Ideal Op-Amp

Introduction

In the last lecture, the operational amplifier (op-amp) was introducted. The circuit model for an op-amp, such as a LM358 consists of a high-gain differential amplifier along with a high input impedance.

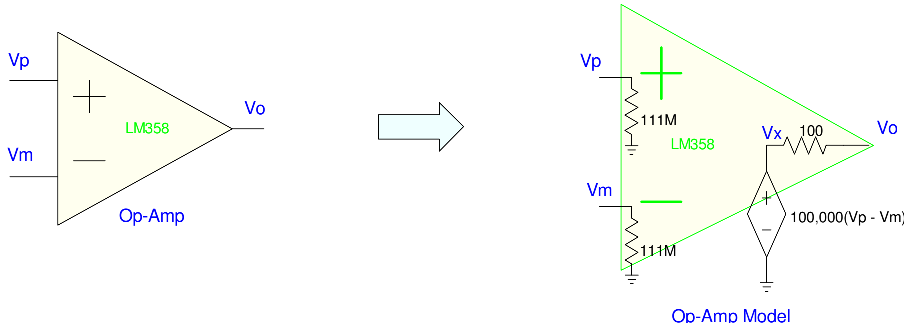

Figure 1: Op-amp symbol and its circuit model

While this model works, its rather clunky and makes circuit analysis somewhat painful even for simple circuits. We need a better tool.

One trick common in engineering is that when you have a difficult problem to solve, change the problem. Change it so that

- The new problem is easier to solve,
- While keeping the flavor of the original problem.

The ideal op-amp model is such a solution.

Ideal Op-Amp

Op-amps are high-gain differential amplifiers with high input and low output impedances. The ideal op-amp model builds upon this and assumes that these terms are either infinity or zero as shown in Table 1.

Term	LM358	Ideal
Input Bias Current	45nA	0nA
(Input Impedance @ 5V)	111M	infinity
Open-Loop Gain	100,000	infinity
Gain Bandwidth Product	1MHz	infinity
Slew Rate	250,000 V/s	infinity
Voltage swing from rail	1.5V	0V
Output Current	20mA	infinity
Output Resistance	100 Ohms	0 Ohms

Table 1: Ideal op-amp model

While this may not seem like much, these assumptions greatly simplify circuit analysis for op-amp circuits while giving fairly accurate answers.

Circuit Analysis with Ideal Op-Amps

When assuming ideal op-amps, there are two main ways of analyzing a circuit

- Voltage Nodes and
- Figure out the voltages that make the circuit work.

With voltage nodes, the goal as usual is to write N equations for N unknown voltages. Usually, one of these equations will be Vp = Vm. This is usually the case whenever there is negative feedback with the op-amp circuit (i.e. whenever the output is an analog voltage rather than a binary signal which is logic 1 or logic 0).

With the second method, you again usually assume Vp = Vm. You then try to figure out what voltages make the currents balance.

It's probably easiest to explain these with some examples.

Example 1: Find the voltages for the following op-amp circuit:

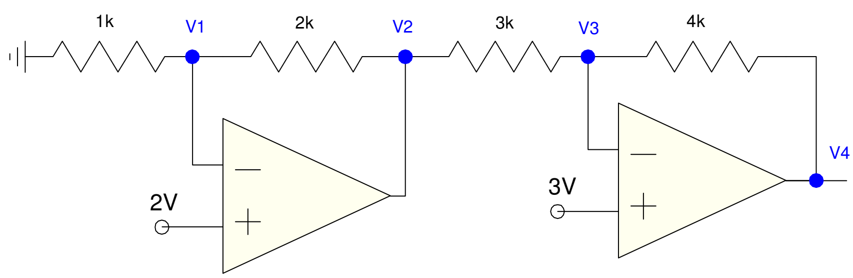

Figure 2: Find the voltages for this op-amp circuit

Let's start with using CircuitLab to find out what the answer is. From CircuitLab

- V1 = 2.000V
- V2 = 6.000V
- V3 = 3.000V
- V4 = -1.000V

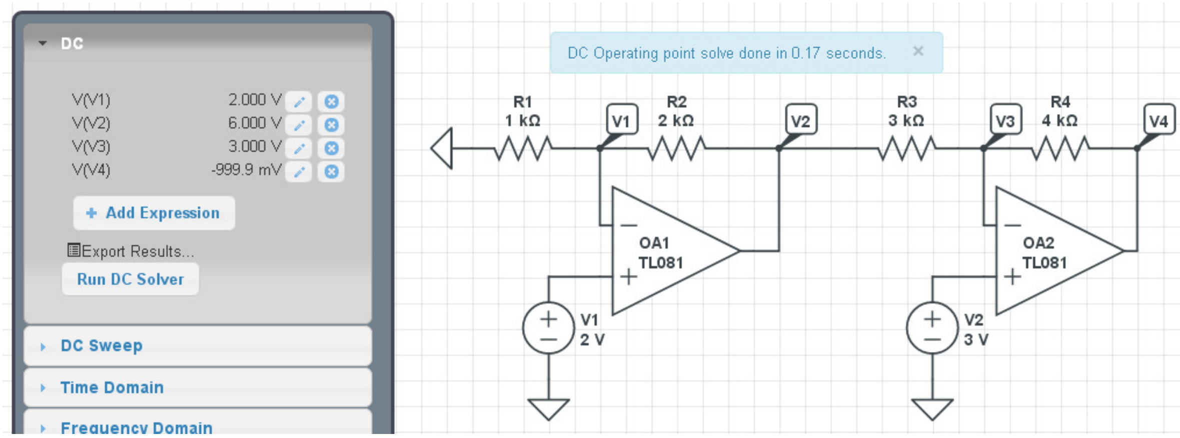

Figure 3: One way to find the voltages: use CircuitLab

A second way is to use voltage nodes. This circuit has four unknown voltages. Hence, four equations are needed. Start with the easy ones:

Vp = Vm.

$$V_1 = 2$$

$$V_3 = 3$$

We now need two more equations. As much as you like, you can't write the voltage node equations at node V2 or node V4. This is for two reasons:

- First, the current coming out of the op-amp is unknown. This makes summing the currents to zero at V2 or V4 difficult.
- Second, if you *could* write the node equations at V2 and V4, they would be redundant. With negative feedback, the op-amp provides whatever current it takes to force Vp = Vm. This is where the first two equations came from. Writing the node equations at V2 and V4 likewise just repeats the first two equations and provides no new information.

Instead of writing the node equations at the output of the op-amps, write the node equations at the inputs.

At node V1:

$$\left(\frac{V_1}{1k}\right) + \left(\frac{V_1 - V_2}{2k}\right) = 0$$

At node V3

$$\left(\frac{V_3 - V_2}{3k}\right) + \left(\frac{V_3 - V_4}{4k}\right) = 0$$

You now have four equations to solve for four unknowns. Time passes and you eventually get the same results that CircuitLab gave.

A third way to solve this circuit is to figure out what voltages make the currents balance.

Start with Vp = Vm. This tells you that

- V1 = 2V and
- V3 = 3V

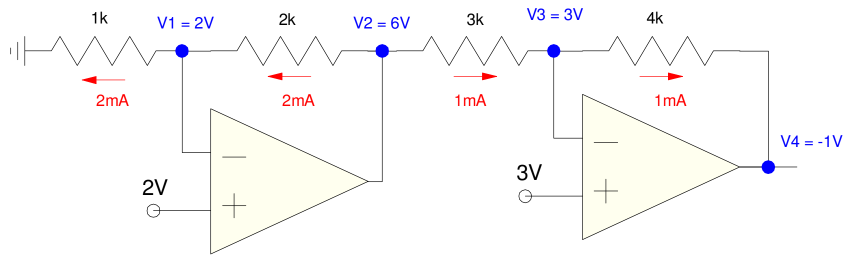

Figure 4: Figure out what voltages make the currents balance.

For the fist op-amp, knowing that V1 = 2V tells you that the current through the 1k resistor is 2mA.

From conservation of current, you also know that the current through the 2k resistor is also 2mA.

From Ohm's law, this tells you that V2 = 6V

$$V_2 = V_1 + 2mA \cdot 2k\Omega$$

Now that you know that V2 is 6V, you know that the current through the 3k resistor is 1mA

$$I = \left(\frac{V_2 - V_3}{3k\Omega}\right) = \left(\frac{6V - 3V}{3k\Omega}\right) = 1mA$$

From conservation of current, the current through the 4k resistor is also 1mA. This then lets you find the voltage at V4

$$V_4 = V_3 - 1mA \cdot 4k\Omega$$

$$V_4 = 3V - 4V = -1V$$

Note that this was much less painful than what we did last lecture. Also note that the answers are *slightly* different from what CircuitLab gives.

- Vp is not *exactly* equal to Vm.

The op-amp gain only 250,000. Likewise, V2 = 2V means that

$$250,000(V_p - V_m) = 2.00V$$

$$V_p - V_m = 8\mu V$$

The answers are close, but slightly wrong. That's to be expected. Op-amps are not ideal. They're closed, but not quite.

Example 2: Determine the voltages

As a second example, consider the circuit of Figure 5. This is a differential amplifier, similar to the AMP04 chip. Find the voltages assuming ideal op-amps.

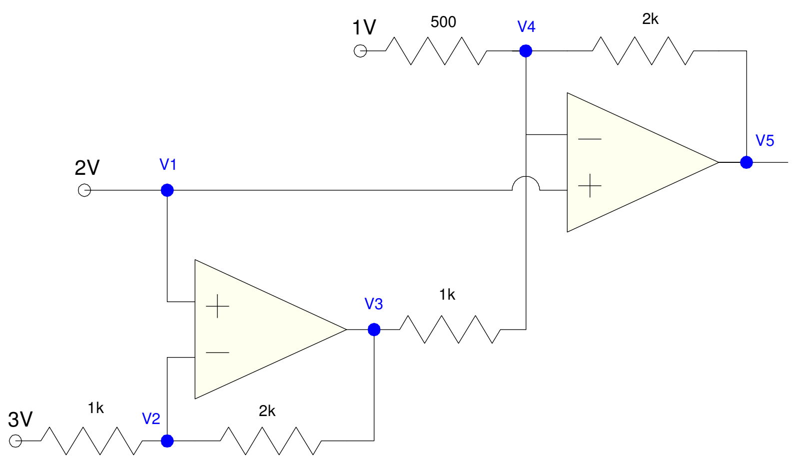

Figure 5: Find the voltages

Once again, let's start with using CircuitLab to find what the answers are. In Figure 6. the voltages found using CircuitLab are given:

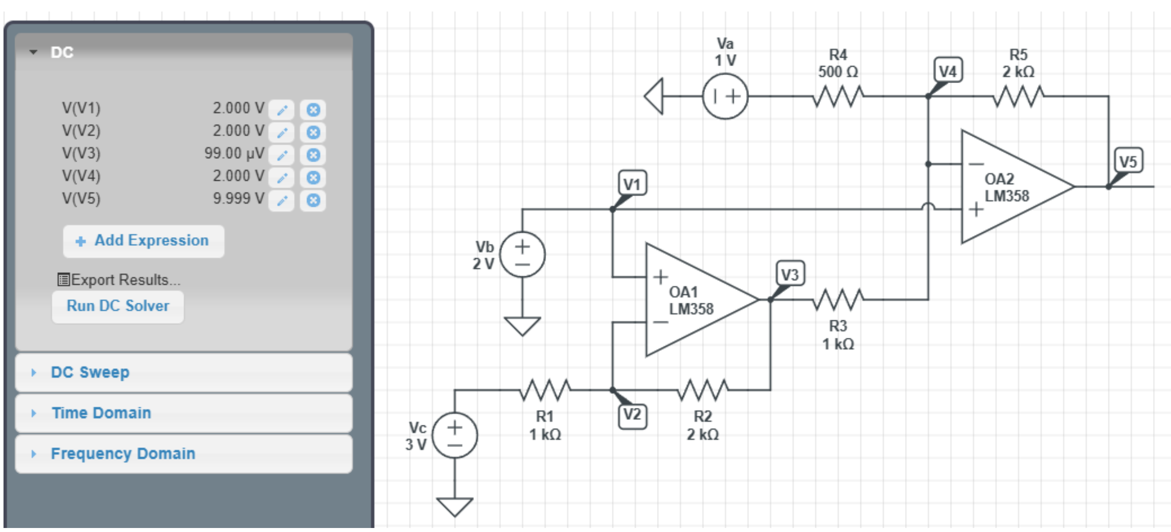

Figure 6: One way to solve this circuit: use CircuitLab

Next, let's try to get the same results by writing the voltage node equations. This circuit has five unknown voltages. We need 5 equations to solve for 5 unknowns.

Voltage Nodes: Start with the easy ones.

$$V_1 = 2$$

$$V_2 = V_1$$

$$V_4 = V_1$$

Two more equations are needed. At node V2

$$\left(\frac{V_2 - 3}{1k}\right) + \left(\frac{V_2 - V_3}{2k}\right) = 0$$

at node V4

$$\left(\frac{V_4 - 1}{500}\right) + \left(\frac{V_4 - V_5}{2k}\right) + \left(\frac{V_4 - V_3}{1k}\right) = 0$$

This gives us 5 equations to solve for 5 unknowns. Note that nodes V3 and V5 were not used for the last two equations. As mentioned before, you can't write the node equations at the output of the op-amp. You need to find two other nodes. V2 and V4 in this case.

Figure it Out: Another way to find the voltages is to figure out what voltages make the currents balance. Start with noting that Vp = Vm

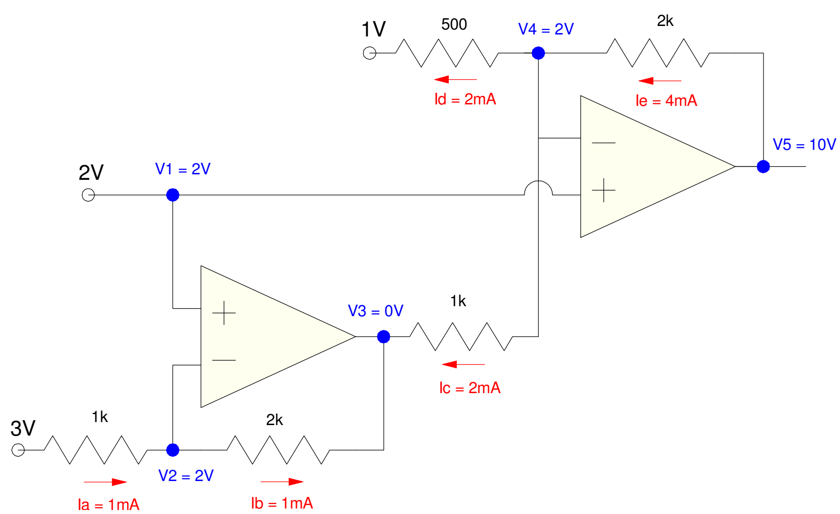

Figure 7: From Vp = Vm, we know V2 = 2V, and V4 = 2V

Starting in the lower left corner, the voltages tell us that Ia = 1mA. From conservation of current Ib is also 1mA, making V3 = 0V.

$$V_3 = V_2 - 2k\Omega \cdot 1mA$$

Knowing V3, you now know that Ic = 2mA and Id = 2mA.

From conservation of current, Ie must be 4mA. This makes V5 equal to 10V.

$$V_5 = V_4 + 2k\Omega \cdot 4mA$$

Example 3: 3-Op Amp Circuit

Next, determine the voltages for the op-amp circuit presented in Figure 8.

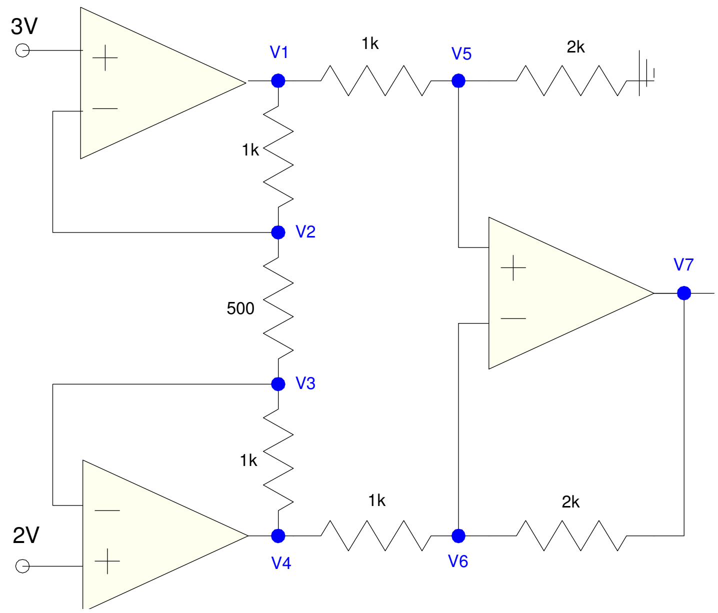

Figure 8: Find the voltages assuming ideal op-amps.

Voltage Nodes: Starting with voltage nodes, there are seven unknown voltages. Seven equations are needed. Start with the easy ones: Vp = Vm

$$V_2 = 3V$$

$$V_3 = 2V$$

$$V_5 = V_6$$

Now write the voltage node equations. At node V2

$$\left(\frac{V_2 - V_1}{1k}\right) + \left(\frac{V_2 - V_3}{500}\right) = 0$$

At node V3

$$\left(\frac{V_3 - V_2}{500}\right) + \left(\frac{V_3 - V_4}{1k}\right) = 0$$

At node V5

$$\left(\frac{V_5 - V_1}{1k}\right) + \left(\frac{V_5}{2k}\right) = 0$$

a node V6

$$\left(\frac{V_6 - V_4}{1k}\right) + \left(\frac{V_6 - V_7}{2k}\right) = 0$$

Given time, you can now solve seven equations and seven unknowns.

Figure it Out: For the second method, note that Vp = Vm. This gives figure 9.

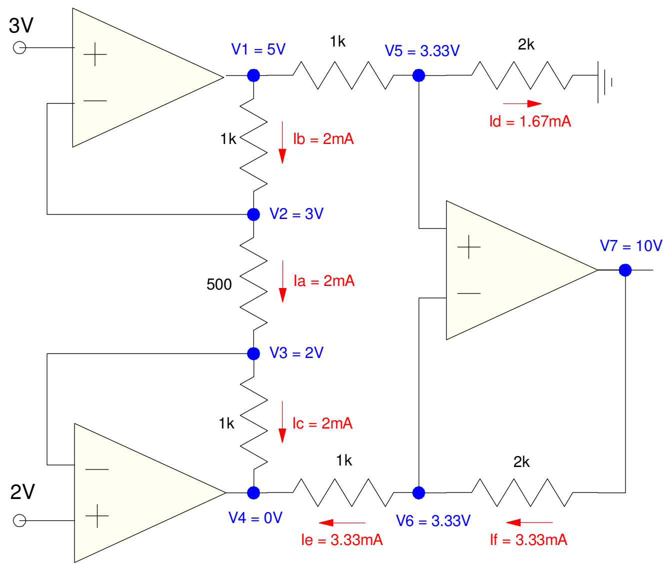

Figure 9: Figure out what voltages make the currents balance.

Start with

V2 = 3V

V3 = 2V

From V2 and V3, we known that Ia = 2mA

$$I_a = \left(\frac{V_2 - V_3}{500}\right) = 2mA$$

From conservation of current, Ib and Ic are also 2mA.

From V2 and Ib, V1 is 5V and V4 is 0V

$$V_1 = V_2 + 1k \cdot I_b = 5V$$

$$V_4 = V_3 - 1k \cdot I_c = 0V$$

From V1 = 5V, Id is 1.67V and V5 is 3.33V

Once we know that V5 is 3.333V, we also know that V6 = 3.33V.

Finally, since V6 = 3.333V, Ie = If = 3.333mA. This makes V7 = 10V.

This was *much much* easier than circuit analysis using the op-amp models.

When is the Ideal Op-Amp Model Valid?

Finally, when can you use the ideal op-amp model? This depends upon the op-amp you're using. If we're using an LM358 or similar op-amp, the ideal op-amp model works when

- The input current (45nA) or the input impedance (111M Ohms) don't matter, and
- The output current limit (20mA) isn't exceeded.

For the input impedance to be insignificant, keep resistors 1M or smaller

- R << 111M
- R < 1M Ohm

For the output current limit of 20mA to not matter, assuming you're limited to 10V at the output, keep the resiances more than

$$R > \frac{10V}{20mA} = 500\Omega$$

As long as you keep the resistors used in the range of

$$500\Omega \le R \le 1M\Omega$$

the ideal op-amp model should work with less than 1% error.

Summary

When analyzing op-amp circuits, assuming ideal op-amps greatly simplifies circuit analysis while giving results that are very close - as long as the resistors used are in the range of 500 to 1M Ohm.

Two main methods are used for analyzing circuits with ideal op-amps

a) Voltage nodes

- Start with Vp = Vm
- Write remaining N equations for N unknowns

b) Figure it out

- Currents have to balance
- Vp = Vm *usually* holds