Super-Loops

Introduction

In the previous lecture, the circuit-analysis technique known as *current loops* was presented. Here, the idea is

- Given a circuit with N windows
- Define N current loops, one in each window,
- Then write N equations for N unknown currents using conservation of voltage.

Normally, this works without a hitch. For some circuits, however, you run into a problem. Take for example the circuit of Figure 1.

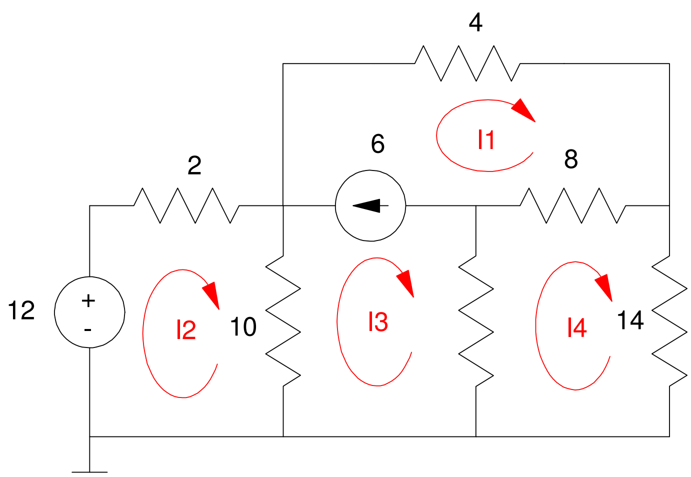

Figure 1: To solve for 4 unknown currents, 4 equations are needed

For this circuit, there are four windows. Hence, we need to write 4 equations to solve for 4 unknowns. Three equations are fairly easy to write:

Current Source

$$I_1 - I_3 = 6$$

Loop I2:

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

Loop I4:

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

There's now a problem.

- I can't write the loop equation around I1 since I don't know the voltage drop across the 6A current source.
- I also can't write the loop equation around I3, again because I don't know the voltage drop across the 6A current source.

So, where does the 4th equation come from?

Superloops

A superloop is a closed path that includes currents from several windows. There are several ways to define a valid superloop.

To find a valid superloop, note that the 4 Ohm resistor does not appear anywhere yet. For the 4th equation, define a loop which includes the 4 Ohm resistor and does not include the current source.

One option is to use the loop in blue in Figure 2:

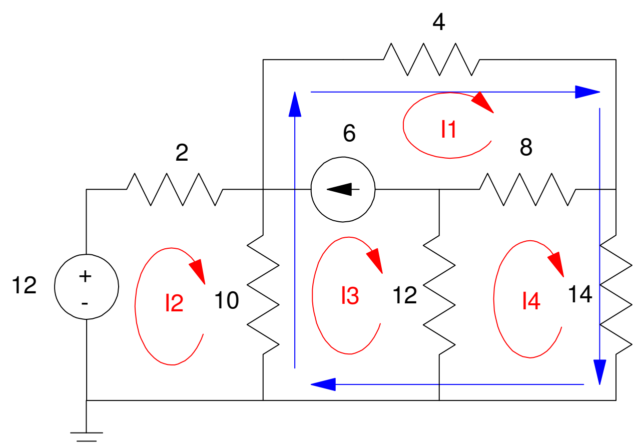

Figure 2: One valid definition of a superloop

With this definition of the superloop, the 4th equation becomes

$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$

A second option is to use a slightly different path for the superloop as shown in Figure 3:

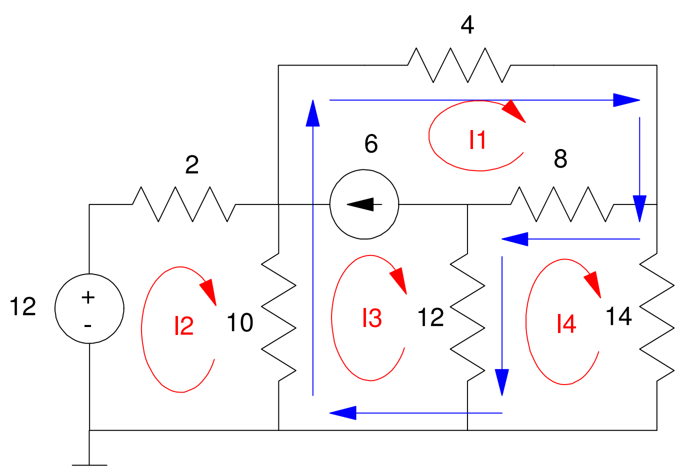

Figure 3: Another valid definition of a superloop

With this definition, the 4th equation becomes

$$10(I_3 - I_2) + 4(I_1) + 8(I_1 - I_4) + 12(I_3 - I_4) = 0$$

Both equations are valid and can be used as the 4th current loop equation.

Once you get 4 equations for 4 unknowns, you can solve for the unknown currents. Using the superloop defined in Figure 2 (repeated here for convenience):

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

$$I_1 - I_3 = 6$$

$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$

become

$$12I_2 - 10I_3 = 12$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

$$I_1 - I_3 = 6$$

$$4I_1 - 10I_2 + 10I_3 + 14I_4 = 0$$

Put in matrix form:

$$\begin{bmatrix} 0 & 12 & -10 & 0 \\ -8 & 0 & -12 & 34 \\ 1 & 0 & -1 & 0 \\ 4 & -10 & 10 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 6 \\ 0 \end{bmatrix}$$

Throw into MATLAB and solve

```
-->A = [0,12,-10,0; -8,0,-12,34; 1,0,-1,0; 4,-10,10,14]

    0.     12.  - 10.     0.
  - 8.     0.   - 12.     34.
    1.     0.   - 1.      0.
    4.  - 10.     10.     14.

-->B = [12;0;6;0]

    12.
    0.
    6.
    0.
```

```
-->inv(A)*B

I1     3.5712271
I2    -1.0239774
I3    -2.4287729
I4    -0.0169252
```

Note that current can be negative. A negative current just means the direction we assumed for I1..I4 was wrong.

This checks with CircuitLab (Figure 4).

- I(R1) matches up with I2 (calculations)
- I(R5) matches up with I4 (calculations)
- I(R6) matches up with I1 (calculations)
- I(R3) maches up with I2 - I3 (calculations)

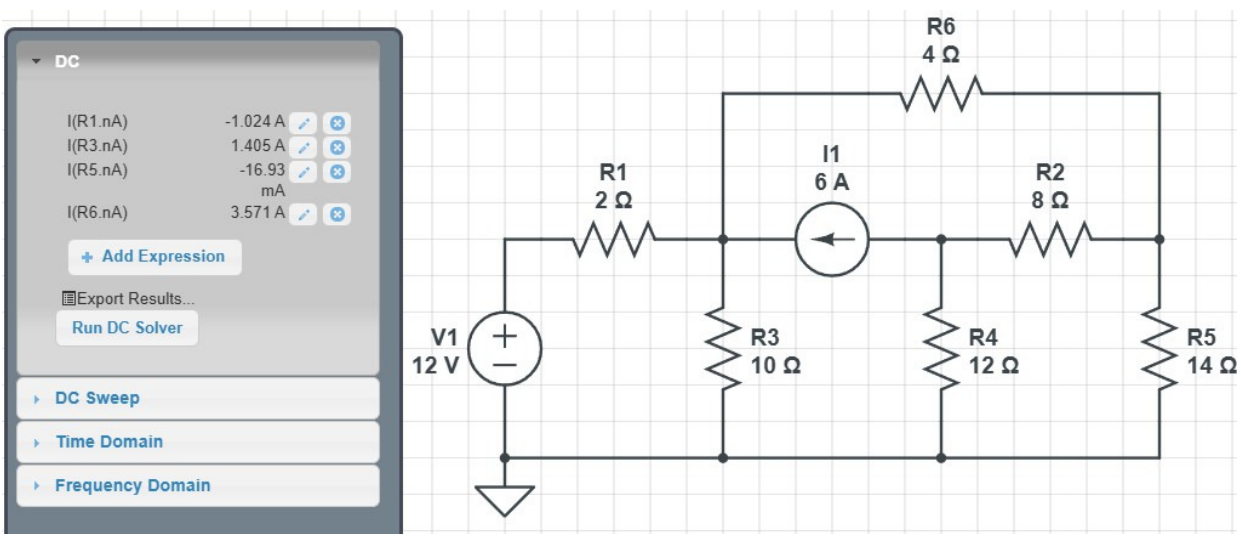

Figure 4: CIrcuitLab simulation to verify calculations

Note that some paths for the 4th equation are not valid. For example, suppose you use the superloop of Figure 5:

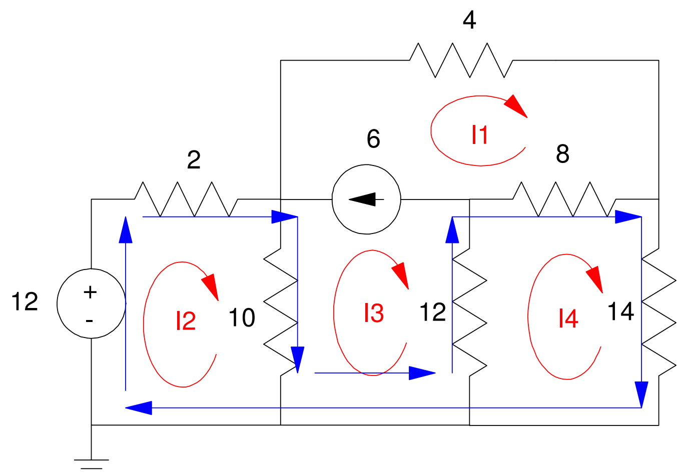

Figure 5: Example of an invalid superloop (shown in blue)

With this superloop, the 4th equation becomes

$$-12 + 2(I_2) + 10(I_2 - I_3) + 12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

This makes the 4 equations to solve for 4 unknowns:

(1) $12I_2 - 10I_3 = 12$

(2) $-8I_1 - 12I_3 + 34I_4 = 0$

(3) $I_1 - I_3 = 6$

(4) $-8I_1 + 12I_2 - 22I_3 + 34I_4 = 12$

Putting these in matrix form and solving in MATLAB gives the following results:

```
-->A = [0,12,-10,0; -8,0,-12,34; 1,0,-1,0; -8,12,-22,34]

    0.     12.   - 10.    0.
  - 8.     0.    - 12.    34.
    1.     0.    - 1.     0.
  - 8.     12.   - 22.    34.

-->B = [12;0;6;12]

    12.
    0.
    6.
    12.

-->inv(A)*B
        !--error 19
Problem is singular.
```

A is not invertible. MATLAB is trying to tell you that you don't have 4 independent equations - meaning there is insufficient information to determine I1..I4.

The problem is the 4 Ohm resistor never appears in any of the 4 equations. This means information is missing. This show up in MATLAB as a singular matrix.

You need to use a different (valid) path for the 4th equation.

Super Loops with Dependent Sources

If you have dependent sources, it just means you need more equations:

- N equations for each of the N current loops,
- Plus one equation for each dependent source

For example, find I1..I4

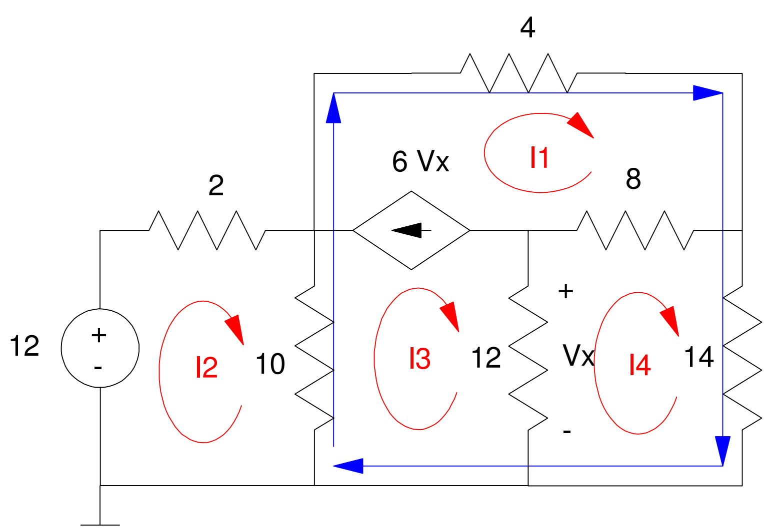

Figure 6: Circuit with a Dependent Source

Step 1: Define the currents (shown in red)

Step 2: Determine how many equations you need. There are five unknowns (I1, I2, I3, I4, Vx). Write 5 equations for 5 unknowns.

Step 3: Start writing equations. Start with the easy ones:

$$V_x = 12(I_3 - I_4)$$

$$6V_x = I_1 - I_3$$

Write the loop equations around I2 and I4

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

For the last equation, use a super-loop (shown in blue)

$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$

This gives 5 equations to solve for 5 unknowns. Grouping terms:

$$12I_3 - 12I_4 - V_x = 0$$

$$I_1 - I_3 - 6V_x = 0$$

$$12I_2 - 10I_3 = 12$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

$$4I_1 - 10I_2 + 10I_3 + 14I_4 = 0$$

Write these in matrix form

$$\begin{bmatrix} 0 & 0 & 12 & -12 & -1 \\ 1 & 0 & -1 & 0 & -6 \\ 0 & 12 & -10 & 0 & 0 \\ -8 & 0 & -12 & 34 & 0 \\ 4 & -10 & 10 & 14 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Throwing these equations into MATLAB and solve:

```
A = [0,0,12,-12,-1 ; 1,0,-1,0,-6 ; 0,12,-10,0,0];
A = [A ; -8,0,-12,34,0 ; 4,-10,10,14,0]

    0.     0.      12.   - 12.   - 1.
    1.     0.    - 1.      0.    - 6.
    0.     12.   - 10.     0.      0.
  - 8.     0.    - 12.     34.     0.
    4.   - 10.     10.     14.     0.

B = [0;0;12;0;0]

    0.
    0.
    12.
    0.
    0.

I = inv(A)*B
```

```
I1    1.0219378
I2    1.3210662
I3    0.3852794
I4    0.3764369
Vx    0.1061097
```

This matches up with results from CircuitLab shown in Figure 7.

Note that

- I(R1) matches up with I2 (calculated)
- I(R5) matches up with I4 (calculated)
- I(R6) matches up with I1 (calculated)

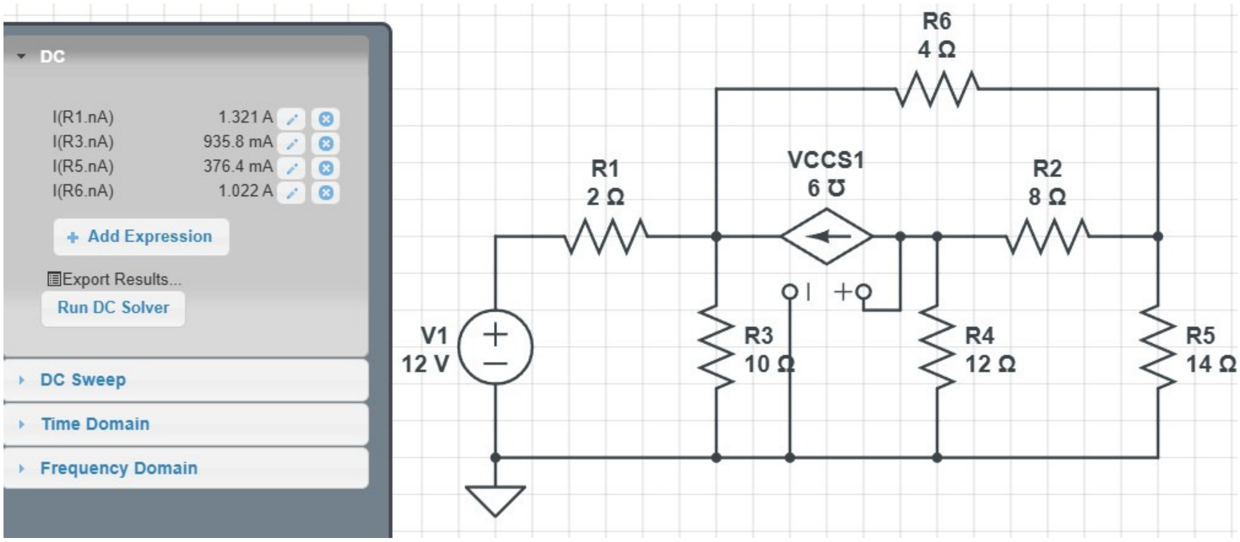

Figure 7: CircuitLab results for a voltage-controlled current source

Summary

With current loops, the goal is to write N loop equations to solve for N unknown currents

Sometimes, you can't write a given loop equation

- Typically when there's a current source
- Voltage is unknown
- You likewise can't sum the voltages to zero

Super-Loops allow you to write the remaining equations

- The sum of the voltages around *any* closed path must be zero
- Often times, there are several super-loop equations you could use.