Current Loops (Current Mesh)

Kirchoff's Current Loops (KCL)

Introduction

Voltage Nodes are one way to come up with N equations for N unknowns for a given circuit. Voltage Nodes is based upon the principle of conservation of current. The idea is to sum the currents to zero at each voltage node to come up with N equations to solve for N unknowns.

Another technique uses conservation of voltage. With conservation of voltage, the sum of all voltages around a closed-path must be zero. Using this property, you can also come up with another set of N equations for N unknowns.

This second technique is called

- *Current Loops*
- *Kirchoff's Current Loops or*
- *Kirchoff's Voltage Law.*

That is the topic of this lecture.

Step 1: Counting the Number of Current Loops

The first step in writing current loop equations is determining how many loop equations you need. One way to think of this is to draw the circuit on paper without any crisscrossing lines and count the number of 'windows' in the circuit.

For example, the following circuits have three and four 'windows'. Hence, you need to write three equations to solve for three unknowns.

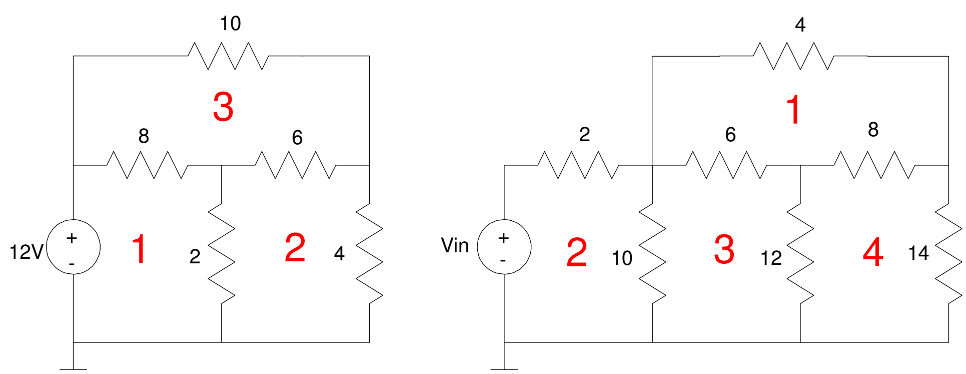


Figure 1: A circuit with 3 windows (left) and 4 windows (right).

Note that for some topologies, you can't draw the circuit on a flat piece of paper without crossing wires (for example, a resistor cube with one set of diagonals connected with a 13th resistor.) In these cases, you probably need to use voltage node techniques.

The number of 'windows' tell you how many current loop equations you need to write.

Step 2: Label Currents

Once the number of windows is determined,

- Define the direction of current in each window, and
- Label each current.

The direction is somewhat arbitrary. I personally like to make each current go clockwise in each window. If you choose this for each window, a pattern emerges when you write the current-loop equations that let you check for sign errors.

For example, for the previous circuit, one valid definition of current direction and labels is shown in Figure 2.

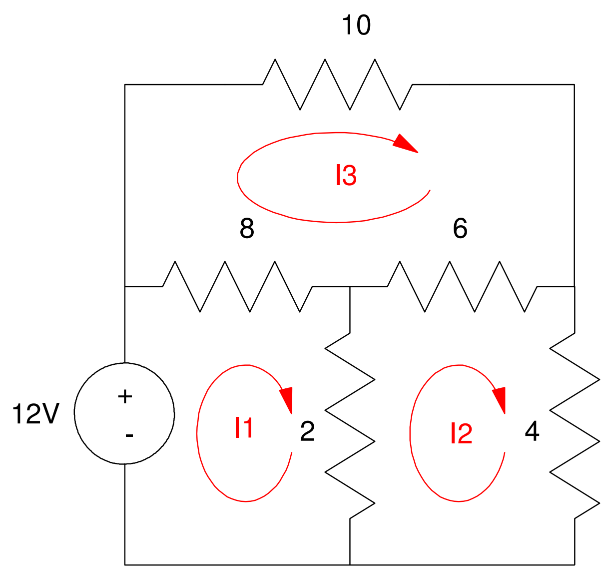


Figure 2: Step 2: Define the loop currents and label them

Another definition of current loops is shown in Figure 4.

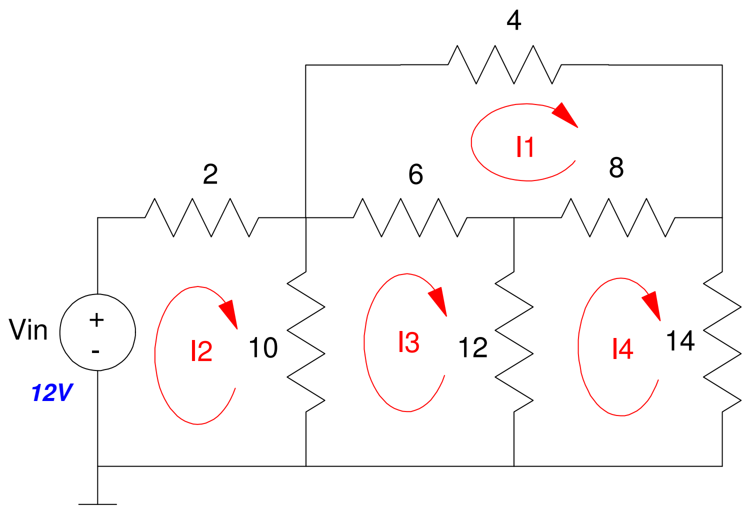


Figure 4: Definition of current loops for a circuit with 4 windows.

Step 3: Write N equations for N unknowns.

Going back to Figure 3, there are three current loops. To solve for these three currents, three equations are needed.

Start with loop I1. If you start in the lower-left corner and go around loop I1, summing all the voltages you encounter, you get

$$-12 + (I_1 - I_3)8 + (I_1 - I_2)2 = 0 \quad (1)$$

Around loop I2:

$$(I_2 - I_1)2 + (I_2 - I_3)6 + (I_2)4 = 0 \quad (2)$$

Around loop I3:

$$(I_3 - I_1)8 + (I_3)10 + (I_3 - I_2)6 = 0 \quad (3)$$

Referring to Figure 4, the loop equations would be:

$$4(I_1) + 8(I_1 - I_4) + 6(I_1 - I_3) = 0 \qquad \textit{loop I1}$$

$$-12 + 2(I_2) + 10(I_2 - I_3) = 0 \qquad \textit{loop I2}$$

$$10(I_3 - I_2) + 6(I_3 - I_1) + 12(I_3 - I_4) = 0 \qquad \textit{loop I3}$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0 \qquad \textit{loop I4}$$

Note that there is a pattern here:

When you go around loop I1

- All of the I1 terms are positive
- All of the other terms are negative

When you go around loop I2

- All of the I2 terms are positive
- All of the other terms are negative

In addition, the unit all match up. For each loop equation, you are adding

Volts + Volts + Volts = 0

This lets you quickly check for errors. For example, if the last equation was

$$12(I_4 - I_3) + 8(I_4 - I_1) + (I_4) = 0$$

you'd know right away there is an error. The units are

Volts + Volts + Amps = 0

This doesn't work: you can't add volts to amps. There is a mistake in this equation.

Step 4: Solve

Once you have N equations for N unknowns, solve. For Figure 2, the three loop equations

$$-12 + (I_1 - I_3)8 + (I_1 - I_2)2 = 0$$

$$(I_2 - I_1)2 + (I_2 - I_3)6 + (I_2)4 = 0$$

$$(I_3 - I_1)8 + (I_3)10 + (I_3 - I_2)6 = 0$$

simplify to

$$10\,I_1 - 2\,I_2 - 8\,I_3 = 12$$

$$-2\,I_1 + 12\,I_2 - 6\,I_3 = 0$$

$$-8\,I_1 - 6\,I_2 + 24\,I_3 = 0$$

Placing in matrix form

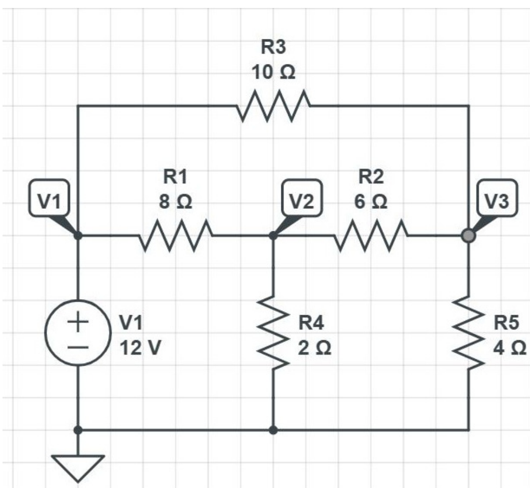
$$\begin{bmatrix} 10 & -2 & -8 \\ -2 & 12 & -6 \\ -8 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

You can then solve using Matlab

In Matlab:

```
A = [10,-2,-8;-2,12,-6;-8,-6,24]

    10.   - 2.    - 8.
  - 2.      12.   - 6.
  - 8.    - 6.      24.

B = [12;0;0]

    12.
    0.
    0.

inv(A)*B

I1    2.0655738
I2    0.7868852
I3    0.8852459
```

You can then check your answers with CircuitLab



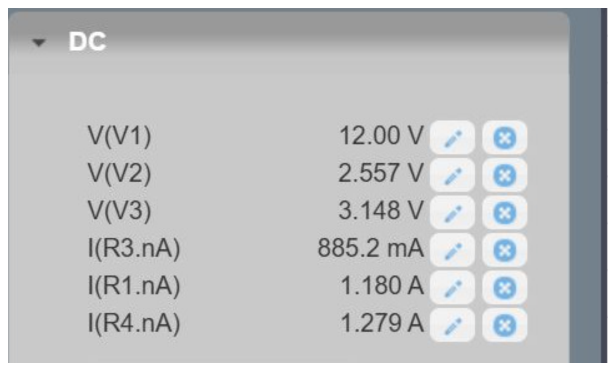


Figure 5: Checking results in CircuitLab. The currents match calculations

For the second circuit with four current loops

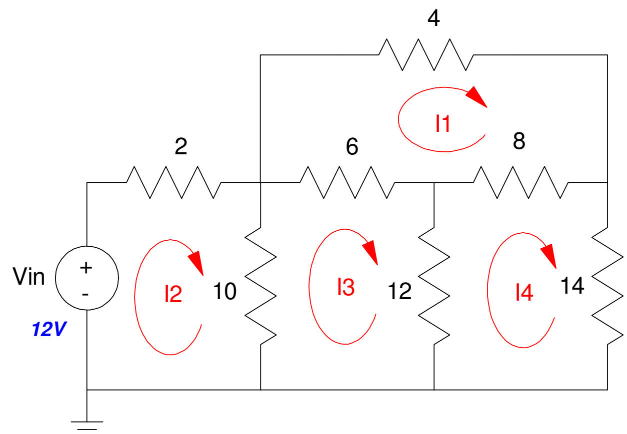


Figure 6: 4-Window Circuit (repeated)

the current loop equations were:

$$4I_1 + 8(I_1 - I_4) + 6(I_1 - I_3) = 0$$

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$10(I_3 - I_2) + 6(I_3 - I_1) + 12(I_3 - I_4) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

Grouping terms:

$$18I_1 - 6I_3 - 8I_4 = 0$$

$$12I_2 - 10I_3 = 12$$

$$-6I_1 - 10I_2 + 28I_3 - 12I_4 = 0$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

Place in matrix form:

$$\begin{bmatrix} 18 & 0 & -6 & -8 \\ 0 & 12 & -10 & 0 \\ -6 & -10 & 28 & -12 \\ -8 & 0 & -12 & 34 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Then solve using Matlab:

```
A = [18,0,-6,-8; 0,12,-10,0; -6,-10,28,-12; -8,0,-12,34]
B = [0;12;0;0]
inv(A)*B

I1:    0.5164104
I2:    1.7860913
I3:    0.9433096
I4:    0.4544411
```

Again, you can check your answers using CircuitLab

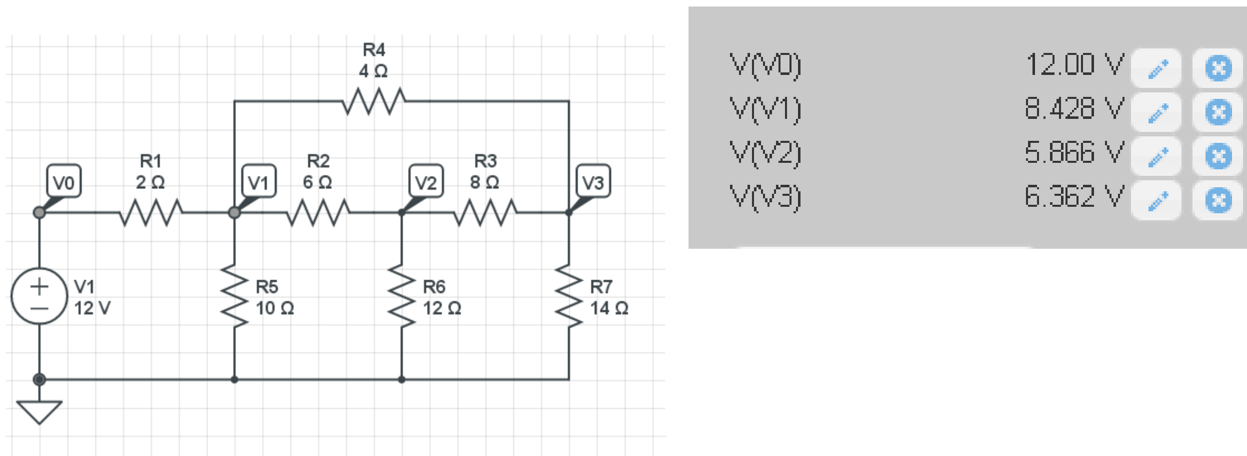


Figure 7: Checking answers in CircuitLab

With CircuitLab, the individual currents are not shown. Instead, combinations are there:

$$I_1 = \left(\frac{V_1 - V_3}{R_4}\right) = 0.5165A$$

$$I_2 = \left(\frac{V_0 - V_1}{R_1}\right) = 1.7860A$$

$$I_2 - I_3 = \left(\frac{V_1}{R_5}\right) = 0.8428A$$

$$I_4 = \left(\frac{V_3}{R_7}\right) = 0.4544A$$

These match with calculations.

Current Loops with Voltage Sources

Voltage sources don't cause any problems with writing the current loops equations. Instead of using V = IR, just use the voltage.

- If you encounter the + sign first, add the voltage.
- If you encounter the - sign first, subtract the voltage.

For example, consider the circuit in Figure 8.

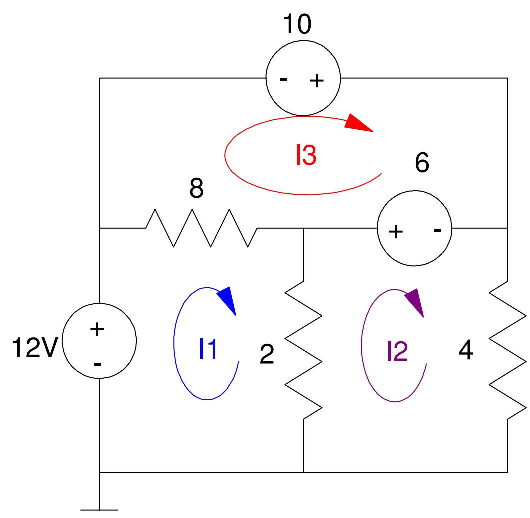


Figure 8: Current loops where there are voltage sources

The loop equations become:

I1: $-12 + 8(I_1 - I_3) + 2(I_1 - I_2) = 0$

I2: $2(I_2 - I_1) + 6 + 4(I_2) = 0$

I3: $-10 + 6 + 8(I_3 - I_1) = 0$

Note that as you go around loop I3

- You encounter the - sign on the 10V source first, so you subtract 10.
- You encounter the + sign on the 6V source first, so you add 6.

Summary

Current Loop Equations are based upon the conservation of voltage

- The sum of all voltages around a closed-path must be zero

To solve for N unknown currents, you write N current loop equations

You can then solve N equations for N unknowns using techniques from Math 129