# Voltage and Current Division

## Introduction

In the previous lecture, we looked at simplifying resistor networks

- Resistors in series add
- Resistors in parallel add as the sum of the inverses, inverted

In this lecture, we add a power supply

- Voltage source or
- Current source

We'll then look at how to compute the voltages and currents for this circuit.

This lecture looks at two techniques:

- Votlage Division and
- Current Division

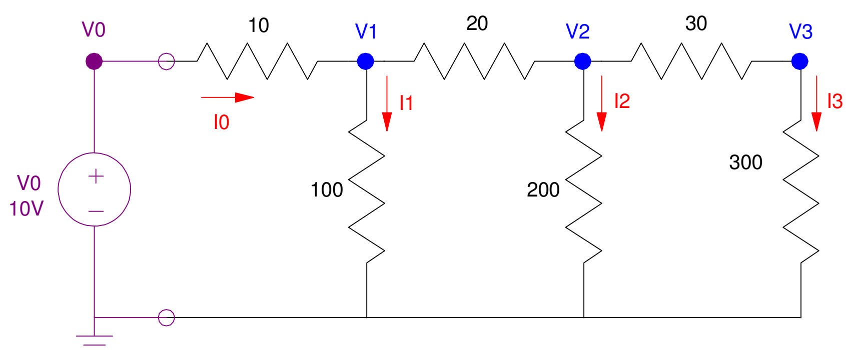


Figure 1: Find the voltages and currents for a given circuit

## Voltage Division

Voltage division applies when you have resistors in series as in Figure 2. In this case, the voltage Vin splits among the resistors.

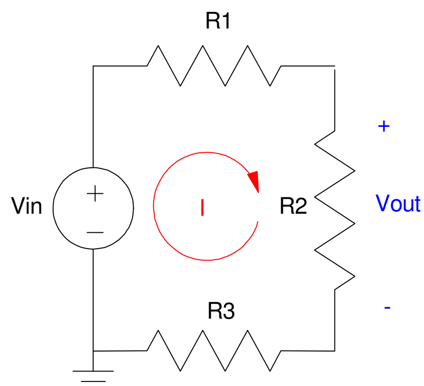


Figure 2: When resistors are in series, the voltage Vin splits among each resistor.

To see how the voltage splits, first find the loop current, I

$$I = \frac{V_{in}}{R_1+R_2+R_3}$$

The voltage across R2 is then from Ohm's law

$$V_{out} = I \cdot R_2$$

$$V_{out} = \left(\frac{R_2}{R_1+R_2+R_3}\right) V_{in}$$

or

$$\boxed{V_{out} = \left(\frac{\text{The resistance you're measuring across}}{\text{The total resistance}}\right) V_{in}}$$

Practice Problem: Find the voltages V1, V2, V3, and V4

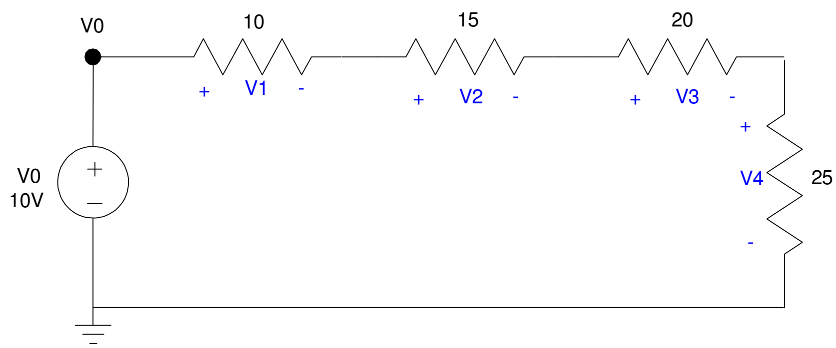


Figure 3: Practice Problem - Find the voltags using voltage division

Solution:

$$V_x = \left(\frac{R_x}{R_1+R_2+R_3+R_4}\right) V_{in}$$

$$V_1 = \left(\frac{10\Omega}{10\Omega+15\Omega+20\Omega+25\Omega}\right) 10V = 1.429V$$

$$V_2 = \left(\frac{15\Omega}{10\Omega+15\Omega+20\Omega+25\Omega}\right) 10V = 2.143V$$

$$V_3 = \left(\frac{20\Omega}{10\Omega+15\Omega+20\Omega+25\Omega}\right) 10V = 2.857V$$

$$V_4 = \left(\frac{25\Omega}{10\Omega+15\Omega+20\Omega+25\Omega}\right) 10V = 3.571V$$

Note that conservation of voltage holds

$$V_0 = 10V = V_1 + V_2 + V_3 + V_4$$

## Voltage Division: DC to DC Converter

One use of voltage division is to reduce the votlage. For example, design a circuit which converts 13.2V to 5.0V.

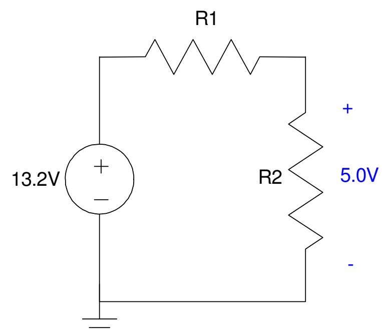


Figure 4: Use voltage division to convert 13.2V down to 5.0V

From voltage division

$$V_{out} = \left(\frac{R_2}{R_1+R_2}\right)V_{in}$$

$$5V = \left(\frac{R_2}{R_1+R_2}\right)13.2V$$

Since this is one equation with two unknowns, one of the unknowns is arbitrary. Let R2 = 5k Ohms. Then

$$R_1 + R_2 = 13.2k\Omega$$

$$R_1 = 8.2k\Omega$$

While this works, it has problems. Suppose you connect a 5k Ohm load to the circuit in an attempt to draw 1mA @ 5V as in Figure 5.

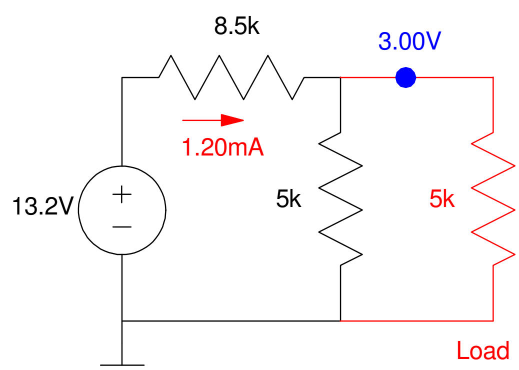


Figure 5: If you add a 5k Ohm load to the previous circuit, the output voltage drops to 3.00V

First, if you connect a load to the 5V output, the voltage drops. For example, add a 5k Ohm load to this circuit in an attempt to draw 1mA @ 5V, the output voltage drops to 3.00V.

Second, this DC to DC converter is only 11% efficient. The load receives 1.8mW (3.0V @ 5k Ohms) whereas the source delivers 15.8mW (13.2V @ 1.2mA).

Better DC to DC convertes exist. These are covered in Electronics.

## Voltage Computations Using Voltage Division

For some circuits, such as the one in Figure 6, you can use voltage division to find the node voltages.

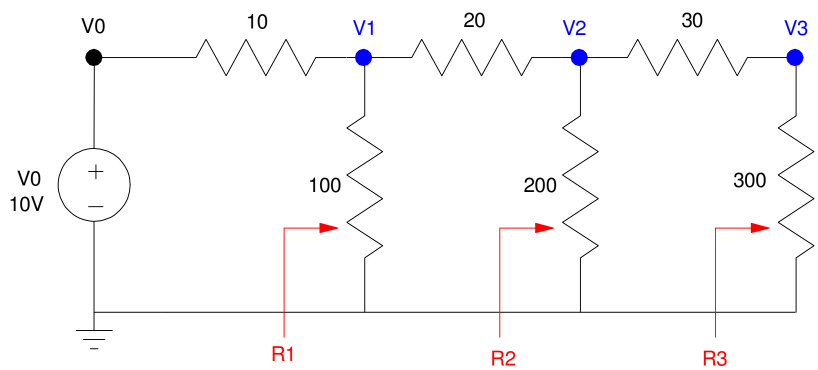


Figure 6: Find the voltages using voltage division.

First, find the resistances looking right at R1, R2, and R3. Using Matlab

```
>> R3 = 300

R3 =   300

>> R2 = 1 / (1/200 + 1/(R3+30))

R2 =  124.5283

>> R1 = 1 / (1/100 + 1/(R2+20))

R1 =   59.1049
```

Once these are known, you can find the voltage V1 using voltage division

```
>> V0 = 10;
>> V1 = R1 / (R1 + 10)*V0

V1 =    8.5529
```

Once you know V1, you can find V2 using volage division

```
>> V2 = R2 / (R2 + 20)*V1

V2 =    7.3694
```

Once you know V2, you can find V3 using voltage division

```
>> V3 = R3 / (R3 + 30)*V2

V3 =     6.6994
```

Finally, you can check your answers using CircuitLab

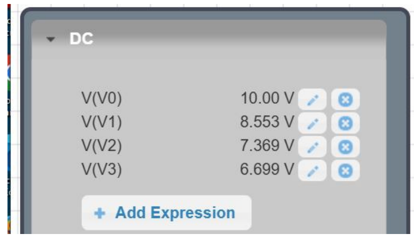


Figure 7: Checking the voltages using CircuitLab

## Voltage Division and Ohm Meters

Voltage division is actually how Ohm-meters work. About the only things you can measure directly are

- Distance,
- Voltage, and
- Time.

In order to measure resistance, a voltage divider is used where R1 is a known resistance and RL is the unknown resistance you're trying to measure.

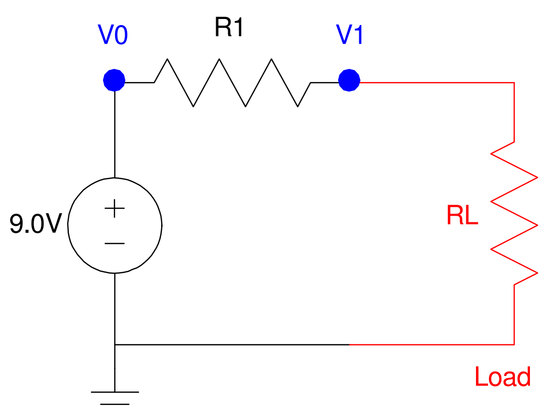


Figure 8: Ohm-meter. If you know R1 and measure V0 and V1, you can compute RL

From voltage division

$$V_1 = \left(\frac{R_L}{R_L+R_1}\right) V_0$$

Doing some algebra

$$R_L = \left(\frac{V_1}{V_0-V_1}\right) R_1$$

For example, if you want to know what the net impedance of the following circuit is, an Ohm-meter would read it as

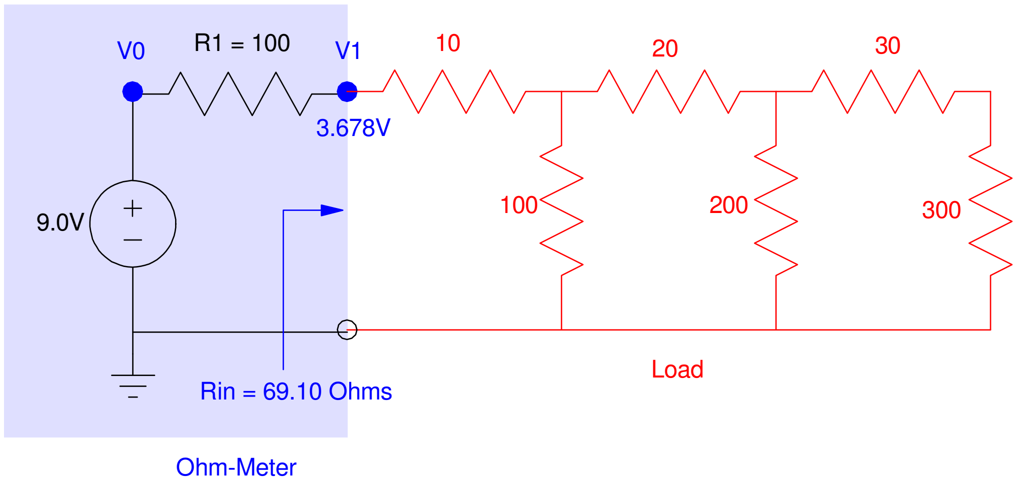


Figure 10: Using an Ohm-Meter to find the resistance

Knowing V0, V1, and R1, the load resistance can be computed

$$R_L = \left(\frac{V_1}{V_0 - V_1}\right) R_1$$

$$R_L = \left(\frac{3.678V}{9V - 3.678V}\right) 100\Omega$$

$$R_L = 69.10\Omega$$

# Current Division

Whereas voltage division applies circuits with resistors in series, current division applies to circuits with resistors in parallel. Take the circuit of Figure 11 for example.

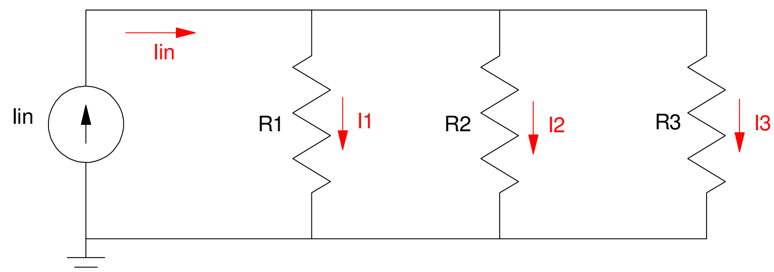


Figure 11: Find currents I1 to I3 using current division

Current Iin splits into I1, I2, and I3. The proportion that goes into each resistor is a function of the resistances.

From resistors in parallel, the total resistance is

$$R_{net} = R_1 || R_2 || R_3$$

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

The votlage is thus

$$V = R_{net} \cdot I_{in}$$

The current I1 is likewise

$$I_1 = \frac{V}{R_1}$$

$$I_1 = \left(\frac{R_{net}}{R_1}\right) I_{in}$$

or

$$I_1 = \left(\frac{\left(\frac{1}{R_1}\right)}{\left(\frac{1}{R_{net}}\right)}\right) I_{in}$$

$$I_1 = \left(\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right) I_{in}$$

Ditto for I2 and I3

$$I_2 = \left(\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right) I_{in}$$

$$I_3 = \left(\frac{\frac{1}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right) I_{in}$$

This gives the equation for current division (note: the admittance of R1 is 1/R1)

**The current through Rx is the admittance of Rx over the total admittance times the input current**

Example: Find currents I1, I2, and I3 using current division

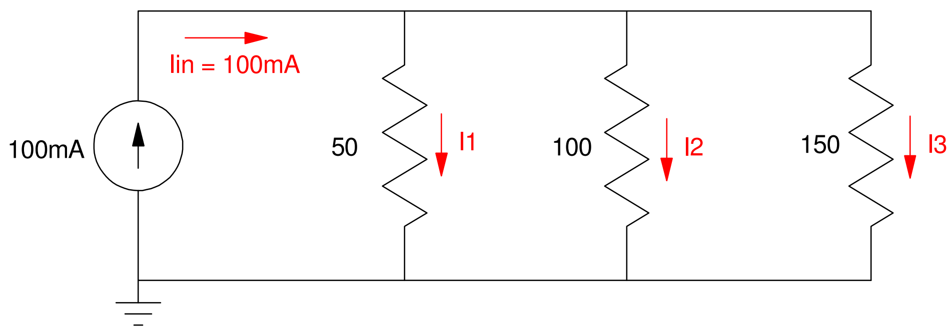


Figure 12: Find currents I1 to I3 using current division

Current I1 is

$$I_1 = \left(\frac{\frac{1}{50}}{\frac{1}{50}+\frac{1}{100}+\frac{1}{150}}\right) 100mA$$

$$I_1 = 54.545mA$$

Current I2 is

$$I_2 = \left(\frac{\frac{1}{100}}{\frac{1}{50}+\frac{1}{100}+\frac{1}{150}}\right) 100mA$$

$$I_2 = 27.568mA$$

Current I3 is

$$I_3 = \left(\frac{\frac{1}{150}}{\frac{1}{50}+\frac{1}{100}+\frac{1}{150}}\right) 100mA$$

$$I_3 = 18.378mA$$

Note that conservation of current holds

$$I_1 + I_2 + I_3 = 100mA$$

You can also check your answers using CircuitLab

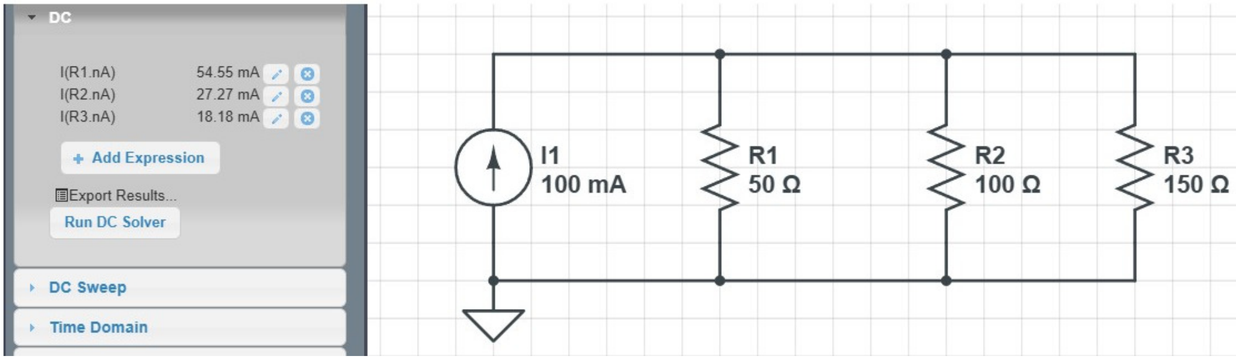


Figure 13: Finding the currents using CIrcuitLab. The values match calculations

## Potentiometers:

Time to switch to a new topic. Potentiometers.

A potentiometer is a resistor where a third terminal (termed the wiper) picks off the voltage part way across the resistor. For example, in the picture shown below a resistor consisting of a long wire wound around a ceramic torous connects the two outer leads. A wiper connects a center tap part way along the length of the potentiometer.

Figure 14: Example of a high-power potentiometer. A wiper taps into the middle of a resistor.

The symbol for a potentiometer reflects how it is built:

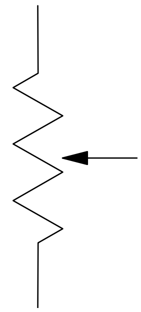

Figure 15: Symbol for a potentiometer. A resistor with an arrow indicating the center tap

There are two main uses for potentiometers:

**Potentiometers Used for Gain Adjustment:** By connecting one end of the potentiometer to a signal and the other end to ground the wiper picks off anywhere from 0% to 100% of the input signal.

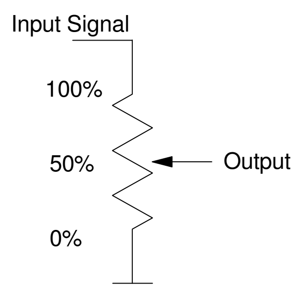


Figure 16: Potentiometer used to attenuate an input signal from 0% to 100%

Note that you do need to take into account the effect of loading the pot. For example, assume you connect a 10kΩ load to a 1k potentiometer. The circuit for this is as follows:

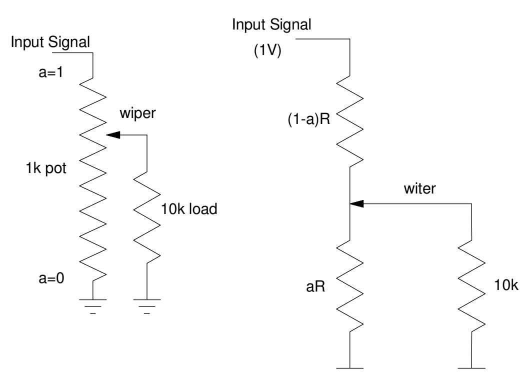


Figure 17: Loading effects cause the input / output relationship to be nonlinear

If the load is infinite, the voltage at the wiper is by voltage division:

$$V_{out} = \left(\frac{R_1}{R_1+R_2}\right) V_{in}:$$

$$V_{out} = \left(\frac{aR}{aR+(1-a)R}\right) V_{in}$$

$$V_{out} = a \cdot V_{in}$$

The output voltage is a fraction of the input as determined by the wiper setting, 'a'.

If the load is 10k, then R1 is aR||10k

$$V_{out} = \left(\frac{R_1}{R_1+R_2}\right) V_{in}$$

$$R_1 = aR || R_L = \left(\frac{aR \cdot R_L}{aR+R_L}\right)$$

You can plot this in MATLAB with the following code:

```
R = 1000;
RL = 10000;
a = [0:0.01:1]'

Vin = 1;

R1 = a*R * RL ./ (a*R + RL);
R2 = (1-a)*R;

Vout = R1 ./ (R1 + R2);

plot(a,a,a,Vout);
xlabel('Pot Setting (a)');
ylabel('Gain');
```

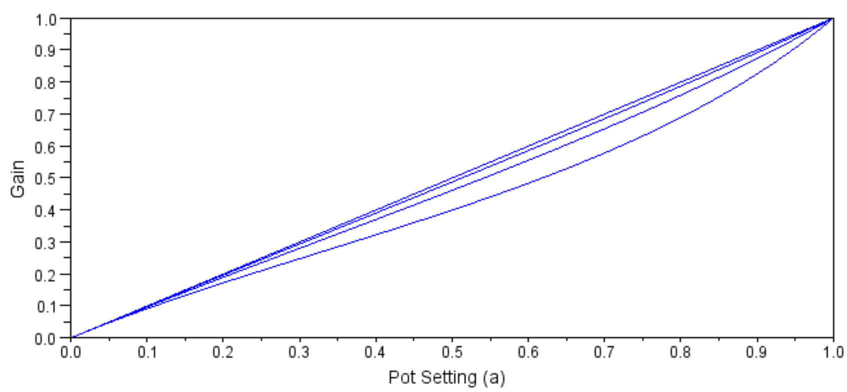


Figure 18: Gain of a potentiometer with a load of (infinite, 10x, 3x, 1x) the resistance of the potentiometer

Note that the with a load placed at the wiper, the gain still goes from 0 to 1 regardless of the load. As long as the load is 3x the resistance of the potentiometer, the gain is almost liner.

## Potentiometers Used as a Variable Resistor

A second use of potentiometers is to provide an adjustable resistor in a circuit. By shorting the wiper to one end, the resistance between the ends is adjustable from 0% to 100% of the total resistance.

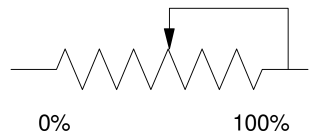


Figure 19: Potentiometer used as a variable resistor

Note that it is not necessary to short the wiper to one end, but this is customary. When a potentiometer fails, usually the wiper breaks. With the above configuration, the resistance becomes 100% when the wiper breaks. If you just connect one end to the wiper (without connecting to the other end), the resistance becomes infinite when the wiper breaks. As the design engineer, it's your choice as to what you want to happen upon failure.

## Summary

Voltage division is for resistors in series. The resistance across a given resistor is

$$V_x = \left(\frac{R_x}{R_1+R_2+R_2+R_4}\right) V_{in}$$

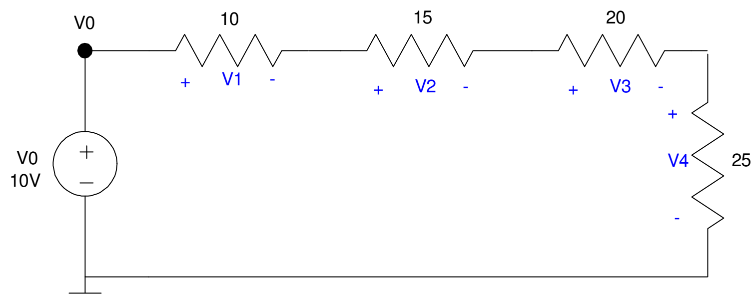


Figure 20: Voltage Division for resistors in series

Current division applies to resistors in parallel. The current through a given resistor is from

$$I_x = \left(\frac{\frac{1}{R_x}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right) I_{in}$$

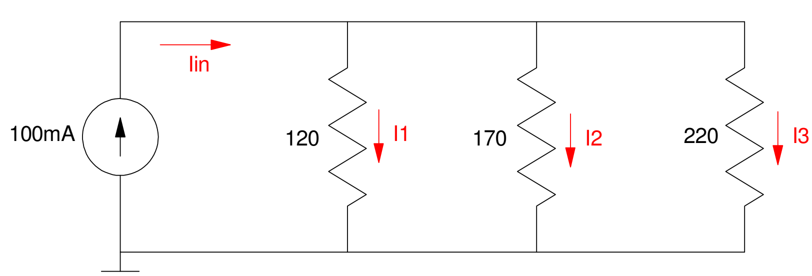


Figure 21: Current division for resistors in parallel