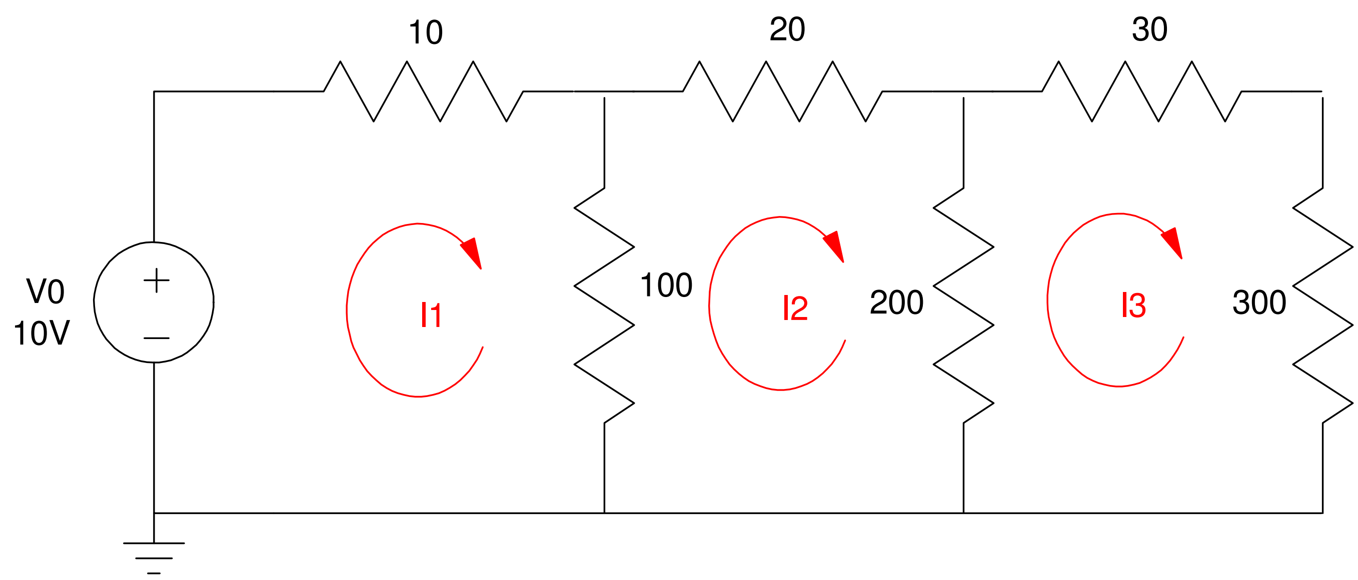
- The sum of voltages around a closed-path must be zero
- Va + Vb + Vc = 0



Current Loops with Resistor Circuits

Current-Loops works with resistor circuits

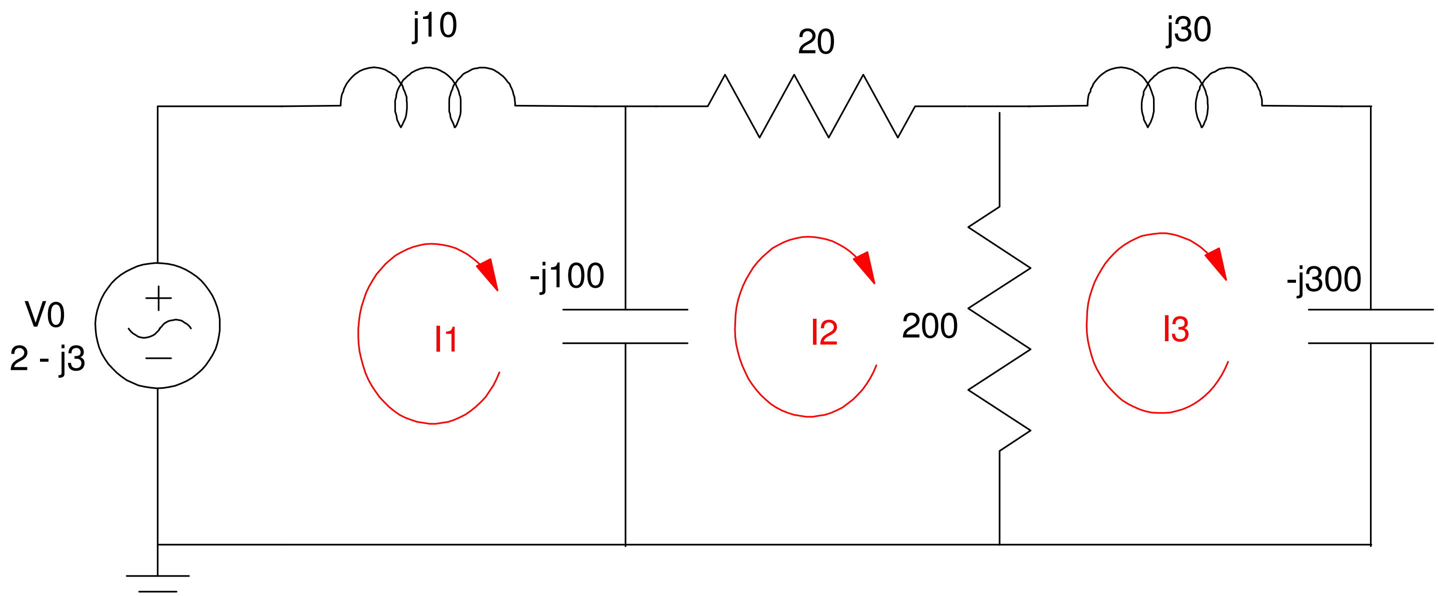
- Write N equations for N unknowns
- Voltages and currents are real numbers
- All numbers are real



Current Loops with RLC Circuits

Current-Loops also works for RLC circuits

- Use phasors
- Write N equations for N unknowns
- Using complex numbers



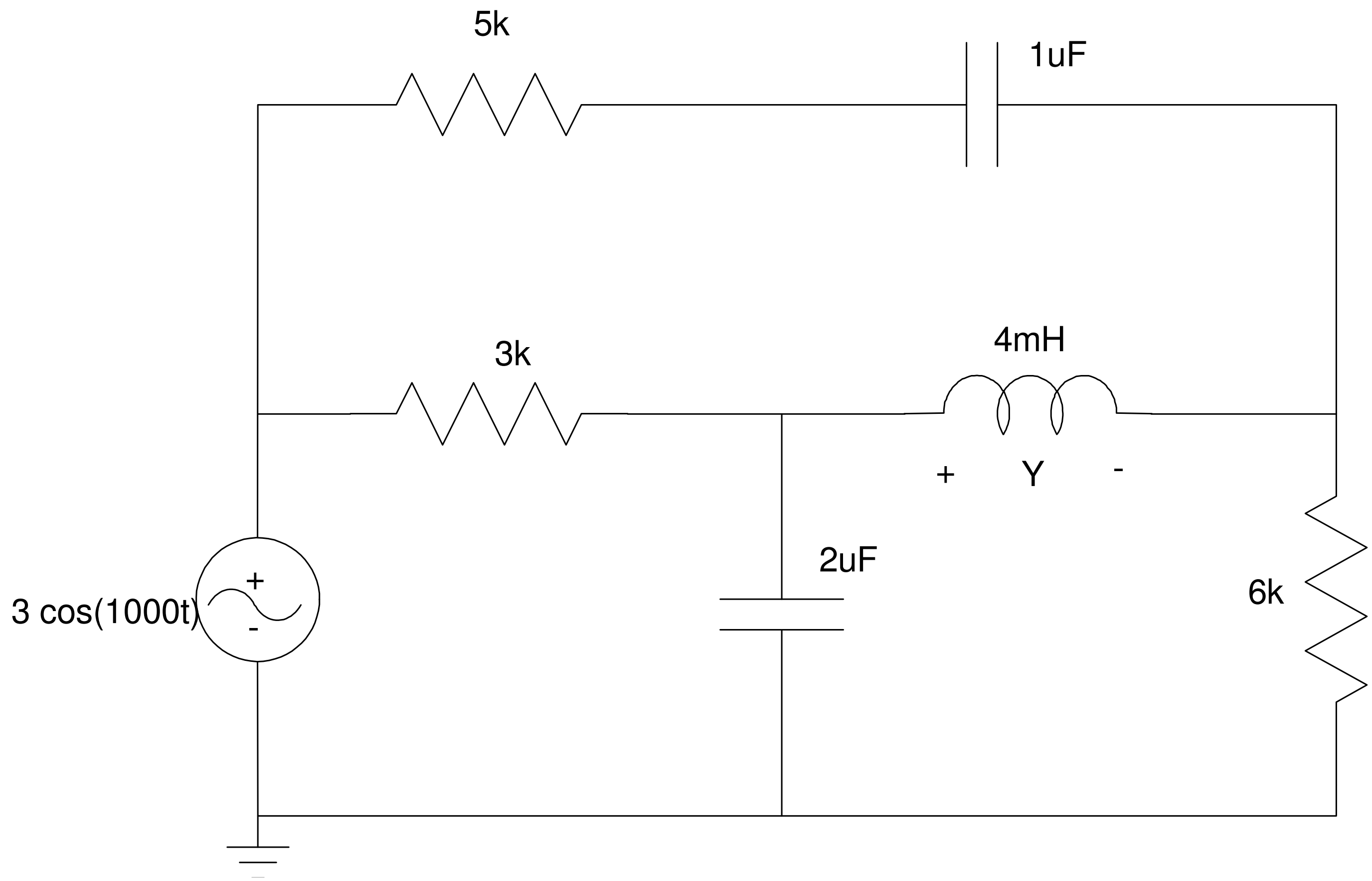
Phasors (recap)

When signals are sinusoids, use phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Current	$i(t) = a\cos(\omega t) + b\sin(\omega t)$	$I = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example 1:

Determine the currents in the following circuit



Step 1:

- Define the currents
- Convert to Phasors

V = 3 cos(1000t)

$$\omega = 1000$$

$$V = 3 + j0$$

C = 1uF

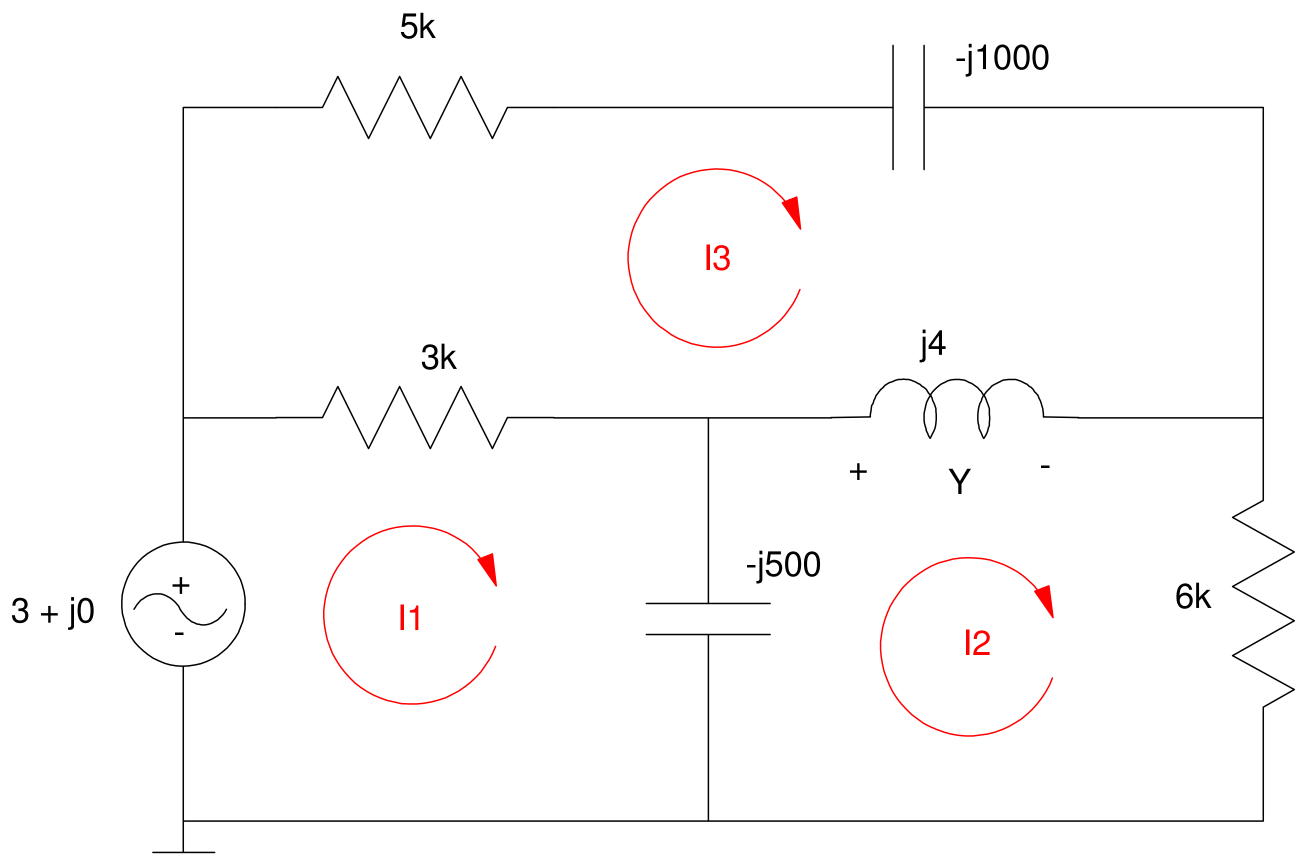
$$Z = \frac{1}{j\omega C} = -j1000$$

C = 2uF

$$Z = \frac{1}{j\omega C} = -j500$$

L=4mH

$$Z = j\omega L = j4$$



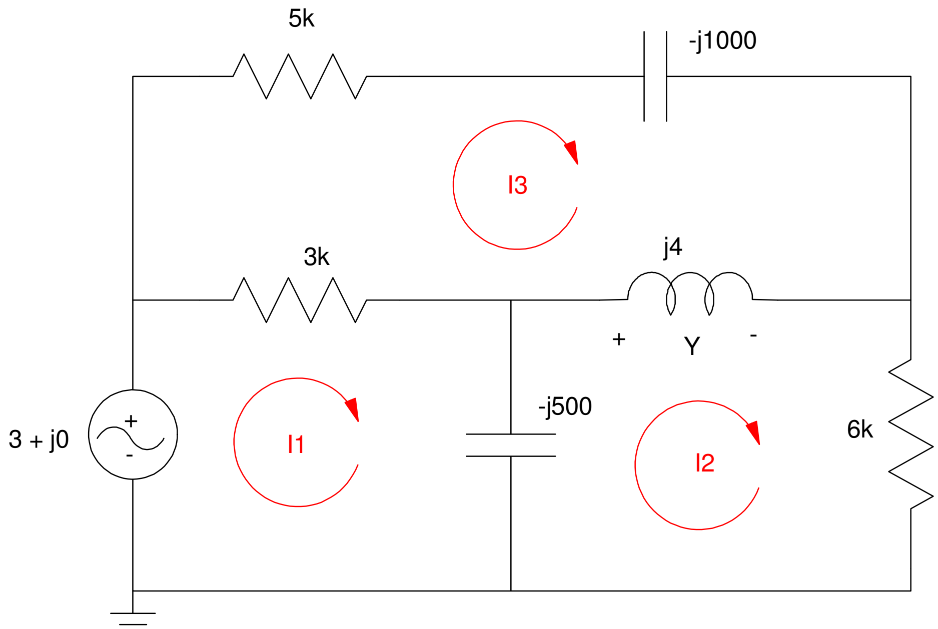
Step 2: Write the current loop equations

3 equations for 3 unknowns

$$-3 + 3000(I_1 - I_3) - j500(I_1 - I_2) = 0$$

$$-j500(I_2 - I_1) + j4(I_2 - I_3) + 6000 I_2 = 0$$

$$-j1000(I_3) + 5000(I_3) + j4(I_3 - I_2) + 3000(I_3 - I_1) = 0$$



Step 3: Solve

Group terms:

$$(3000 - j500)I_1 + (j500)I_2 + (-3000)I_3 = 3$$

$$(j500)I_1 + (6000 - j496)I_2 + (-j4)I_3 = 0$$

$$(-3000)I_1 + (-j4)I_2 + (8000 - j996)I_3 = 0$$

```
A = [3000-500j, 500j, -3000 ;
     500j,6000-496j , -4j;
     -3000, -4j, 8000-996j];

I_mA = inv(A) * [3;0;0] * 1000

I1(mA)    1.4008 + 0.4598i
I2(mA)    0.0475 - 0.1125i
I3(mA)    0.4962 + 0.2342i
```

Step 4: Convert back to time

In phasor form:

```
I1(mA)    1.4008 + 0.4598i
I2(mA)    0.0475 - 0.1125i
I3(mA)    0.4962 + 0.2342i
```

In the time-domain

$$i_1(t) = 1.4008\cos(1000t) - 0.4598\sin(1000t) \ \ \text{mA}$$

$$i_2(t) = 0.0475\cos(1000t) + 0.1125\sin(1000t) \ \ \text{mA}$$

$$i_3(t) = 0.4962\cos(1000t) - 0.2342\sin(1000t) \ \ \text{mA}$$

CircuitLab Results

1 cycle = 6.3ms

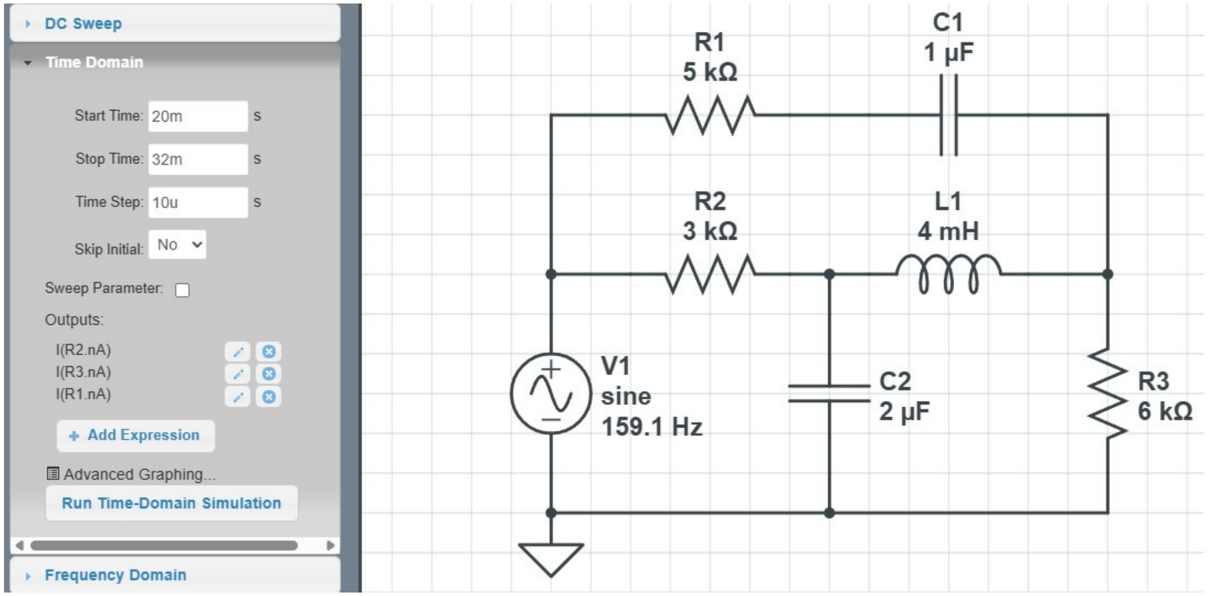
- 1 / 159.15Hz

Plot for 2 cycles

- 12ms

Start at 20ms

- Let the transient die out
- Phasor analysis gives steady-state solution



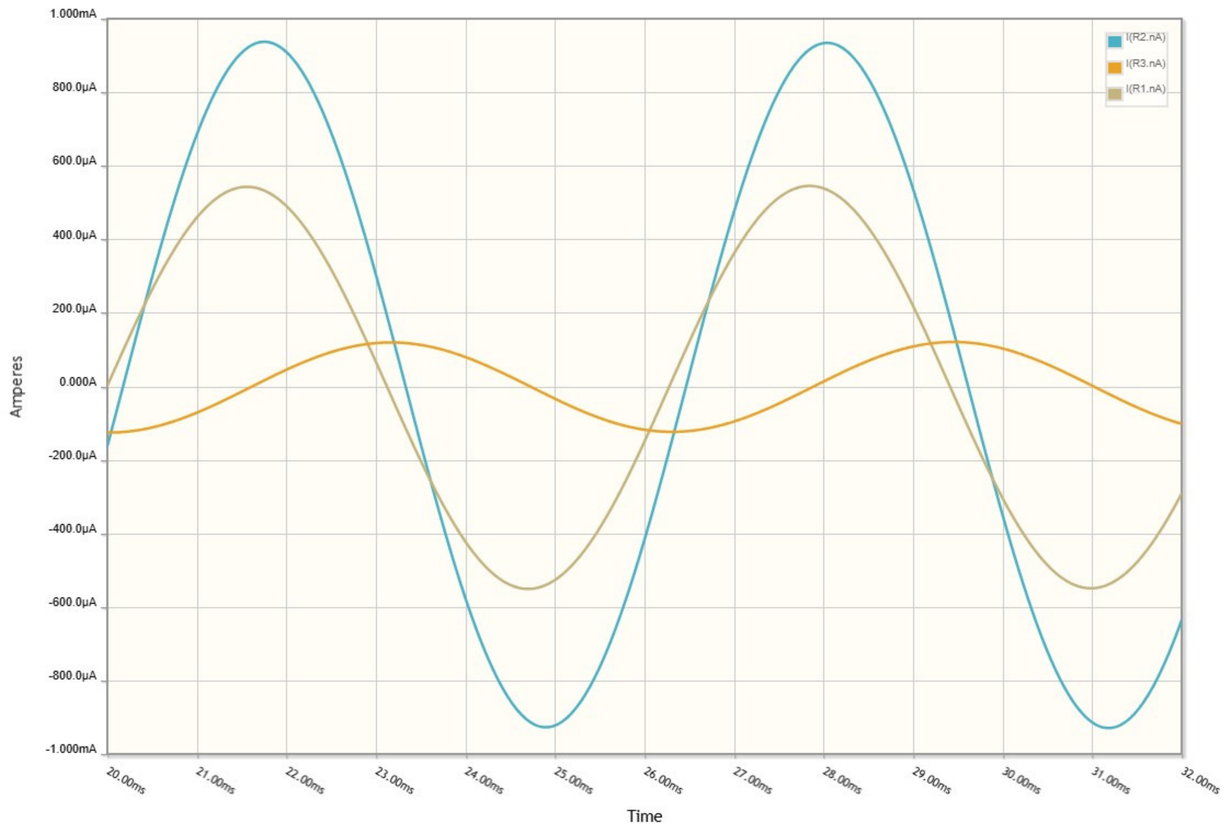
CircuitLab Result

Matlab

- |I1| = 1.474mA
- |I2| = 0.122mA
- |I3| = 0.549mA
- |I1-I3| = 0.9323mA

CircuitLab

- |I3| = 0.5450mA
  - I(R1) = I3
- |I2| = 0.122mA
  - I(R2) = I2
- |I1-I3| = 0.934mA
  - I(R3) = I1 - I3



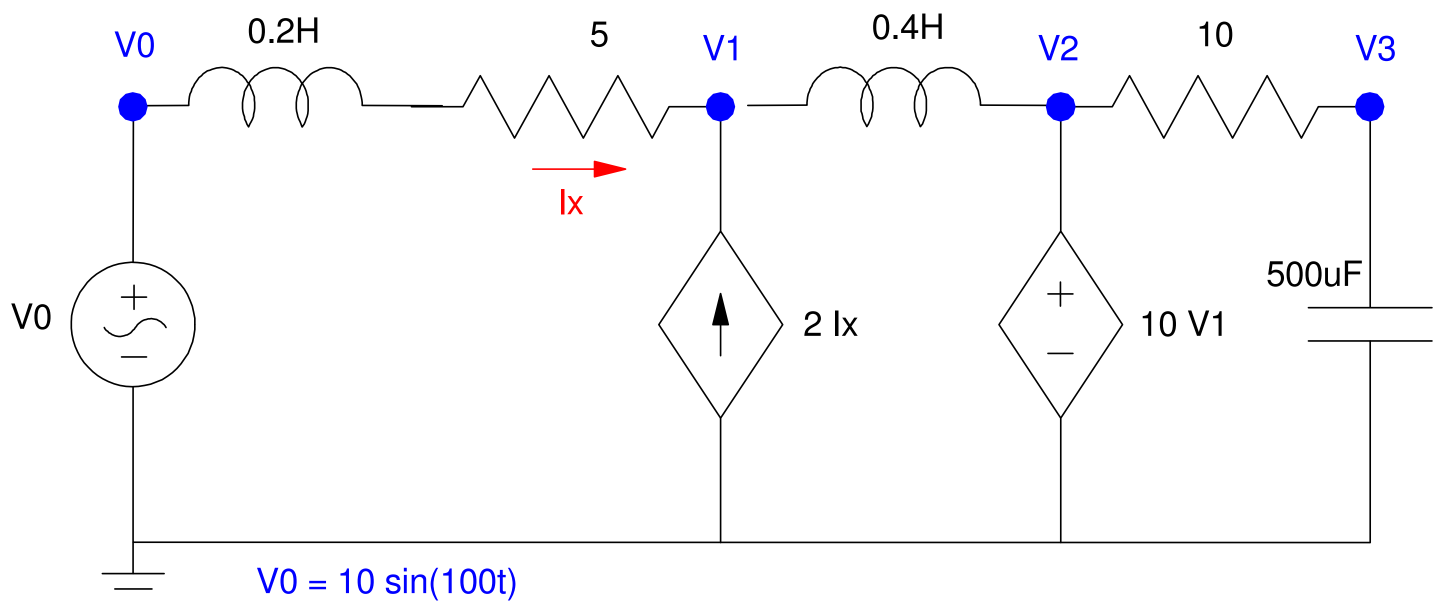
Results match

- There is an angle as well - this shows up as the phase shift in these graphs

Current Loops with Dependent Sources

Not surprisingly, it also works with dependent sources

Example: Determine the currents in the following circuit



Step 1:

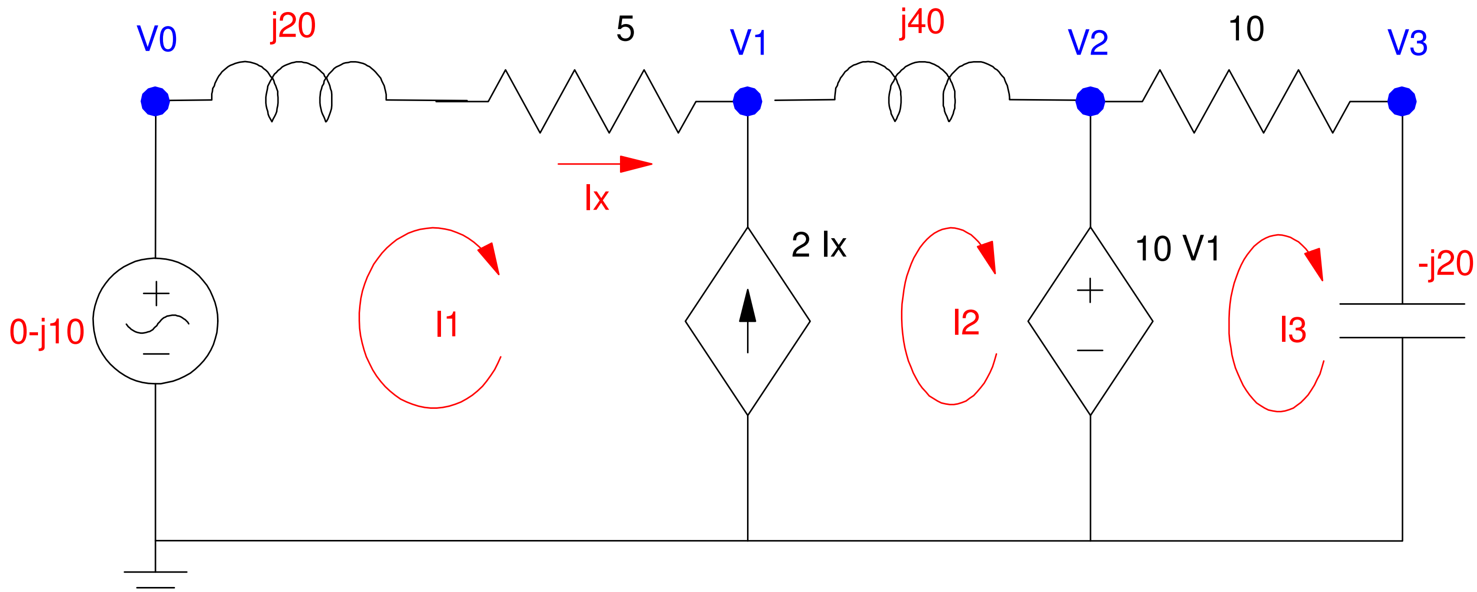
Define the loop currents

Convert to the phasor domain: (shown in blue)

$$a\cos(\omega t) + b\sin(\omega t) \rightarrow a - jb$$

$$L \rightarrow j\omega L$$

$$C \rightarrow \frac{1}{j\omega C}$$



Step 2: N equations for N uknowns

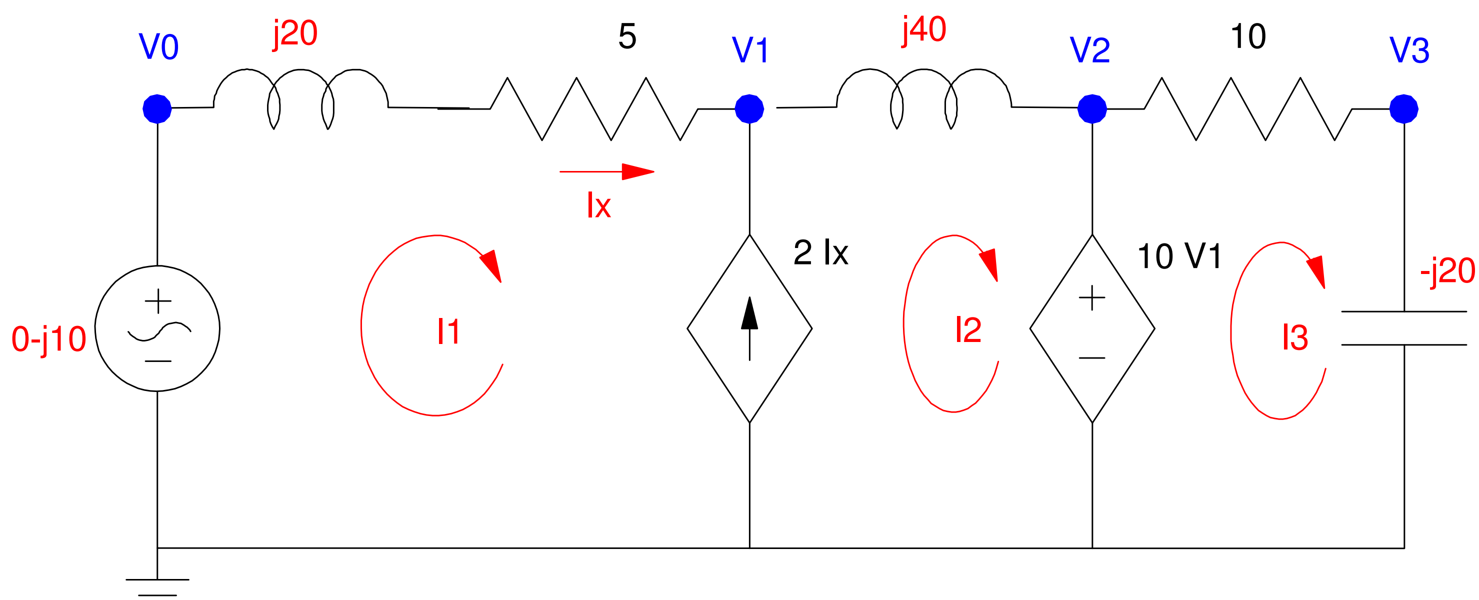
$$I_x = I_1$$

$$V_1 = j40 \cdot I_2 + (10 - j20)I_3$$

$$2I_x = I_2 - I_1$$

$$-10V_1 + 10I_3 - j20I_3 = 0$$

$$-(0 - j10) + (5 + j20)I_1 + j40I_2 + (10 - j20)I_3 = 0 \quad \textit{super loop}$$



Step 3: Solve

Group terms

$$I_x - I_1 = 0$$

$$j40 \cdot I_2 + (10 - j20)I_3 - V_1 = 0$$

$$2I_x - I_2 + I_1 = 0$$

$$-10V_1 + (10 - j20)I_3 = 0$$

$$(5 + j20)I_1 + j40I_2 + (10 - j20)I_3 = 0 - j10$$

Place in matrix form

$$\begin{bmatrix} (-1) & 0 & 0 & (1) & 0 \\ 0 & (j40) & (10 - j20) & 0 & (-1) \\ (1) & (-1) & 0 & (2) & 0 \\ 0 & 0 & (10 - j20) & 0 & (-10) \\ (5 + j20) & (j40) & (10 - j20) & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_x \\ V_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -j10 \end{bmatrix}$$

Solve in Matlab

```
>> a1 = [-1,0,0,1,0];
>> a2 = [0,40j,10-20j,0,-1];
>> a3 = [1,-1,0,2,0];
>> a4 = [0,0,10-20j,0,-10];
>> a5 = [5+20j,40j,10-20j,0,0];
>> A = [a1;a2;a3;a4;a5];
>> B = [0;0;0;0;-10j];
>> I = inv(A)*B
```

```
I1  -0.9600 - 0.7200i
I2  -2.8800 - 2.1600i
I3  -7.0400 - 1.2800i
Ix  -0.9600 - 0.7200i
V1  -9.6000 +12.8000i
```

Step 4: Convert back to the time domain

Phasor domain

```
I1  -0.9600 - 0.7200i
I2  -2.8800 - 2.1600i
I3  -7.0400 - 1.2800i
Ix  -0.9600 - 0.7200i
V1  -9.6000 +12.8000i
```

Time Domain

$$i_1(t) = -0.96\cos(100t) + 0.72\sin(100t)$$

$$i_2(t) = -2.88\cos(100t) + 2.16\sin(100t)$$

$$i_3(t) = 7.04\cos(100t) + 1.28\sin(100t)$$

Checking in CircuitLab

Matlab

- |I1| = 1.2000
- |I2| = 3.6000
- |I3| = 7.1554

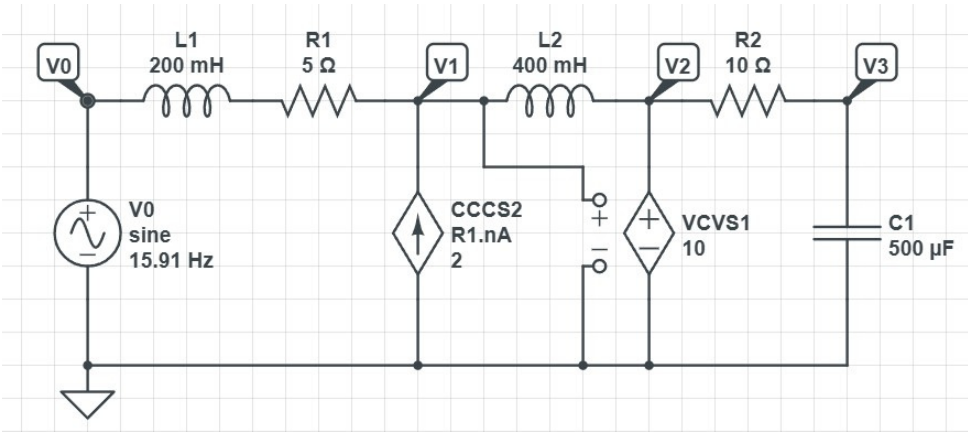
CircuitLab

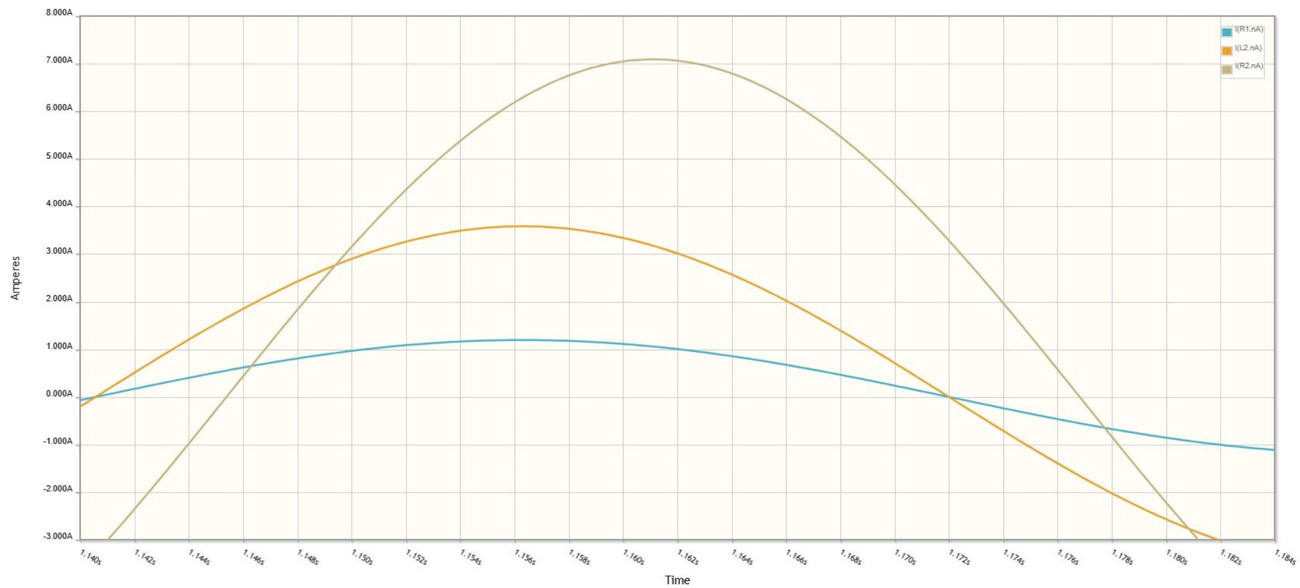
- Start at t=1 seconds
- End at t=1.2 seconds
- Step Size = 0.2ms

Result

- |I1| = 1.192A
- |I2| = 3.582A
- |I3| = 7.090A

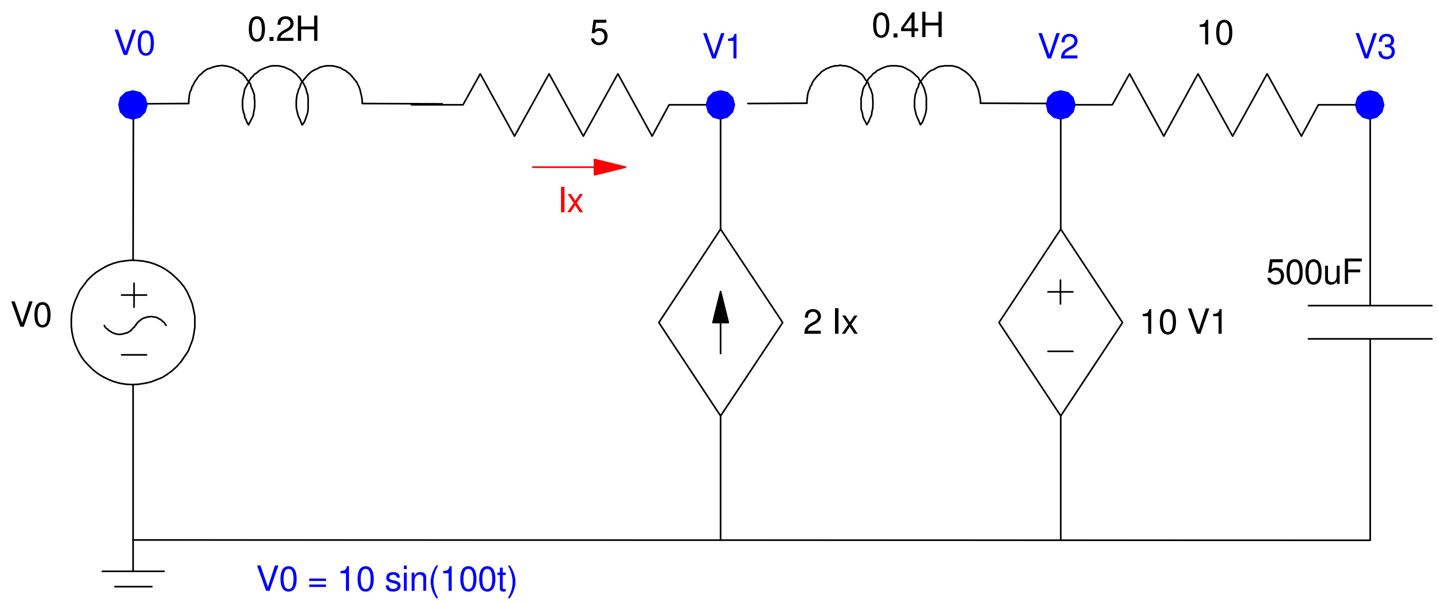
Matches Matlab





What happens if the frequency changes?

- Phasor impedances change
- Whole new problem



Summary

Current Loops work the same

- for DC inputs
- for AC inputs

The only difference is you're now dealing with complex numbers.

Matlab is able to solve N equations for N unknowns

- with real numbers, and
- with complex numbers

This is where Matlab really shines