Thevenin Equivalent and Load Lines

So far we've looked at two methods to solve a circuit: current loops and voltage nodes. You can solve any circuit using these techniques.

A third option, Thevenin Equivalents, is a little more tricky - but it can make some circuits a lot easier to analyze if you can get the hang of it.

Load Lines

The idea behind Thevenin equivalents is that the voltage-current relationship for a linear circuit follows a straight line. Likewise, any circuit which has the same voltage-current relationship will behave exactly the same.

For example, consider the following circuit:

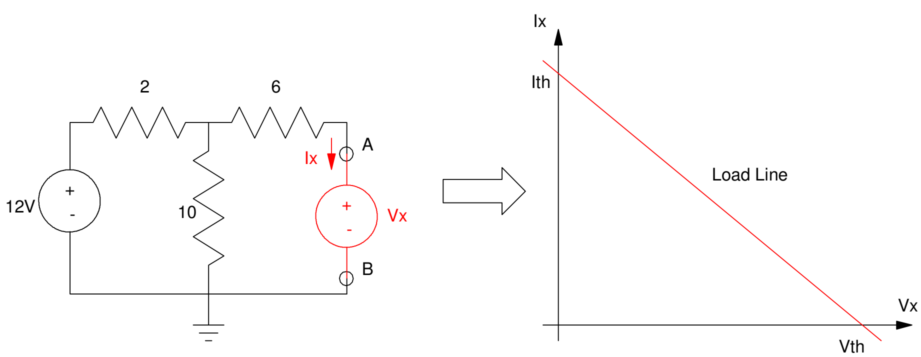

Figure 1: The load-line shows how the current at the output varies with the voltage

Since it's a linear circuit, the V/I relationship at the load (Vx, Ix) will follow a straight line as shown on the right. As far as the load is concerned, any circuit with the same load line is indistinguishable.

You can see this in CircuitLab. If you add a voltage source at the output of a circuit and sweep it, the current will follow a straight line - termed the load line.

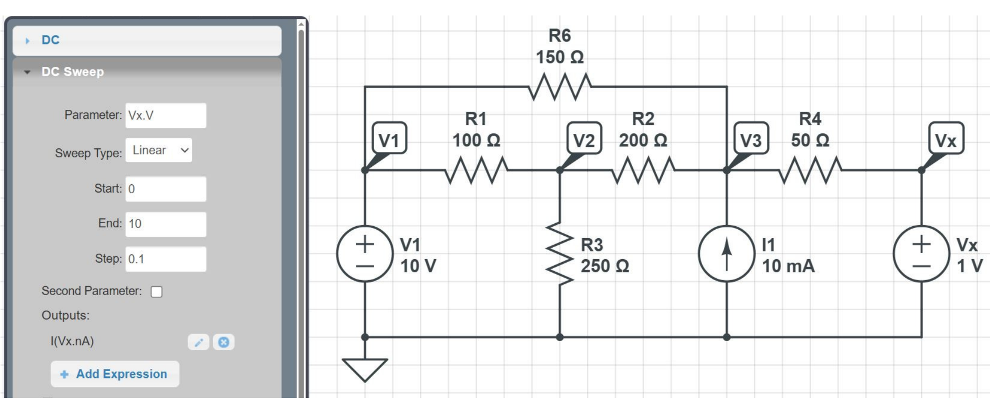

Figure 2: Given a circuit, add a voltage source at the output (Vx). Then plot the current as Vx changes.

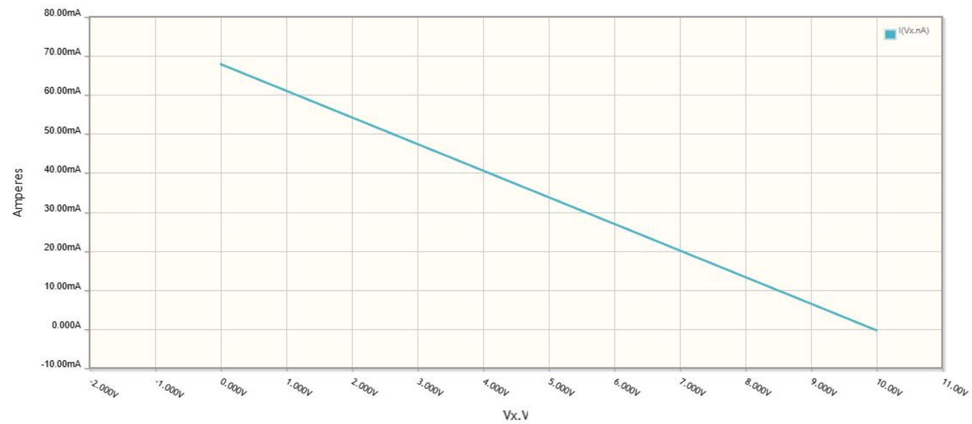

Figure 3: Resulting current (Ix) vs voltage (Vx) as Vx is swept. This VI curve is called the load line.

Thevenin and Norton Equivalent

Since all circuits with the same load line are indistinguishable, let's use the simplest one possible:

- A voltage source and resistance in series (termed a Thevenin equivalent), or
- A current source and resistance in parallel (termed a Norton equivalent).

The trick is to find the values of Vth, Rth, or Ith

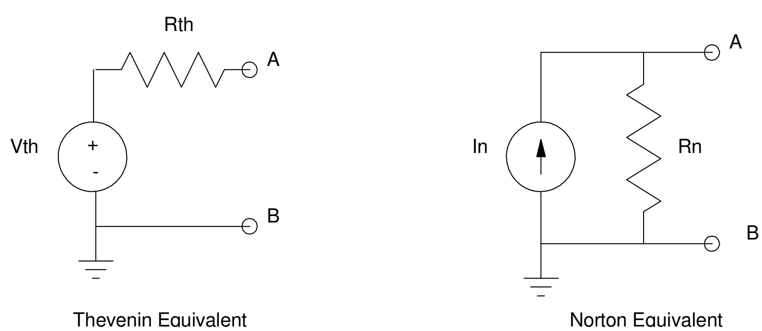

Figure 4: The Thevenin and Norton equivalent are the simplest circuit which have the same load-line.

Note that these circuits are equivalent to the circuit you're analyzing. This means that if you do the same thing to both circuits, you should get the same result. This leads to the following procedure to find the Thevenin and Norton equivalents:

- Vth: Measure the open-circuit voltage of your circuit.
- Rth = Rn: Turn off all sources (V = 0, I = 0). Measure the resulting resistance.
- In: Measure the short-circcuit current

It's probably easiest to illustrate this through examples.

Example 1: Given a load line, determine the Thevenin and Norton equivalent circuits

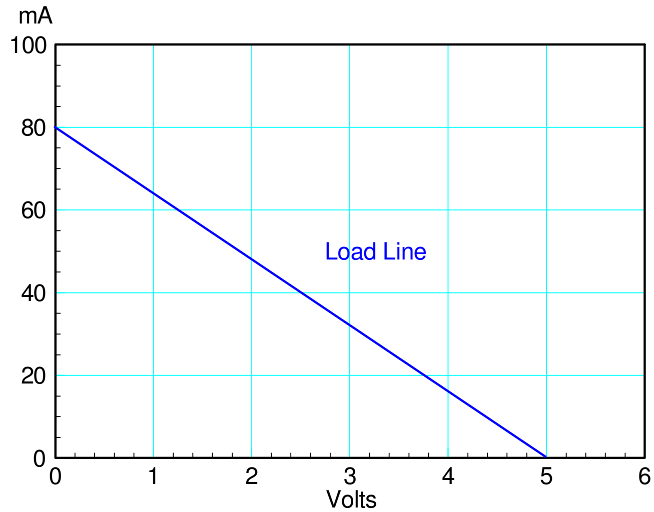

Figure 5: Find the Thevenin and Norton equivalent for the given load line.

Thevenin Equivalent:

The Thevenin voltage is the voltage you measure when the current is zero. This is the X-axis intercept

Vth = 5.00V

The Thevenin resistance is the slope of the VI curve

$$R_{th} = \frac{V(open)}{I(short)}$$

$$R_{th} = \left(\frac{5V}{80mA}\right) = 62.5\Omega$$

This gives you the Thevenin equivalent.

Norton Equivalent:

The Norton current is the short-circuit current

In = 80mA

The Norton resistance is the slope of the VI curve. This is the same as Rth

$$R_n = 62.5\Omega$$

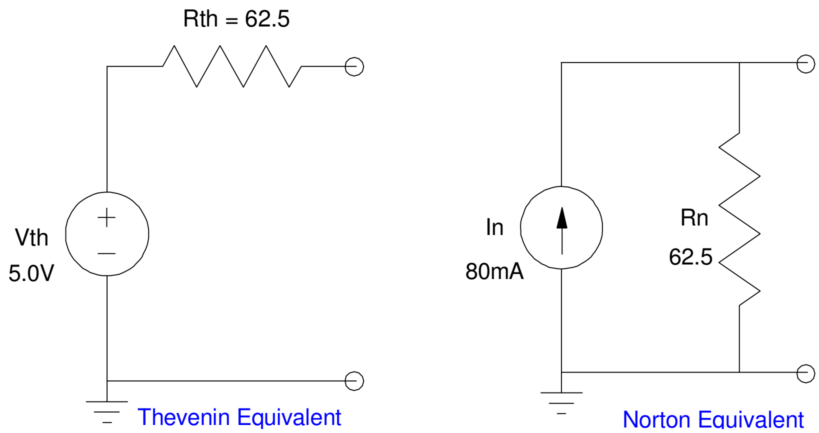

Figure 6: Thevenin and Norton equivalent circuits for the load-line of figure 5

Thevenin and Norton equivalents can also be found for a given circuit. These replace a complex circuit with the simplest circuit which has the same load line. This can be useful. From the load's standpoint, you can't tell what the circuit is: any circuit with the same load line looks the same from the load's standpoint.

For example, consider the circuit of Figure 7. Find the Thevenin and Norton equivalent.

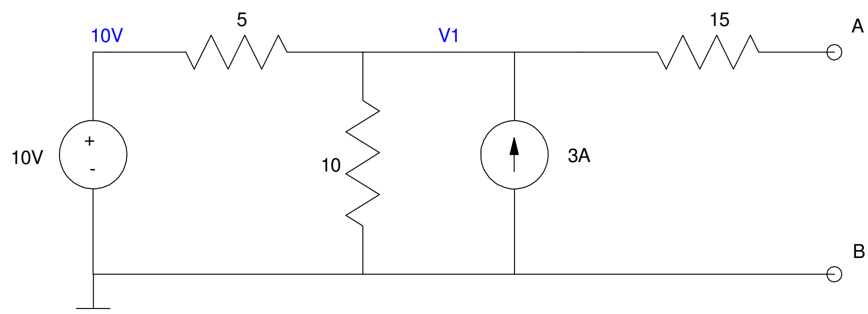

Figure 7: Find the Thevenin and Norton equivalent circuits

Thevenin Equivalent

To find Vth, measure the open-circuit voltage.

This isn't obvious so let's write the voltage node equation

$$\left(\frac{V_1-10}{5}\right)+\left(\frac{V_1}{10}\right)-3=0$$

$$V_{th}=V_1=16.67V$$

This is Vth

To find Rth, you could find the short-circuit current then calculate Rth. To find the short-circuit current, short AB and measure the current. Again, this isn't obvious so write the node equation at V1

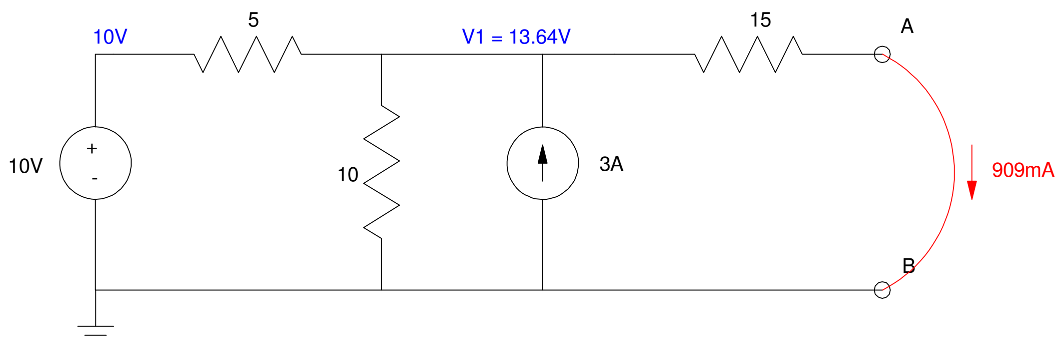

Figure 8: To find I(short), short the output and compute the short-circuit current

$$\left(\frac{V_1-10}{5}\right)+\left(\frac{V_1}{10}\right)-3+\left(\frac{V_1}{15}\right)=0$$

$$V_1=13.64V$$

$$I_{short}=I_N=\frac{13.64V}{15\Omega}=909.1mA$$

This tells you that

$$R_{th}=\frac{V(open)}{I(short)}$$

$$R_{th}=\frac{16.67V}{909.1mA}=18.333\Omega$$

Another way to calculate Rth is to first turn off all sources as shown in Figure 9

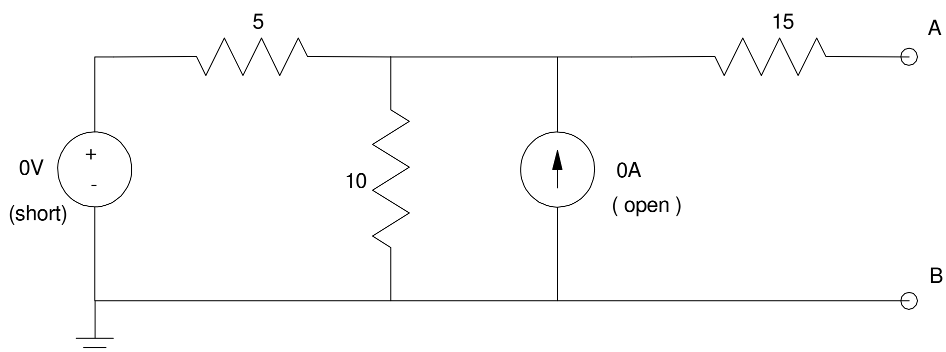

Figure 9: To find Rth, turn off all sources then compute Rab

The resistance is

$$R_{AB}=15+5||10$$

$$R_{AB}=18.333\Omega$$

The resulting Thevenin and Norton equivalents are then as presented in Figure 10.

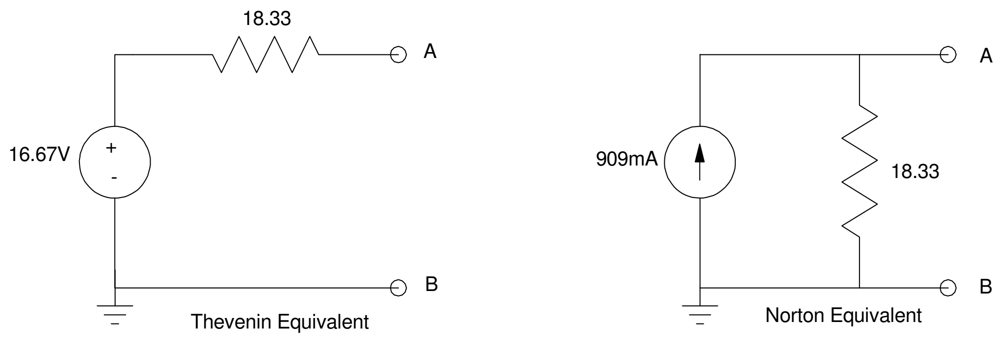

Figure 10: Thevenin and Norton equivalents of Figure 9

Why Use Thevenin and Norton Equivalents?

Sometimes, the Thevenin equivalent helps to explain what's happening with a circuit. For example, suppose you want to convert 12V to 5V. One way to do this is to build a voltage divider as shown in Figure 11.

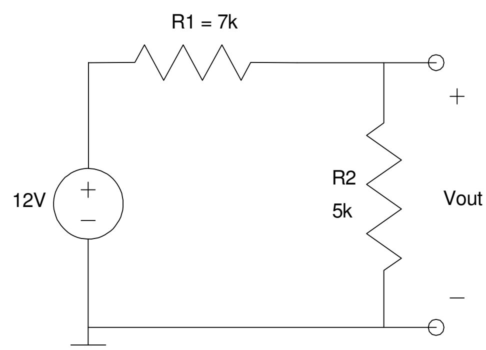

Figure 11: Voltage divider to output 5V from a 12V source

While this circuit looks OK, it has a problem. Any time you connect a load to the output, the 5V output drops. For example, if you attach a 500 Ohm resistor to Vout, the 5V output drops to 0.73V.

Why is that?

The Thevenin equivalent helps show the limitations of this design. Taking the Thevenin equivalent of Figure 11 gives you

Open-Circuit Voltage (Vth)

$$V_{th} = \left(\frac{5k}{5k+7k}\right) 12V = 5V$$

Thevenin Resistance: Turn off the source and measure the resistance looking in:

$$R_{th} = 5k || 7k = 2917\Omega$$

This tells you that the DC to DC converter circuit can be replaced with its Thevenin equivalent. Attach a 500 Ohm resistor to the load and you can see from voltage division that the voltage will drop to 731mV.

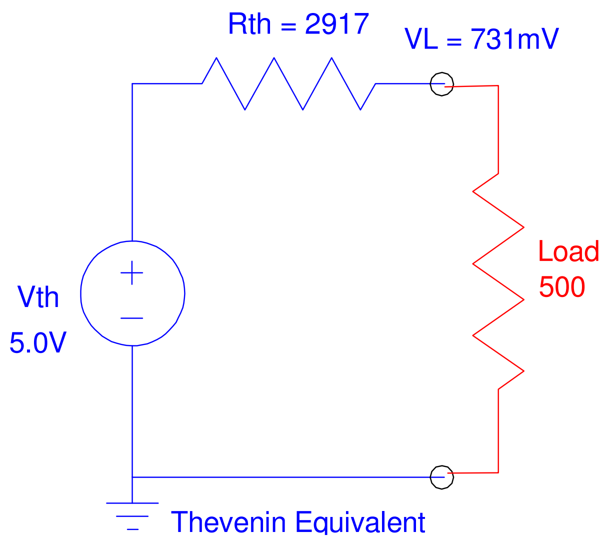

Figure 12: Attempting to draw 10mA @ 5V (500 Ohms) results in VL dropping to 731mV

The net result is that Figure 11 is a DC to DC converter that only works if the load is much larger than 2917 Ohms.

Circuit Simplification using Thevenin and Norton Equivalent

Given a circuit, you can change between Thevenin and Norton equivalents using the relationship

$$R_{Thevenin} = R_{Norton}$$

$$V_{Thevenin} = I_{Norton} \cdot R$$

$$I_{Norton} = \frac{V_{Thevenin}}{R}$$

For example, determine the Thevenin equivalent seen at terminal AB

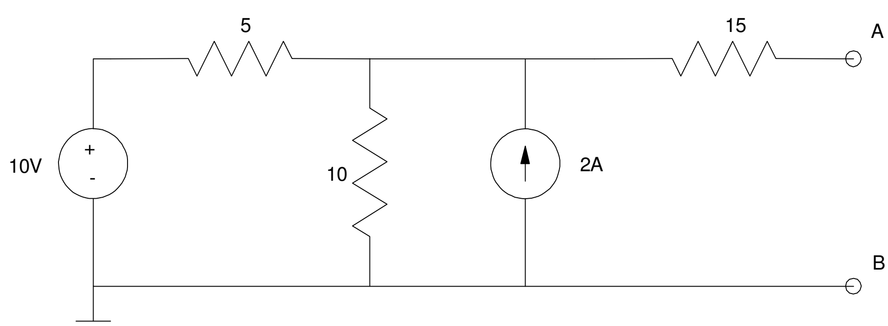

Figure 13: Find the Thevenin equivalent at AB

Step 1: Convert the 10V / 5 Ohm resistor to its Norton equivalent

$$I_N = \frac{10V}{5\Omega} = 2A$$

$$R_N = R_{Th} = 5\Omega$$

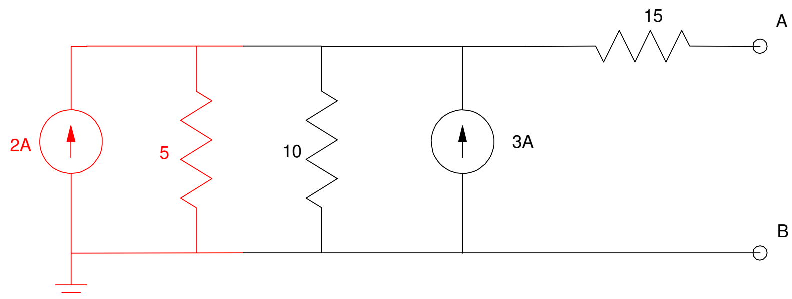

Figure 14: Convert the 10V : 5 Ohm source to its Norton equivalent (shown in red)

Add the resistors in parallel

$$R_{net} = 5||10 = \left(\frac{1}{5} + \frac{1}{10}\right)^{-1} = 3.333\Omega$$

Add the current sources

$$I_{net} = 2 + 3 = 5$$

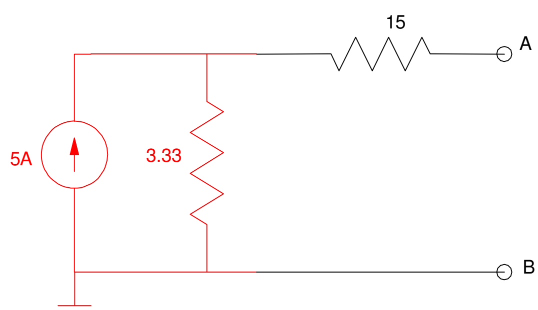

Figure 15: Add the resistors and current sources in parallel:

Convert back to a Thevenin equivalent

$$V_{th} = 5A \cdot 3.3333\Omega = 16.666V$$

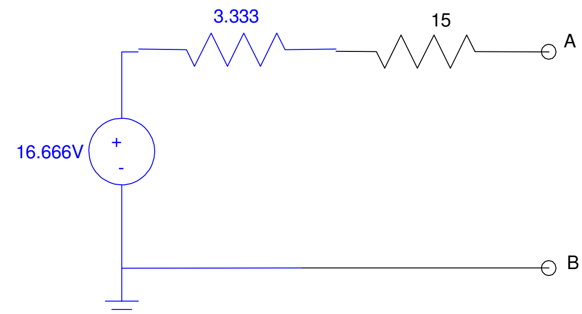

Figure 16: Convert to a Thevenin equivalent (shown in blue)

Add the resistors in series and you have the Thevenin equivalent looking in from terminals AB

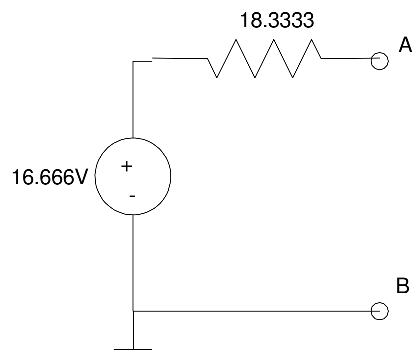

Figure 17: Thevenin Equivalent of Figure 13

Summary

For linear circuits (resistor circuits),

- If you plot the voltage vs. current at the load,
- You get a line

This is termed *the load line*

As far as the load is concerned

- A given circuit looks like
- Any other circuit with the same load line

This allows you to replace a circuit with a simpler model

- Thevenin equivalent or
- Norton equivalent