Resistors in Series and Parallel

In the last lecture, the relationship between voltage and current for a single resistor was presented. This was Ohm's Law:

$$V = IR$$

In this lecture, we'll cover one way to simplify a resistor circuit to obtain a single net resistance. This technique involves

- Combining resistors that are in series, and
- Resistors that are in parallel.

Once combined, you can then check your answers using

- CircuitLab, or
- An Ohmmeter

Resistors in Series:

When resistors are in series, all of the current that goes through one resistor also goes through the other resistors that are in series. For example, in Figure 1, the current through R1 is I. This is also the current through R2 and R3. Hence, these resistors are in series.

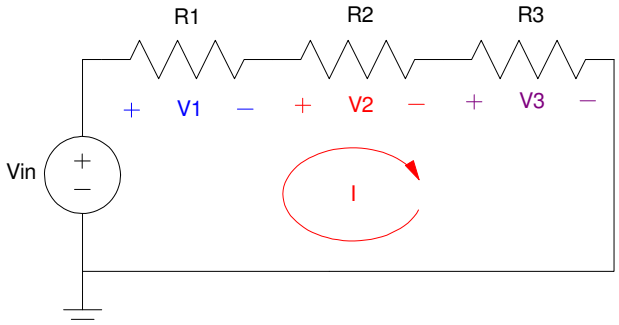

Figure 1: Resistors in Series

The net resistance can be found by computing

$$R_{net} = \frac{V_{in}}{I}$$

Using Kirchoff's voltage law:

$$\text{Vin} = \text{V1} + \text{V2} + \text{V3}$$

$$V_{in} = I \cdot R_1 + I \cdot R_2 + I \cdot R_3$$

$$V_{in} = I \cdot (R_1 + R_2 + R_3)$$

$$V_{in} = I \cdot R$$

This tells you that resistor in series add.

$$\boxed{R_{net} = R_1 + R_2 + R_3 + \ldots}$$

Problem 1: Let R1 = 100 Ohms, R2 = 200 Ohms, R3 = 300 Ohms. Find the total resistance.

Answer: Resistors in series add:

$$R_{net} = R_1 + R_2 + R_3$$

$$R_{net} = 100\Omega + 200\Omega + 300\Omega$$

$$R_{net} = 600\Omega$$

Problem 2: Let R1 = 100 Ohms, R2 = 200 Ohms, and the total resistance be 1000 Ohms. Find R3.

Answer: Resistors in series add:

$$R_{net} = R_1 + R_2 + R_3$$

$$1000\Omega = 100\Omega + 200\Omega + R_3$$

$$R_3 = 700\Omega$$

Resistors in Parallel:

When two or more resistors are in a circuit such that the voltages on one side are the same, and the voltages on the other side are also the same, the resistors are in parallel. An example of this is shown in Figure 2 where all three resistors have voltage Va on one side and Vb on the other side.

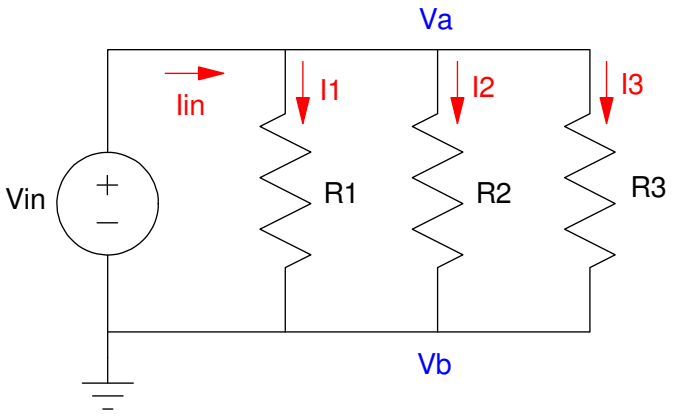

Figure 2: Resistors in parallel

To combine resistors in parallel, find the net resistance as

$$R_{net} = \frac{V_{in}}{I_{in}}$$

Using Kirchoff's current law:

Iin = I1 + I2 + I3

$$I_{in} = \left(\frac{V_{in}}{R_1}\right) + \left(\frac{V_{in}}{R_2}\right) + \left(\frac{V_{in}}{R_3}\right)$$

$$I_{in} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_{in}$$

$$V_{in} = I_{in} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$V_{in} = I_{in} \cdot R$$

This tells you that resistors in parallel combine as the sum of the admittances, inverted:

$$\boxed{R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)^{-1}}$$

Problem 3: Let R1 = 100 Ohms, R2 = 200 Ohms, R3 = 300 Ohms. Find the total resistance.

Solution: Resistors in parallel add as the sum of the inverses inverted.

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$R_{net} = \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{300}\right)^{-1}$$

$$R_{net} = 54.54\Omega$$

Problem 4: Let R2 = 200 Ohms, R3 = 300 Ohms, and the total resistance be 100 Ohms. Find R1.

Solution:

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$100\Omega = \left(\frac{1}{R_1} + \frac{1}{200} + \frac{1}{300}\right)^{-1}$$

$$\frac{1}{100} = \frac{1}{R_1} + \frac{1}{200} + \frac{1}{300}$$

$$R_1 = 600\Omega$$

Simplifying Resistor Circuits

By combining resistors in series and parallel, you can sometimes simplify a resistor network into a single net resistance. For example, consider the following circuit:

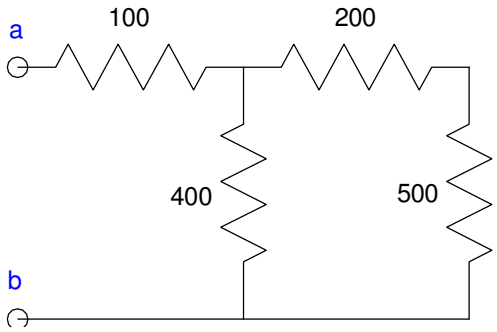

Figure 3: Find the net resistance from a to b (Rab)

The trick here is to find resistors that are in series and are in parallel. Once found, replace them with their equivalent resistance.

For this circuit, the 200 and 500 Ohm resistors are in series. Replace them with their equivalent resistance

$$R_1 = 200\Omega + 500\Omega$$

$$R_1 = 700\Omega$$

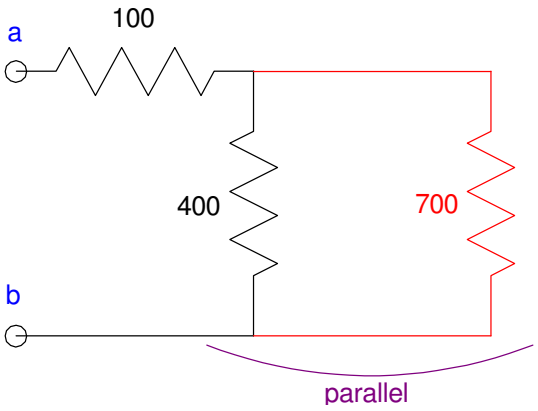

Figure 4: Combine 200 and 500 Ohms in series to get 700 Ohms

Note that R1 is now in parallel with the 400 Ohm resistor. Combining these

$$R_2 = 400\Omega || 700\Omega$$

$$R_2 = 254.5\Omega$$

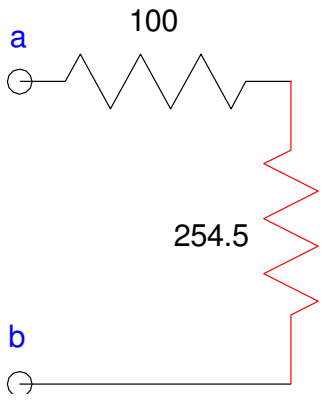

Figure 5: Combining 400 || 700 Ohms gives 254.5 Ohms

R2 is now in series with 100 Ohms. Add these in series

$$R_{ab} = 100\Omega + 254.5\Omega$$

$$R_{ab} = 354.5\Omega$$

One way to do these calculations is using Matlab. Recall the circuit is

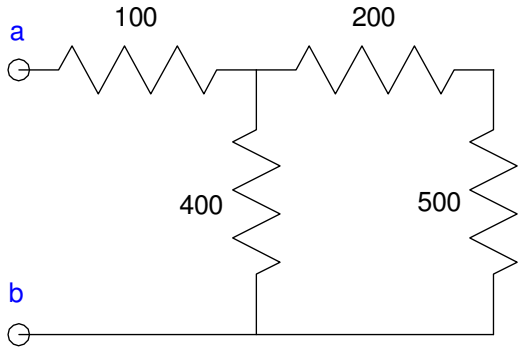

Figure 6: Resistor circuit to simplify

In Matlab

```
>> R1 = 200 + 500

R1 =   700

>> R2 = 1 / (1/R1 + 1/400)

R2 =  254.5455

>> R3 = R2 + 100

R3 =  354.5455
```

Another way to check your answer is CircuitLab. First , build the circuit. Add a 10V source and then compute the current draw

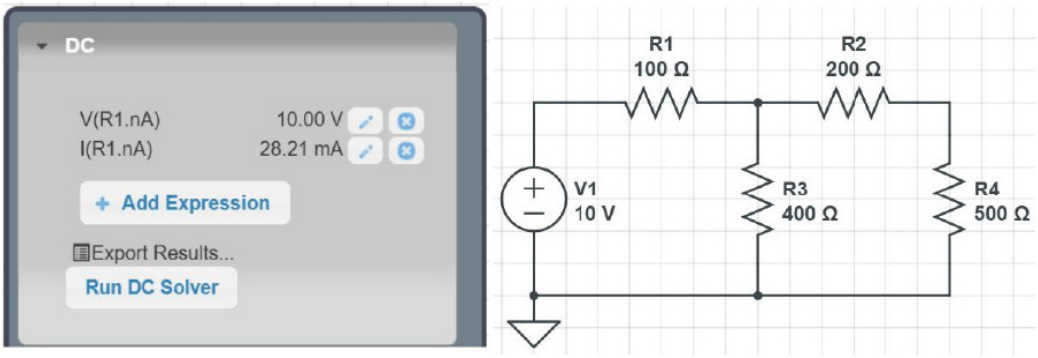

Figure 7: CircuitLab simulation to find the net resistance.

From Ohm's law, the effective resistance can be computed as

$$V = IR$$

$$R = \frac{V}{I} = \frac{10V}{28.21mA}$$

$$R = 354.48\Omega$$

You could also build this circuit in lab and measure

- The resistance Rab using an Ohmmeter, or
- The current draw for a known input V1.

Note that to measure the current, you would normally measure the voltage across R1 and then compute the current draw.

As a sidelight, with CircuitLab you really don't need a subscription to Chegg or the use of AI to check your answers on homework sets. Just input the circuit into CircuitLab and check your answers with CircuitLab's results. The two should match if you're doing it right.

Example 2: Combine resistors in series and parallel to find the net resistance, Rab:

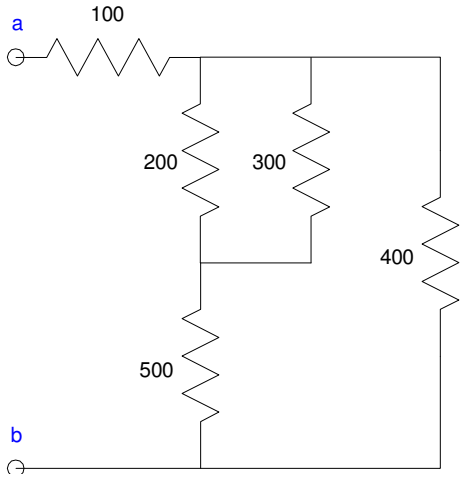

Figure 8: Find the resistance Rab.

Solution:

The 200 and 300 are in parallel

$$200 \parallel 300 = 120 \text{ Ohms}$$

This is in series with 500

$$120 + 500 = 620$$

This is in parallel with 400

$$620 \parallel 400 = 243.137$$

This is in series with 100

$$243.137 + 100 = \mathbf{343.137}$$

In Matlab:

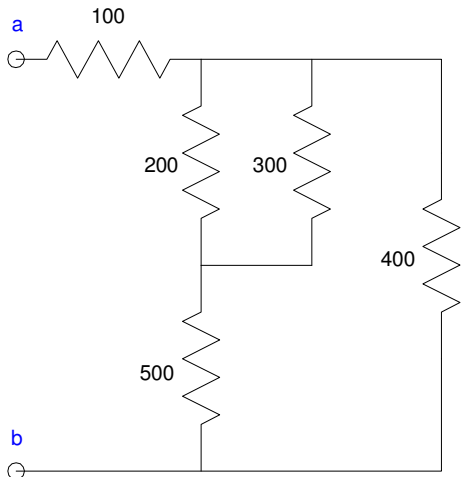

Figure 8: (repeated)

```
>> R1 = 1 / (1/200 + 1/300)

R1 =   120

>> R2 = R1 + 500

R2 =   620

>> R3 = 1 / ( 1/R2 + 1/400)

R3 =  243.1373

>> Rab = R3 + 100

Rab =  343.1373
```

This matches our calculations

CircuitLab

In CircuitLab, build the circuit along with a 10V source. Once built, find the net current this circuit draws

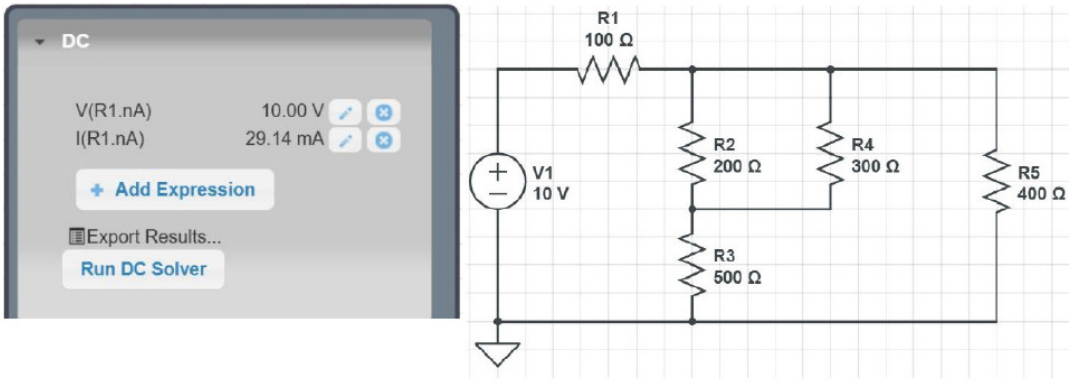

Figure 9: Checking answers using CircuitLab

From Ohm's law

$$V = IR$$

$$R = \frac{V}{I} = \frac{10V}{29.14mA}$$

$$R = 343.17\Omega$$

This again checks with our calculations. Chegg really isn't needed. CircuitLab will tell you if your calculations are correct or not.

Summary

Resistor circuits can sometimes be simplified by combining resistors in series and in parallel.

Series::

- When resistors are in series, all of the current through one resistor goes through the other resistors.
- Resistors in series add.

$$R_{net} = R_1 + R_2 + R_3$$

Parallel:

- When resistors are in parallel, the voltages on one side are all the same as are the voltages on the other side.
- Resistors in parallel add as the sum of the inverses, inverted.

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$