SuperNodes:

Introduction

In the previous lecture, the technique known as Voltage Nodes was used to determine the voltages in a given circuit. Voltage Nodes is based upon conservatio of current: the sum of all currents flowing from a node must be zero. Using conservation of current, N voltage node equations were written to solve for N knowns. Once N equations are found for N unknowns, linear algebra can be used to solve for these N unknowns using Matlab.

For example, in the circuit in Figure 1, there are three voltage nodes (plus ground). To solve for these voltages, three equation are needed to solve for 3 unknpwns.

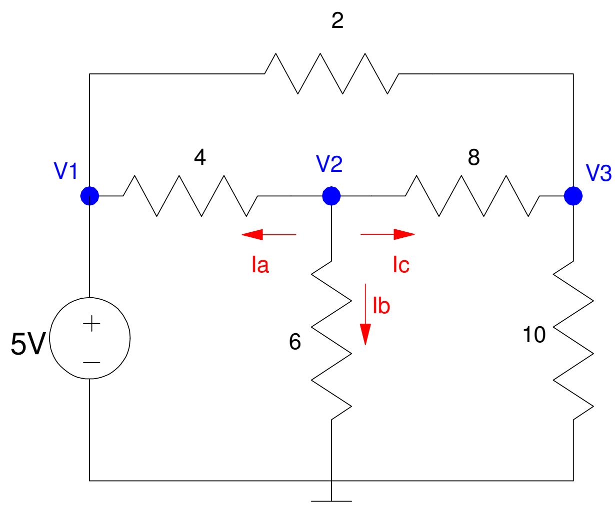


Figure 1: To solve for 3 unknown voltages, write 3 equations for 3 unknowns.

For this circuit, the three equations would be

$$V_1 = 5$$

$$\left(\frac{V_2 - V_1}{4}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_2 - V_3}{8}\right) = 0$$

$$\left(\frac{V_3 - V_1}{2}\right) + \left(\frac{V_3 - V_2}{8}\right) + \left(\frac{V_3}{10}\right) = 0$$

Once you have 3 equations for 3 unknowns, group the terms, express in matrix form, then solve using Matlab.

While this technique usually works, there are circuits where voltage nodes has probems. For example, the circuit in Figure 2 has a voltage source between nodes V1 and V3. This causes problems.

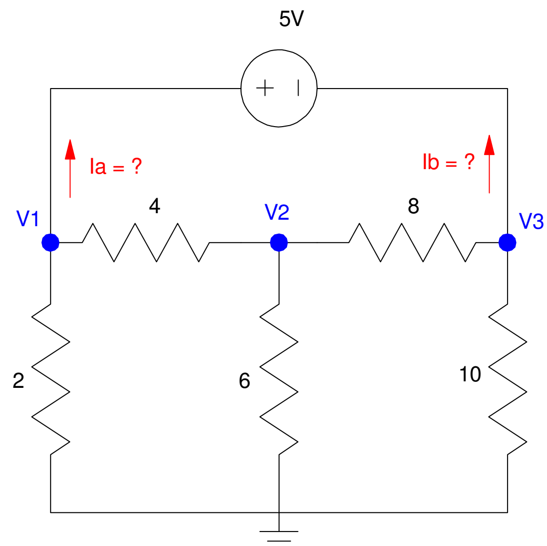


Figure 2: Circuit where voltage nodes has problems

Two equations are fairly easy to write. The votlave source gives

$$V_1 - V_3 = 5 \tag{1}$$

At node #2

$$\left(\frac{V_2 - V_1}{4}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_2 - V_3}{8}\right) = 0 \tag{2}$$

To solve for three unknowns, a third equation is needed. However, the voltage node equations at V1 and V3 don't work:

- At node V1, the current Ia is not known. This prevents you from writing the voltage node equation at node V1.
- At node V3, the current Ib is also not known. This prevents you from writing the voltage node equation at node V2.

So, what do you do in this case?

The answer is to write a third equation using somethingn called a *supernode*. That is the topic of this lecture.

Supernodes

Conservation of current doesn't just apply to a single votlage node: it applies to any closed path. Supernodes use this to come up with a third voltage node equation by defining a closed path that encircles two or mode voltage nodes.

For example, in Figure 3, a closed-path that encircles nodes V1, V2 and V3 is defined.

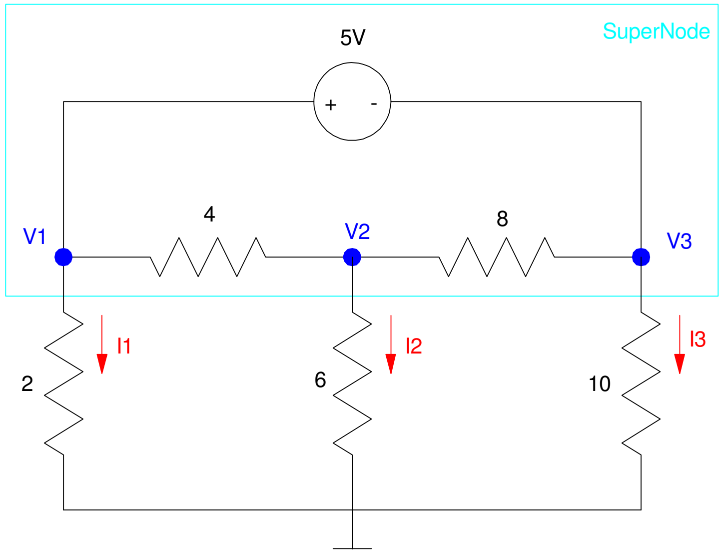


Figure 3: Define a closed path that encircles nodes V1,. V2, and V3. This is termed a *supernode*

From conservation of current, the total current flowing from the supernode must be zero. This provides us with the third equation to solve for the three unknown voltages.

$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 0 \qquad (3)$$

Note that supernodes are not unique. Another perfectly valid supernode is shown in Figure 4.

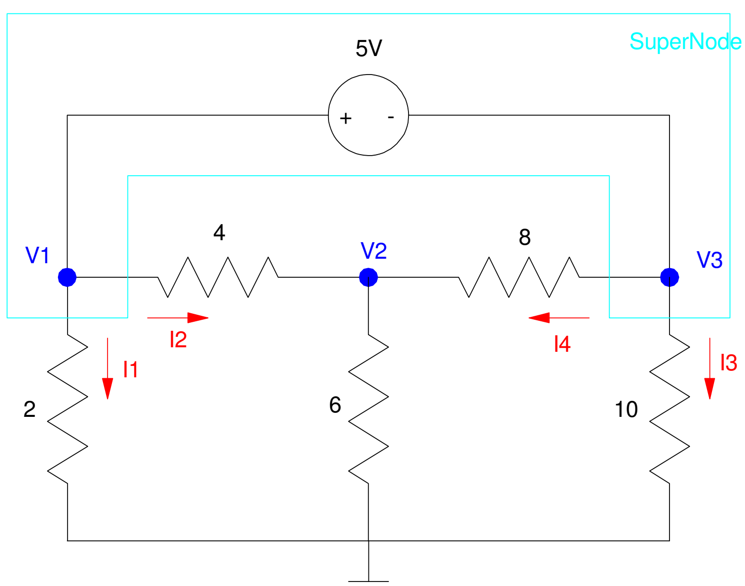


Figure 4: Another valid supernode.

With this supernode, the third equation would be:

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_1 - V_2}{4}\right) + \left(\frac{V_3}{10}\right) + \left(\frac{V_3 - V_2}{8}\right) = 0 \tag{3}$$

Note that with the previous lecture, a pattern was used to check our node equations:

- At node V1, all the V1 terms were positive and all the rest were negative.

With with supernodes, this pattern becomes

- All the voltages contained within the supernode are positive
- The other voltages are negative

Once you have three equations for three unknowns, solve.

Grouping terms:

$$V_1 - V_3 = 5 \tag{1}$$

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 0 \tag{2}$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 0 \tag{3}$$

Place in matrix form:

$$\begin{bmatrix} 1 & 0 & -1 \\ -0.25 & 0.5417 & -0.125 \\ 0.5 & 0.1666 & 0.1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [1,0,-1 ; -0.25,0.5417,-0.125 ; 0.5,0.16666,0.1]

    1.0000         0   -1.0000
   -0.2500    0.5417   -0.1250
    0.5000    0.1667    0.1000

>> B = [5;0;0]

     5
     0
     0

>> V = inv(A)*B

V1     0.9677
V2   -0.4838
V3   -4.0323
```

Using CircuitLab, you can check your answers:

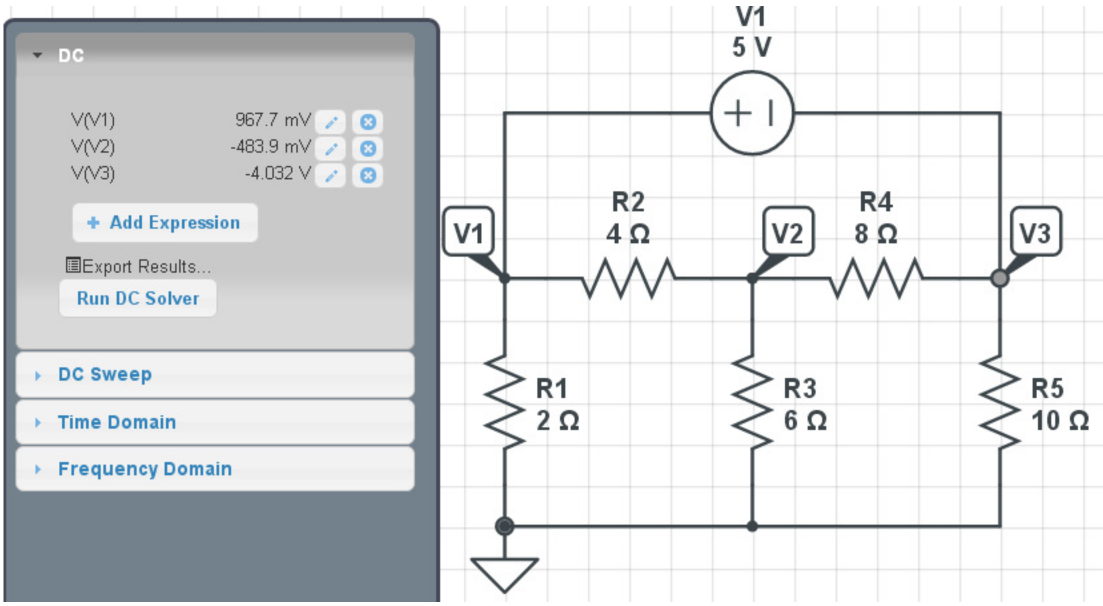


Figure 5: Circuitlab agrees with our computed results.

When is a Supernode Equation Valid?

This brings up a question. There are often several valid supernodes you can use. How do you know whether your supernode equation is valid?

There are several ways to do this.

First, check to make sure that all curcit elements are included in your equations. For example, suppose you have the circuit of Figure 6.

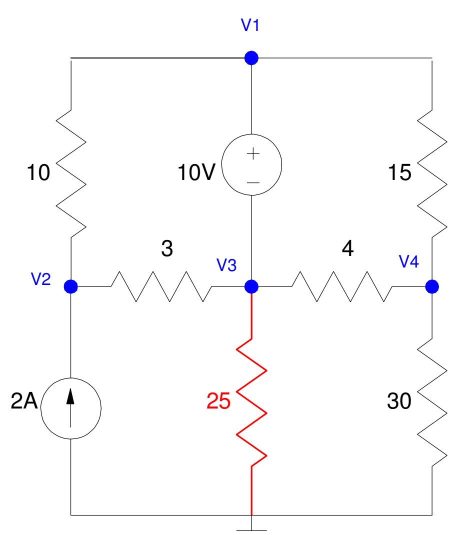


Figure 6: Write 4 equations for 4 unknown votlages.

Three equations are easy to write:

$$V_1 - V_3 = 10 \tag{1}$$

$$-2 + \left(\frac{V_2 - V_1}{10}\right) + \left(\frac{V_2 - V_3}{3}\right) = 0 \tag{2}$$

$$\left(\frac{V_4 - V_1}{15}\right) + \left(\frac{V_4 - V_3}{4}\right) + \left(\frac{V_4}{30}\right) = 0 \tag{3}$$

Note that the 25 Ohm resistor doesn't show up in any of these equations. The 25 Ohm resistor matters: if you change it, the voltages will change. Hence, the supernode equation *must* include this 25 Ohm resistor to be valid.

A second method is to use Matbab. If you have 4 valid equations, then you'll be able to solve for the voltages using a matrix inverse

$$AV = B$$

$$V = A^{-1}B$$

If you get an error in matlab, such as

```
>> V = inv(A)*B
Warning: Matrix is singular to working precision.

V =

   NaN
   NaN
   NaN
   NaN
```

this tells you that your supernode equation is invalid.

What's happening is this. To solve for 4 unknown voltages, you need 4 linearly independent equations. If the 4th equation is redundant (linear combination for the first three),

- There's no new information
- There's not enough information to solve
- The A matrix is not invertable

For example, in Figure 7, two valid and one invalid supernodes are defined.

- For the valid supernodes, the current through the 25 Ohm resistor appears in the supernode equation.
- For the invalid supernode, the 25 Ohm resistor does not.

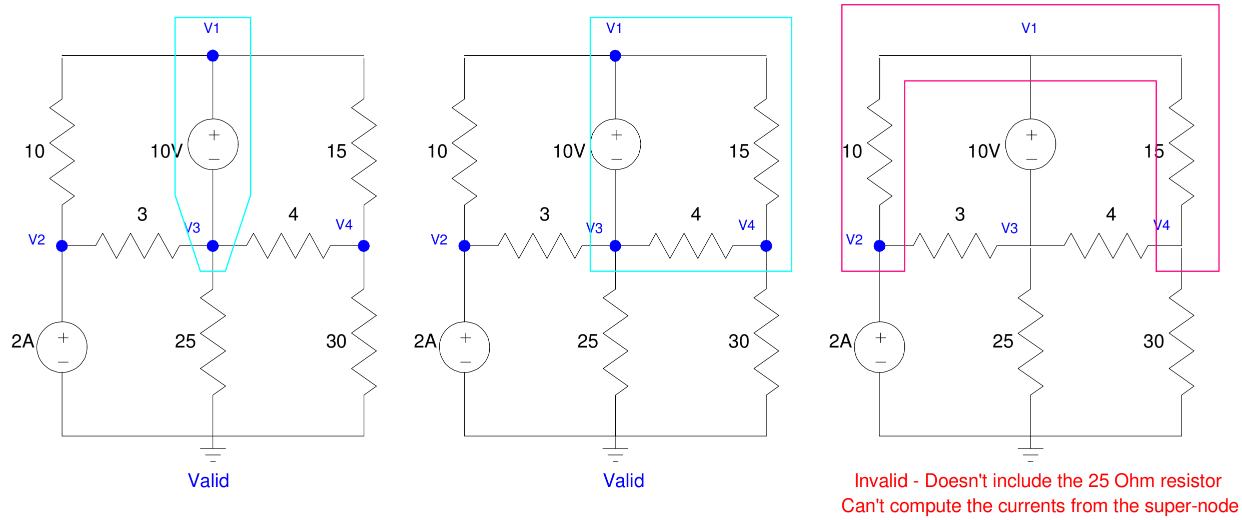


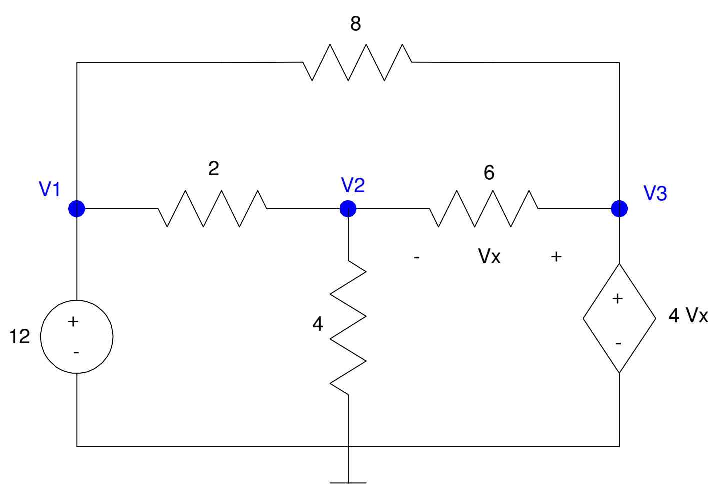
Figure 7: Two valid and one invalid definition of the supernodee.

Voltage Nodes with Dependent Sources

If you have a dependent source in your circuit, the way you handle the node(s) with the dependent source depend upon whether it's a current or a voltage source.

- If it's a current source, sum the currents to zero as normal.
- If it's a voltage source, you can't sum the current to zero at that nodeL the source provides whatever current it takes to hold the voltage. Instead, replace that node voltage with its voltage as defined by the controlled voltage source.

For example, consider the following circuit



Circuit with a Dependent Source

Start with writing Vx in terms of what you know (V1, V2, V3)

$$V_x = V_3 - V_2 \tag{1}$$

Now write three more equations so you can solve N equations for N unknowns. Start with the easyones, the voltage sources

$$V_1 = 12 \tag{2}$$

$$V_3 = 4V_x \tag{3}$$

Now we need one more equation: sum the current from V2 to zero

$$\left(\frac{V_2-V_1}{2}\right)+\left(\frac{V_2}{4}\right)+\left(\frac{V_2-V_3}{6}\right)=0 \tag{4}$$

This gives 4 equations for 4 unknowns. (Note: you could simplify this into 3 equations for 3 unknowns, but I prefer not to do so. With Matlab, it's easy to solve N equations for N unknowns. What's most important is that you get the equations right.)

To solve in Matlab, group terms

$$V_x - V_3 + V_2 = 0 \tag{1}$$

$$V_1 = 12 \tag{2}$$

$$V_3 - 4V_x = 0 \tag{3}$$

$$\left(\frac{-1}{2}\right)V_1+\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_2+\left(\frac{-1}{6}\right)V_3=0 \tag{4}$$

Placing in matrix form

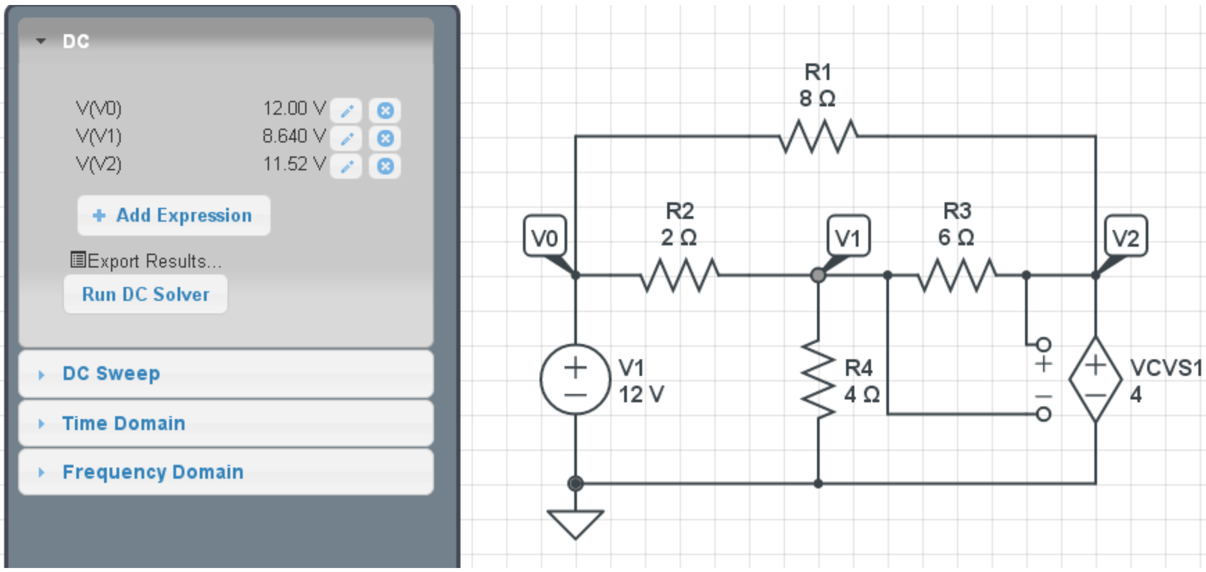
$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ -0.5 & 0.9167 & -0.1666 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_x \end{bmatrix}=\begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> A = [0,1,-1,1 ; 1,0,0,0 ; 0,0,1,-4 ; -0.5,0.9167,-0.1666,0]

         0    1.0000   -1.0000    1.0000
    1.0000         0         0         0
         0         0    1.0000   -4.0000
   -0.5000    0.9167   -0.1666         0

>> B = [0;12;0;0]

     0
    12
     0
     0


>> V = inv(A)*B
```

```
V1    12.0000
V2     8.6385
V3    11.5180
Vx     2.8795
```

Checking in Circuitlab - the voltages match



Circuitlab Results: Voltages match our calculations

Summary

Conservation of current applies to any closed path

- The sum of all currents from a closed region is zero

If you are having problems coming up with N equations for N unknowns, add a super-node

- Define a closed-path
- Such that all currents from the super-node are known

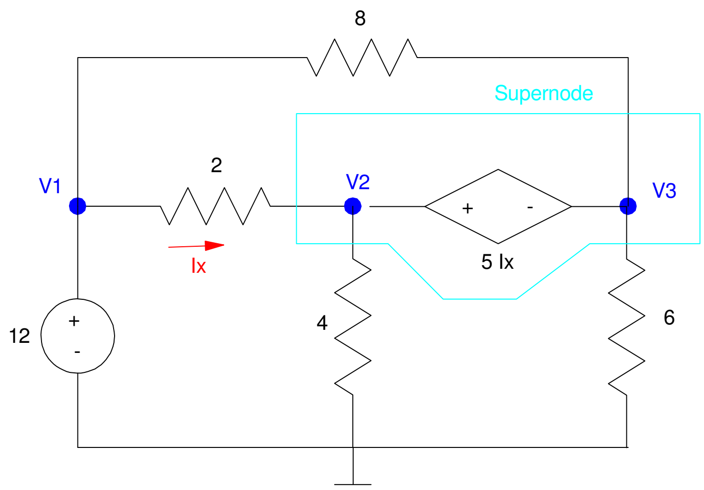
Super-nodes are not unique

- Several closed paths could be used
- Each should give the same result
- It helps the grader if you show what super-node you're using in homework sets and exams

SuperNodes and Dependent Sources

Supernodes apply to dependent sources as well. If you can't determie a certain current, create a supernode so that you only have resistors and current sources connecting to the supernode.

For example, determine the voltages for the following circuit:



Circuti with a dependent source. For this circuit, we'll need to use a supernode.

Start with the controlled source

$$I_x = \left(\frac{V_1 - V_2}{2}\right) \tag{1}$$

Write the easy equations (the voltage sources)

$$V_1 = 12 \tag{2}$$

$$V_2 - V_3 = 5I_x \tag{3}$$

Now we're stumped: we can't write the node equation at V2 or V3 due to now knowing the current through the controlled voltage source. So, create a super-node (show in light blue)

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_3}{6}\right) + \left(\frac{V_3 - V_1}{8}\right) = 0 \tag{4}$$

Now that we have 4 equations with 4 unknowns, we can solve. Grouping terms:

$$V_1 - V_2 - 2I_x = 0 \quad (1)$$

$$V_1 = 12 \quad (2)$$

$$V_2 - V_3 - 5I_x = 0 \quad (3)$$

$$\left(\frac{-1}{2} + \frac{-1}{8}\right)V_1 + \left(\frac{1}{2} + \frac{1}{4}\right)V_2 + \left(\frac{1}{6} + \frac{1}{8}\right)V_3 = 0 \quad (4)$$

Place in matrix form

$$\begin{bmatrix} 1 & -1 & 0 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ -0.625 & 0.75 & 0.2917 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

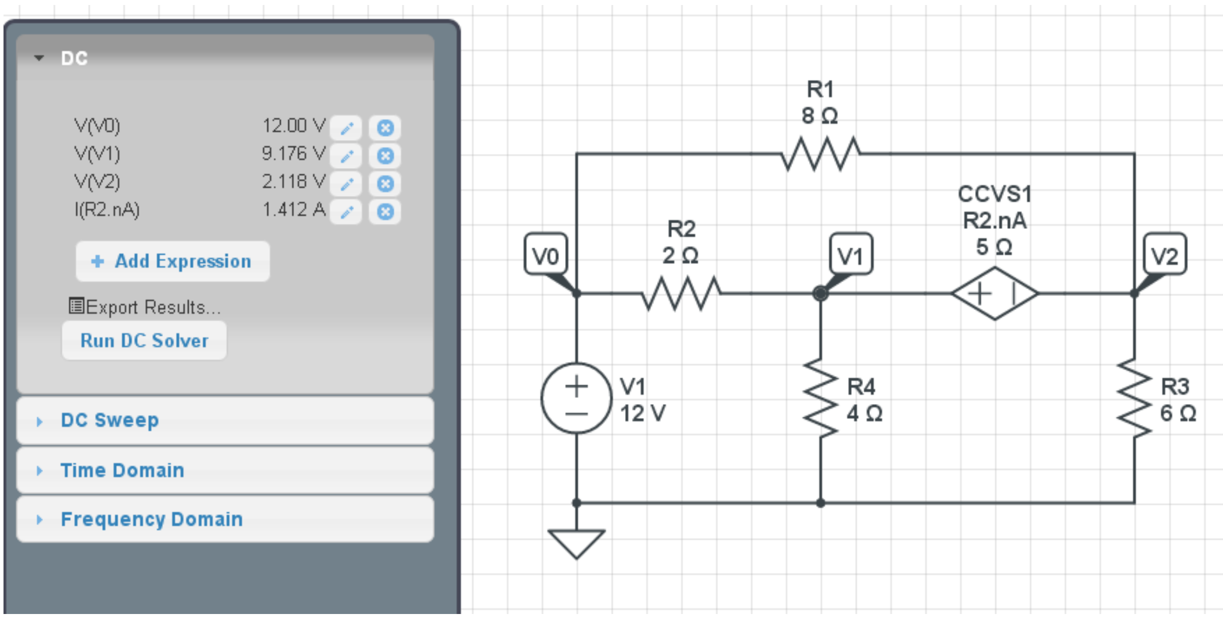
```
>> A = [1,-1,0,-2 ; 1,0,0,0 ; 0,1,-1,-5 ; -0.625,0.75,0.2917,0]

    1.0000   -1.0000         0   -2.0000
    1.0000         0         0         0
         0    1.0000   -1.0000   -5.0000
   -0.6250    0.7500    0.2917         0

>> B = [0;12;0;0]

     0
    12
     0
     0

>> V = inv(A)*B
```

```
V1   12.0000
V2    9.1764
V3    2.1175
Ix    1.4118
```

Verifying in Circuitlab

Matlab Values (repeated)

```
V1   12.0000
V2    9.1764
V3    2.1175
Ix    1.4118
```



Circuitlab verification: voltages match our calculations.