Voltage Nodes

Kirchoff's Current Law (KCL)

Background

In the previous lecture, votlage division was used to find the voltages in a circuit

- For some circuits, this works really well
- For other, it's really tedious
- For still others, voltage division just doesn't work

For example, in the following circuit, voltage division just doesn't work: you can't simplify the circuit by combining resistors in series and parallel.

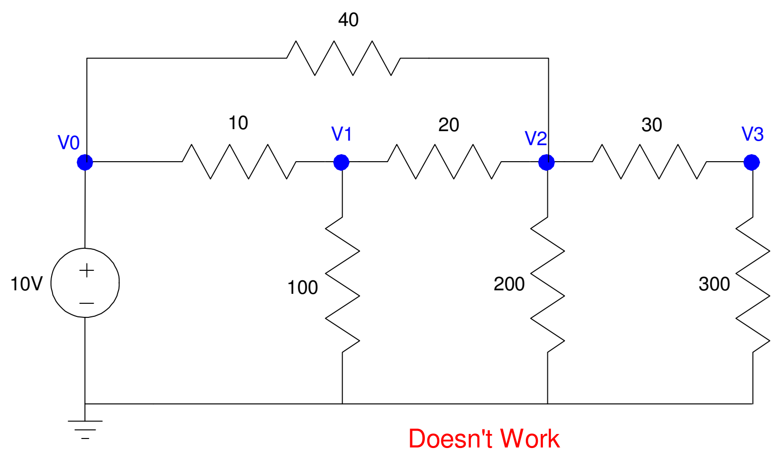

Figure 1: Circuit where voltage division doesn't work.

We need a better tool

To solve a circuit, we primarilly use three tools:

- Kirchoff's voltage law (voltage nodes)
- Kirchoff's current law (current mesh)
- Thevenin equivalents

The goal of the first two is to obtain N equations so solve for N unknowns.

Voltage Nodes

Voltage Nodes uses the notion that the sum of the current to (or from) a node must sum to zero. This holds at each and every node.

The idea is that

- If you have a circuit with N unknown voltages, and
- You can write N equations for these N unknowns,
- Then you can solve these N equations for N unknowns using techniques taught in Linear Algebra.

This a common theme used throughout ECE. Any time you have a problem, if you can reduce it to writing N eqations for N unknowns, you can solve this problem using Matlab and linear algebra.

With Matlab, solving N equations for N unknowns is pretty easy. What matters most is getting the equations right.

For example, find the voltages and currents in the following circuit:

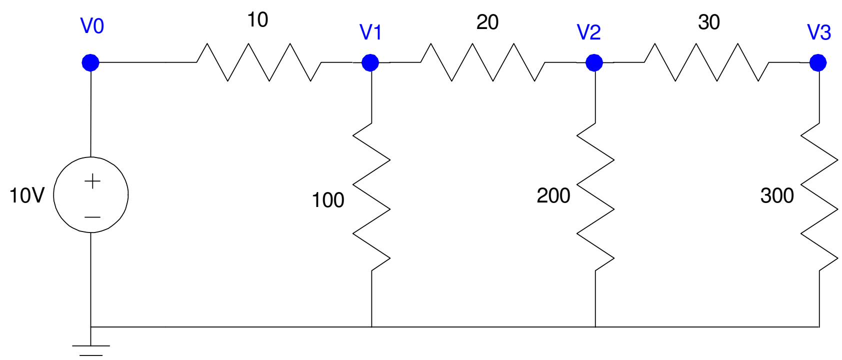

Figure 2: Find the voltages V0 to V3

This is a circuit we've looked at before and solved using CircuitLab and Voltage Division. The results were:

- V0 = 10.000V
- V1 = 8.553V
- V2 = 7.369V
- V3 = 6.699V

Now, let's get the same result using voltage nodes.

Step 1: Define the voltage nodes.

First, define circuit ground as well as each voltage node. Label these voltage nodes. This has already been done in Figure 2 with four voltage nodes (V0 to V3) along with a ground reference.

What this tells me is

- I have four voltage nodes
- I need four equations for four unknowns.

Step 2: Write N equations for N unknowns

Start with the easy equation. The voltage source tells you that

$$V_0 = 10V$$

One down, three equations to go.

Next, apply conservation of current to node #1. Three options are

- The sum of the currents to the node must be zero
- The sum of the currents from the node must be zero, or
- Current in equals current out.

I personnaly prefer summing the currents from the node as shown in Figure 3. This leads to the equation

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

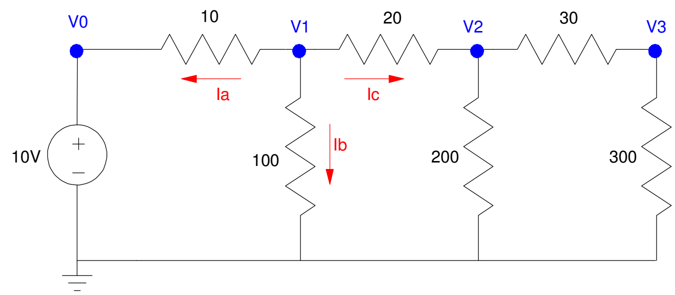

Figure 3: The sum of all currents from node #1 must be zero.

Note: By summing the currents from the node to zero, a there is a pattern with the voltage node equaton:

- All of the V1 terms are positive
- All of the other terms are negative
- All of the units are Ohms (Volts over Ohms)

This lets you check for errors in your equations. Getting the equations right is important. If the equations are wrong, the answers will be wrong.

Next, go to node #2. A Node #2, currents must sum to zero

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

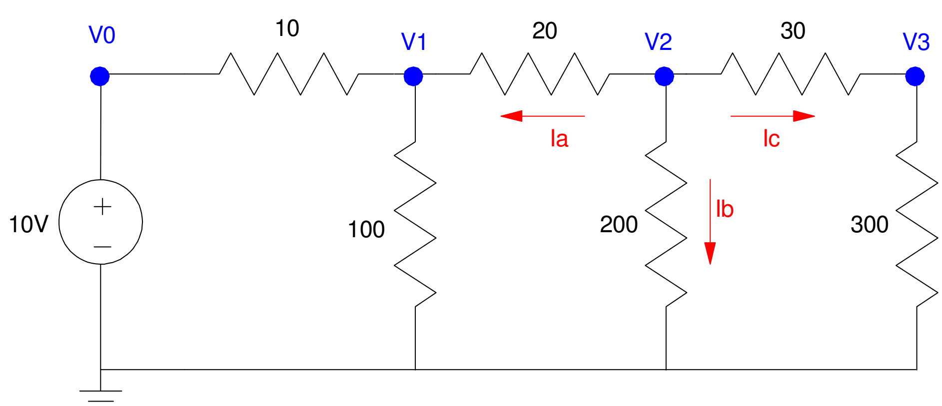

Figure 4: Node #2: The currents from the node must sum to zero

Again note that

- All the V2 terms are positive
- All the other terms are negative
- All units match (ohms + ohms + ohms)

Finally, at node #3 you get the fourth and final equation

$$I_a + I_b = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$

Note again that

- All the V3 terms are positive
- All the other terms are negative
- All units match (ohms + ohms)

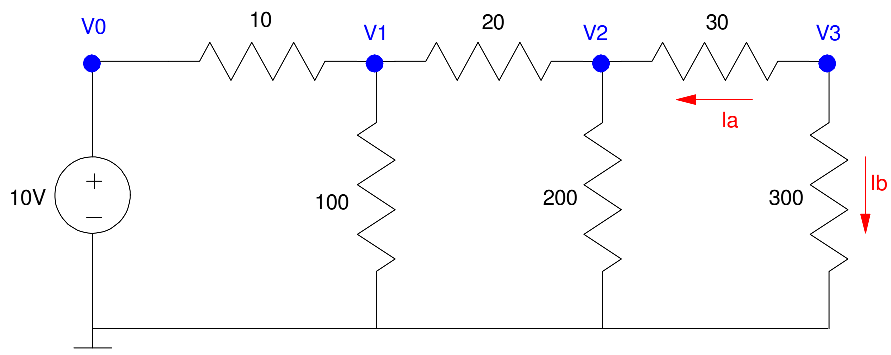

Figure 5: Node #3: The currents from the node must sum to zero

The net result is four equations for four unknowns

$$V_0 = 10$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$

This is typically where you stop on tests. On homework sets where you have access to Matlab, you can solve.

First, group the terms

$$V_0 = 10$$

$$\left(\frac{-1}{10}\right)V_0 + \left(\frac{1}{10}+\frac{1}{100}+\frac{1}{20}\right)V_1 + \left(\frac{-1}{20}\right)V_2 = 0$$

$$\left(\frac{-1}{20}\right)V_1 + \left(\frac{1}{20}+\frac{1}{200}+\frac{1}{30}\right)V_2 + \left(\frac{-1}{30}\right)V_3 = 0$$

$$\left(\frac{-1}{30}\right)V_2 + \left(\frac{1}{30}+\frac{1}{300}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{100}+\frac{1}{20}\right) & \left(\frac{-1}{20}\right) & 0 \\ 0 & \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{200}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

Solve using Matlab

```
a1 = [1,0,0,0];
a2 = [-1/10, 1/10+1/100+1/20, -1/20, 0];
a3 = [0, -1/20, 1/20+1/200+1/30, -1/30];
a4 = [0, 0, -1/30, 1/30+1/300];
A = [a1;a2;a3;a4]

    1.0000         0         0         0
   -0.1000    0.1600   -0.0500         0
         0   -0.0500    0.0883   -0.0333
         0         0   -0.0333    0.0367

B = [10 ; 0 ; 0 ; 0]

    10
     0
     0
     0

V = inv(A) * B

V0   10.0000
V1    8.5529
V2    7.3694
V3    6.6994
```

This is the same result we got using CircuitLab and voltage division.

Voltage Nodes with Current Sources

If your circuit has current sources, it's not an issue with voltage nodes. The current source just defines the current through a certain brance.

For example, consider the circuit of Figure 6. Find the voltages V1, V2, and V3

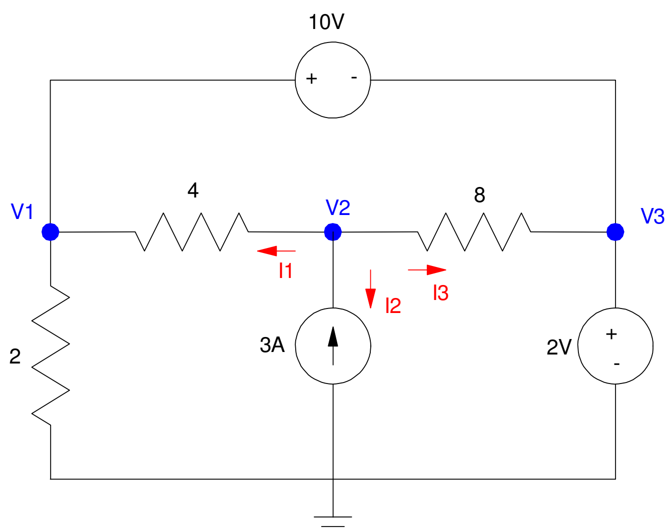

Figure 6: Find the voltages using voltage nodes.

Since there are three voltage nodes: we need to write 3 equations to solve for 3 unknowns.

Start with the easy ones: the voltage sources. Each voltage sources defines the voltage between the + and - terminal:

$$V_+ - V_- = \mathrm{X}$$

For the two in this circuit:

$$V_3 - 0 = 2 \tag{1}$$

$$V_1 - V_3 = 10 \tag{2}$$

That's two equations: we need one more. At V2, the sum of the current from the node must add to zero:

$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_2 - V_1}{4}\right) + (-3) + \left(\frac{V_2 - V_3}{8}\right) = 0 \tag{3}$$

We now have 3 equations to solve for the 3 unknown voltages. Grouping terms:

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 3$$

Place in matrix form:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -0.25 & 0.375 & -0.125 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix}$$

Solving in Matlab:

```
>> A = [0,0,1 ; 1,0,-1 ; -0.25,0.375,-0.125]

         0         0    1.0000
    1.0000         0   -1.0000
   -0.2500    0.3750   -0.1250

>> B = [2;10;3]

     2
    10
     3

>> V = inv(A)*B

V1    12.0000
V2    16.6667
V3     2.0000
```

This checks with Circuitlab:

Circuitlab results agree with our computed results.

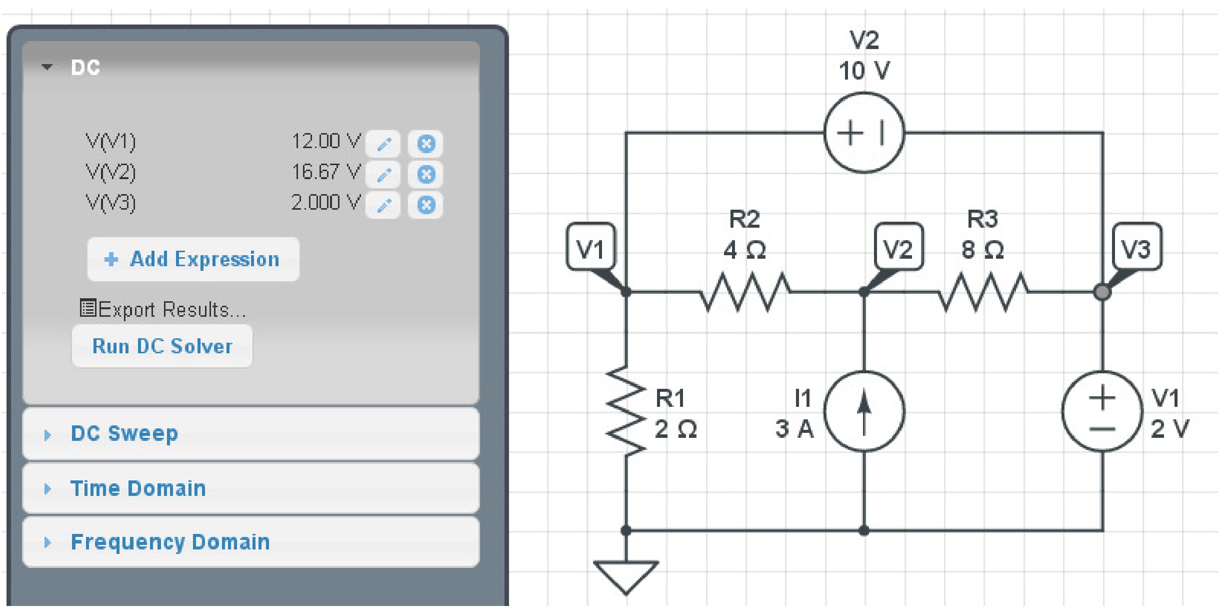

Figure 7: CircuitLab results confirm calculations

Voltage Nodes with Dependent Sources

Dependent sources are either voltage sources or current sources which depend upon another circuit element. These are typically covered in electronics as

- Bipolar Junction Transistor: Current-controlled current source
- Op-Amp Amplifier: Voltage controlled voltage source
- MOSFET: Voltage controlled current source

For this class, treat these as a generic input.

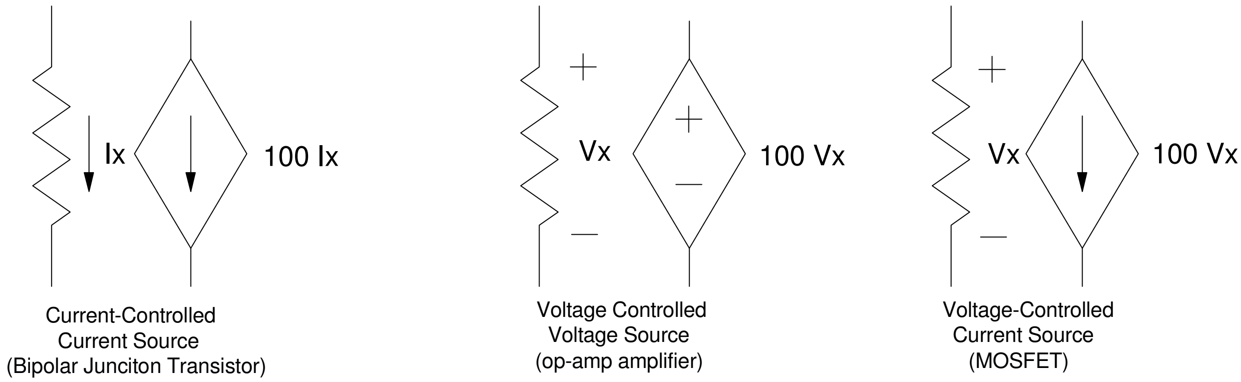

Figure 8: Dependent Sources

When a circuit has a dependent source, you wind up with one more equation:

- N equations for N unknown voltages, plus
- One more equation for each dependent source

Essentially, you need another equation to define Vx or Ix.

For example, consider the circuit of Figure 9.

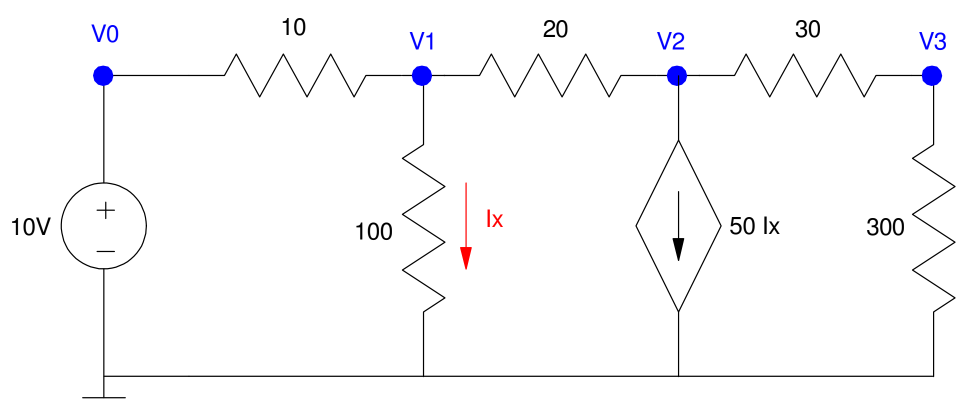

Figure 9: Circuit with a dependent source

This circuit has four votlage nodes and one dependent source. Hence, we'll need five equations for five unknowns. Start with the easy ones.

10V Source:

$$V_0 = 10$$

Ix:

$$I_x = \left(\frac{V_1}{100}\right)$$

That's two down, three to go. For the final three, write the voltage node equations.

Node V1:

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

Node V2:

$$\left(\frac{V_2 - V_1}{20}\right) + 50I_x + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Node V3:

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$

Group the terms, place in matrix form, and solve with Matlab.

Dependent sources aren't an issue. They just give you one more equation. With Matlab, solving 5 equations for 5 unknowns isn't a big deal.

Summary

Voltage Nodes is one way to compute the voltages in a circuit

- Define N voltage nodes
- Write N equations for N unknowns
- Solve using Matlab

This is based on conservation of current

- The sum of the currents from a node must be zero
- Ia + Ib + Ic = 0

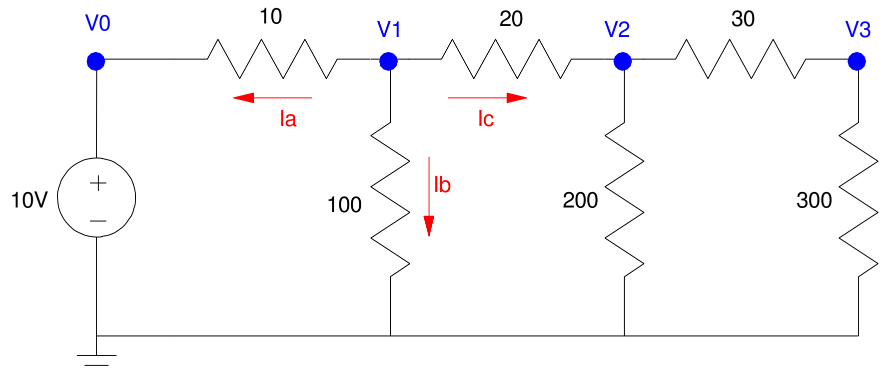

Figure 10: Voltage Nodes uses conservation of current to write N equations for N unknowns