

Energy Conversion

Power Grid Analysis

Objective

- Transformer Operation
- Efficiency of a power grid
- Why we use AC instead of DC
- Why there are transformer substations
- DC, AC Single Phase, AC Three Phase.

Matlab Functions

- `plot()`, `inv()`, least-squares solution for M equations with N unknowns ($M \gg N$)

Motors and Transformers

ECE 331 Energy Conversion deals with modeling and analyzing transformers and different types of motors. Both rely upon electromagnetic phenomena:

- If you apply current to a coil, you produce magnetic flux (think of it as magnetic current)
- If you have a changing magnetic flux flowing through a coil, you produce current

A transformer consists of two coils of wire connected by an iron core (iron is used since it conducts magnetic flux 100,000 times better than just about anything else).

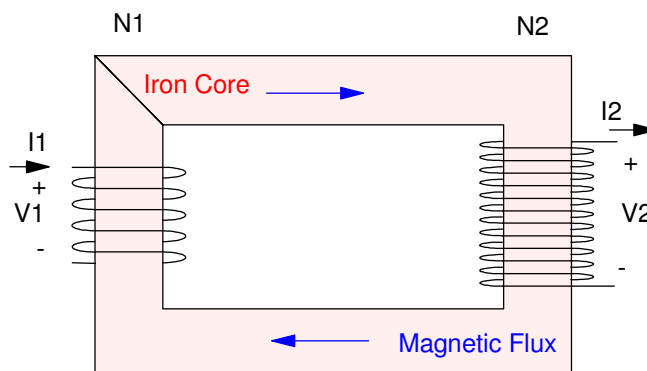


Figure 1: Transformer with a turn-ratio of $N1 : N2$

To model a transformer, you need two circuit components:

- R : A large amount of copper wire is needed to create the coils. This wire has resistance
- jX : Inherent in an inductor (and motor) are coils. These produce inductance, with an impedance of $j\omega L$. If you are connected to a power grid, the frequency is fixed at 60Hz, like it or not. This lets you simplify the complex impedance of a transformer (or motor) as jX .

In short, if you can analyze RL circuits, you can analyze circuits with transformers and motors.

Transformer Equations

Assume you apply current to side 1 of the above transformer. The magnetic flux induced into the iron core is

$$\phi = N_1 I_1 \quad \text{amp-turns}$$

If this is an AC current, the flux will be time-varying and produce current on side 2:

$$\phi = N_1 I_1 = N_2 I_2$$

giving the current relationship

$$I_2 = \left(\frac{N_1}{N_2} \right) I_1$$

By conservation of energy, power in must equal power out

$$V_1 I_1 = V_2 I_1$$

Substituting:

$$V_2 = \left(\frac{N_2}{N_1} \right) V_1$$

Transformers convert one voltage to another by the turns-ratio

If you connect an impedance to side 2 of the transformer, the impedance on side 2 by definition is

$$Z_2 = \frac{V_2}{I_2}$$

Transferring this to side one gives

$$Z_2 = \frac{V_2}{I_2} = \frac{\left(\frac{N_2}{N_1} \right) V_1}{\left(\frac{N_1}{N_2} \right) I_1}$$

$$\left(\frac{N_1}{N_2} \right)^2 Z_2 = \frac{V_1}{I_1} = Z_1$$

$$Z_1 = \left(\frac{N_1}{N_2} \right)^2 Z_2$$

Impedances seen through a transformer scale by the square of the turns ratio

This is useful for many applications. For example, suppose you have an amplifier with an output impedance of 100 Ohms driving an 8-Ohm speaker:

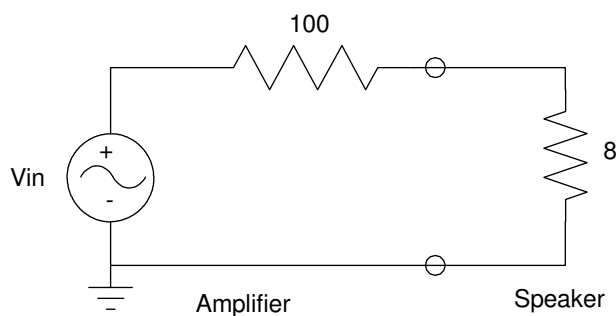


Figure 2: Amplifier with 100 Ohm output impedance driving an 8-Ohm speaker

By voltage division, only 7.4% of the voltage gets to the speaker

$$V_s = \left(\frac{8}{8+100} \right) V_{in}$$

This is a problem since you have a 7.4% efficient amplifier. If you connect a transformer with a 10:1 ratio to the speaker, however, the 8 Ohm speaker now looks like an 800 Ohm load

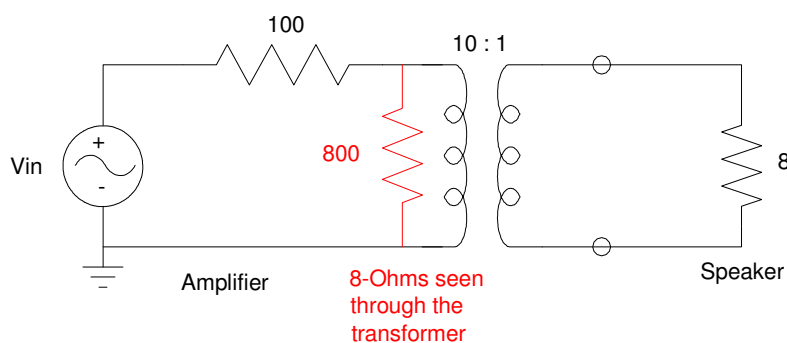


Figure 3: By adding a transformer to the output, the load now looks like 800 Ohms

Note from this circuit:

- If you have a radio which weighs a ton, it probably has a transformer in it (the iron core is what weighs so much).
- By adding a transformer, the efficiency increases to 89% (800 / 900).
- But, to deliver 12V to the speaker, you need to have an amplifier which outputs 120V, and
- Transformers don't work at DC - and neither will this amplifier.

Transformer Modeling and Testing

Assume a transformer is connected to the power grid (60Hz). To model the transformer, you need to account for several things:

- The copper windings have some inductance and resistance. These are called *the line losses*
- The iron core will also dissipate heat through hysteresis and eddy currents. These are called *the core losses*

A common model for a transformer is likewise:

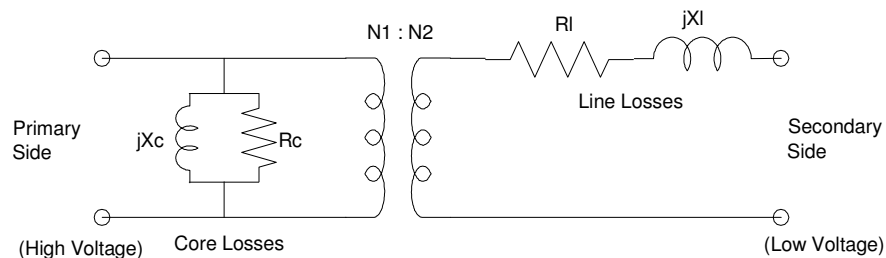


Figure 4: Transformer Model

To determine the parameters for a transformer, run two different tests:

- Open Circuit Test: (This measures R_c and jX_c) Leave the secondary side open. Apply rated voltage to the primary side and measure the voltage, current, and power.
- Short Circuit Test: (This measures R_l and jX_l) Short the primary side. Apply a small voltage to the secondary side and measure the voltage, current, and power.

Often times in power systems, a parameter called *power factor* is used:

$$\text{Power Factor} = \frac{\text{real(power)}}{\text{abs(V)} \cdot \text{abs(I)}}$$

This is a measure of the complex impedance. If the impedance is real, the power factor is 1.000. If the impedance is complex,

$$R + jX = Z \angle \theta$$

the power factor is the cosine of the angle of Z

$$pf = \cos \theta$$

The power is also

$$P = \frac{V^2}{Z} \cdot pf = |V| \cdot |I| \cdot pf$$

The impedance as a function of power and power factor is:

$$Z = \left(\frac{V^2 \cdot pf}{P} \right) \angle \arccos(pf)$$

Example: A 13kV : 240V transformer is tested with the following results:

	V	Power	pf
Open-Circuit Test	V1 = 13kV	100 W	0.02
Short-Circuit Test	V2 = 10V	200 W	0.95

Table 1: Open-circuit and short-circuit test results for a hypothetical transformer

Open-Circuit Test: (this tells you the core impedance)

$$P = \frac{V^2}{Z_0} \cdot pf$$

$$Z_c = \left(\frac{V^2 \cdot pf}{P} \right) \angle \arccos(pf)$$

$$Z_c = Z_0 \angle 88.85^\circ$$

$$Z_c = 678 + j33.79k\Omega \quad (\text{series model})$$

$$\frac{1}{Z_c} = \frac{1}{R_c} + \frac{1}{jX_c} \quad (\text{parallel model})$$

$$\frac{1}{Z_c} = 5.935 \cdot 10^{-7} + -j2.9583 \cdot 10^{-5}$$

$$R_c = \left(\frac{1}{\text{real part}} \right) = \left(\frac{1}{5.935 \cdot 10^{-7}} \right) = 1.68M\Omega$$

$$jX_c = \left(\frac{1}{\text{imag part}} \right) = \left(\frac{1}{2.9583 \cdot 10^{-5}} \right) = j33.81k\Omega$$

Short Circuit Test: (this tells you the line impedance)

$$P = \frac{V^2}{Z_0} \cdot pf$$

$$Z_l = \left(\frac{V^2 \cdot pf}{P} \right) \angle \arccos(pf)$$

$$Z_l = 0.475 \angle 18.19^\circ$$

$$Z_l = 0.4513 + j0.1483 \quad (\text{series model})$$

$$R_l = 0.4513\Omega$$

$$jX_l = j0.1483\Omega$$

Matlab Code:

```
>> % Open Circuit Test
>>
>> Z0 = (13000 ^ 2) / 100 * 0.02

    33800

>> Zc = Zo * exp(j*acos(0.02))

    6.7600e+002 +3.3793e+004i

>> Rc = 1 / real(1/Zc)

    1.6900e+006

>> Xc = -1 / imag(1/Zc)

    3.3807e+004

>>
>> % Short Circuit Test
>>
>> Zo = 10^2 / 200 * 0.95

    0.4750

>> Zo = Zo * exp(j*acos(0.95))

    0.4512 + 0.1483i

>> Rl = real(Zo)

    0.4512

>> Xl = imag(Zo)

    0.1483
```

Transmission Line Analysis:

In 2024, North Dakota produced 3,238 GWh of electricity:

Source	GWh	% of Total
Gas	250	7.7%
Hydro	126	3.9%
Wind / Solar	1,187	36.7%
Coal	1,675	51.7%
Total	3,238	100%

Table 2: 2024 energy production in North Dakota
<http://www.eia.gov/state/?sid=nd>

At \$0.11 / kWh, this equates to \$356 million. A large percentage of this energy is transmitted east to Minneapolis and Chicago. This leads to the following problem: How do you transmit large amounts of energy over a distance of 1000km (Minneapolis) or 2000km (Chicago)?

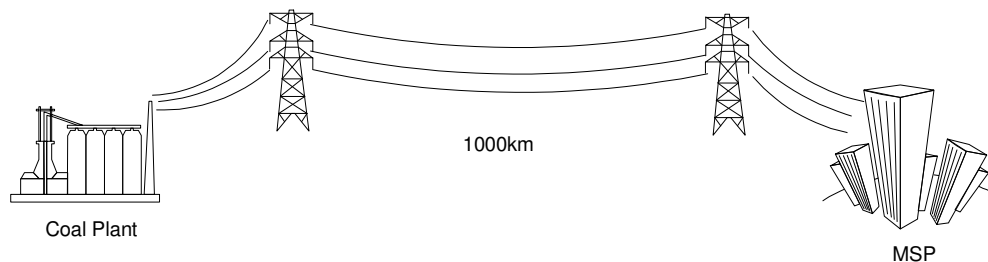


Figure 5: Problem for North Dakota: Transmit large amounts of power 1000+ km away

Option 1: DC Power Grid

Thomas Edison's vision for the U.S. power grid was to have DC generators (called Dynamos) produce DC power, carried to the residential users over DC power lines. The problem with DC is it is very hard to convert from one voltage to another (possible but difficult - especially on a large scale). This means what you generate is what you get.

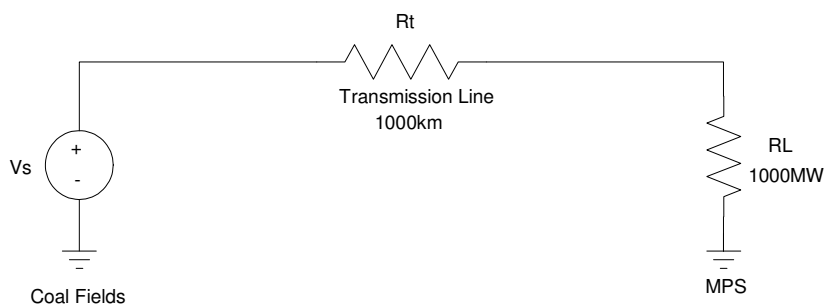


Figure 6: With a DC Power Grid, what you generate is what you use (120VDC)

Assume for the moment that this is a DC power grid with the customers (Minneapolis) using 120VDC. The current required is then

$$I = \frac{1000MW}{120V} = 83.33MA$$

Suppose the transmission line was an aluminum wire, 50mm diameter. The resistivity of aluminum is (www.wikipedia.com)

$$\rho = 2.82 \cdot 10^{-8} \Omega m$$

An aluminum wire, 50mm diameter, 1000km in length has a resistance of

$$R = \frac{\rho L}{A}$$

$$R = \frac{(2.82e-8 \Omega m)(1e6 m)}{\pi(0.025m)^2} = 14.36\Omega$$

14 Ohms may sound like a small resistance, but consider that it is carrying 83MA. The power dissipated by this wire (termed transmission line losses) would be

$$P_t = I^2 R = 9.97 \cdot 10^{16} W$$

The efficiency of this power grid is almost zero

$$\eta = \frac{\text{power to load}}{\text{total power}}$$

$$\eta = \frac{1000MW}{9.97e16W + 1000MW}$$

$$\eta = 0.00000001\%$$

This is for a single transmission line, however. If you add more transmission lines, the line resistance drops as 1/n where 'n' is the number of transmission lines. For the transmission losses to be 100MW (for an efficiency of 90%), you would need 997,000,000 transmission lines.

$$n = \frac{9.97 \cdot 10^{16} W}{100MW} = 997,000,000$$

It is impractical to ship DC power at 120V over a long distance.

Option 2: AC Power Grid

The problem with the above design is the current is *much* too high. If you use a transformer to reduce the current on the line by 1000x (meaning the voltage increases 1000x), the line losses drop by the square of this (1,000,000x). Now you can transmit power more efficiently.

So, assume

- The voltage at the coal plant is immediately bumped up to 345kV
- This voltage is stepped down to 13kV at a substation near the city, and then
- Stepped down to 120V close to the residence:

A circuit diagram for this system is as follows:

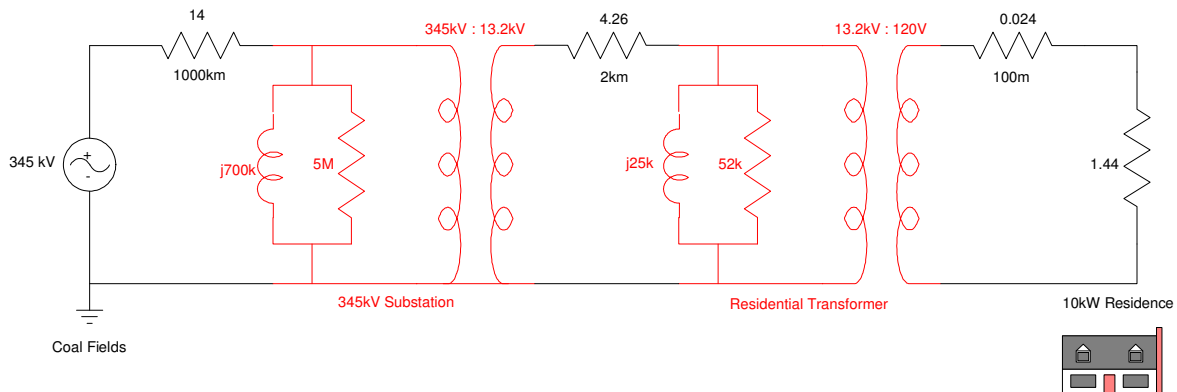


Figure 7: Circuit model for transporting power 1000km using transformers

Step 1: Remove the transformers and convert the impedances and voltages as

$$V_2 = \left(\frac{N_2}{N_1} \right) V_1$$

$$Z_2 = \left(\frac{N_2}{N_1} \right)^2 Z_1$$

For example, the voltage becomes:

$$\left(\frac{120V}{345,000V} \right) \cdot 345kV = 120V$$

The 1000km transmission line becomes

$$\left(\frac{120V}{345,000V} \right)^2 \cdot 14\Omega = 0.00000169\Omega$$

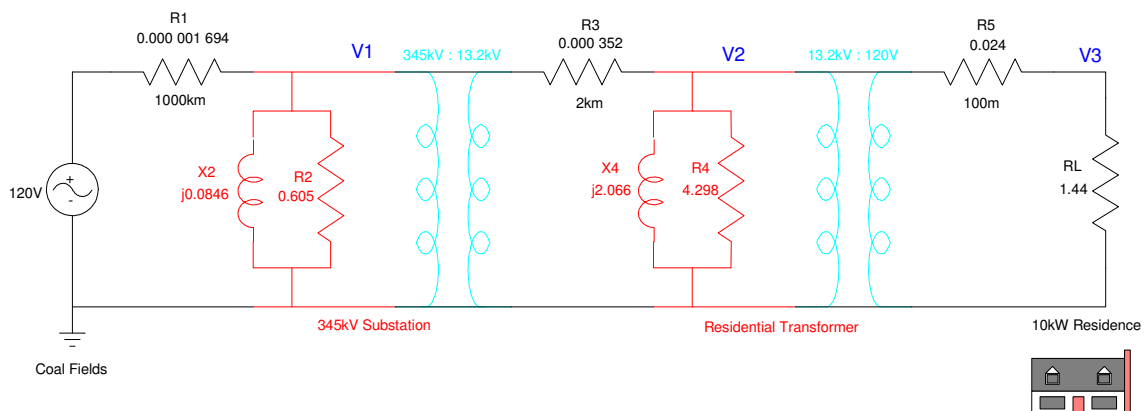


Figure 8: Remove the transformers to analyze the circuit

Step 2: Write the voltage node equations:

$$\left(\frac{V_1 - 120}{0.00001694} \right) + \left(\frac{V_1}{j0.0846} \right) + \left(\frac{V_1}{0.605} \right) + \left(\frac{V_1 - V_2}{0.0003022} \right) = 0$$

$$\left(\frac{V_2 - V_1}{0.0003022} \right) + \left(\frac{V_2}{j2.066} \right) + \left(\frac{V_2}{4.298} \right) + \left(\frac{V_2 - V_3}{0.024} \right) = 0$$

$$\left(\frac{V_3 - V_2}{0.024} \right) + \left(\frac{V_3}{1.44} \right) = 0$$

Step 3: Solve in Matlab.

```
R1 = 14 * (120/345000) ^2;
R2 = 5e6 * (120/345000) ^2;
X2 = j*700e3 * (120/345000) ^2;
R3 = 4.26 * (120/13200) ^2;
R4 = 52000 * (120/13200) ^2;
X4 = j*25000 * (120/13200) ^2;
R5 = 0.024;
RL = 1.44;

a11 = (1/R1 + 1/X2 + 1/R2 + 1/R3);
a12 = -1/R3;
a13 = 0;
a21 = -1/R3;
a22 = 1/R3 + 1/X4 + 1/R4 + 1/R5;
a23 = -1/R5;
a31 = 0;
a32 = -1/R5;
a33 = 1/R5 + 1/RL;

A = [a11, a12, a13 ; a21, a22, a23 ; a31, a32, a33]

    593243.81 - 11.808036i    - 2840.3756         0
    - 2840.3756             2882.2749 - 0.484i    - 41.666667
         0                  - 41.666667         42.361111

B = [120 / R1 ; 0 ; 0]

    70848214.
         0.
         0.

V = inv(A) * B

    119.99948 + 0.0024983i
    119.9608 + 0.0229322i
    117.99423 + 0.0225563i

abs(V)

    119.99948
    119.9608
    117.99423
```

With the transformers, you are able to transfer 117.99V to the customer.

To find the efficiency of the system, compute the losses:

$$-->P1 = \text{abs}(120 - V(1))^2 / R1$$

$$3.8459186$$

$$-->P2 = \text{abs}(V(1))^2 / R2$$

$$23804.793$$

$$-->P3 = \text{abs}(V(1) - V(2))^2 / R3$$

$$5.4355736$$

$$-->P4 = \text{abs}(V(2))^2 / R4$$

$$3348.5804$$

$$-->P5 = \text{abs}(V(2) - V(3))^2 / R5$$

$$161.14165$$

$$-->P_L = \text{abs}(V(3))^2 / R_L$$

$$9668.4988$$

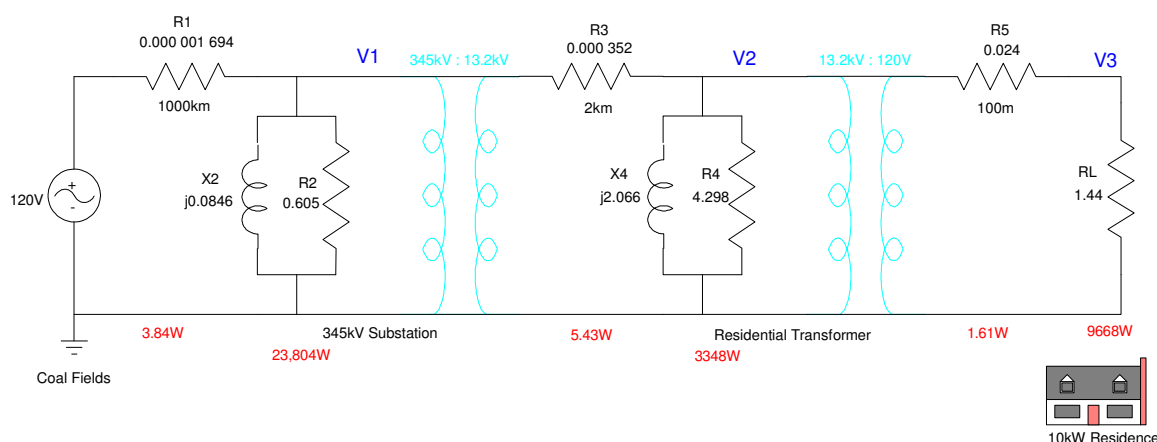


Figure 9: Resulting power dissipation for each element

If you include the core losses:

$$\eta = \left(\frac{P_L}{P_L + P_1 + P_2 + P_3 + P_4 + P_5} \right)$$

$$\eta = 26.1\%$$

If you ignore the core losses (i.e. assume these losses are shared by thousands of customers), the efficiency is

$$\eta = \left(\frac{P_L}{P_L + P_1 + P_3 + P_5} \right)$$

$$\eta = 98.2\%$$

Note:

- If you ignore the core losses, 98% of the power gets to the customer.
- If you include the core losses, 26% of the power gets to the customer.

It is impractical for a utility to run a power line to a single customer 1000km away. Its more efficient to have that customer run a diesel generator for really remote locations.

The losses for the transformers are shared by many customers - so the 24kW isn't all that bad if it feeds 1,000 customers.

It costs the utility money even if the customers are not using any power. It takes energy (and money) to keep the grid powered up.

Note: This is why we have high-voltage transmission lines running across the state.

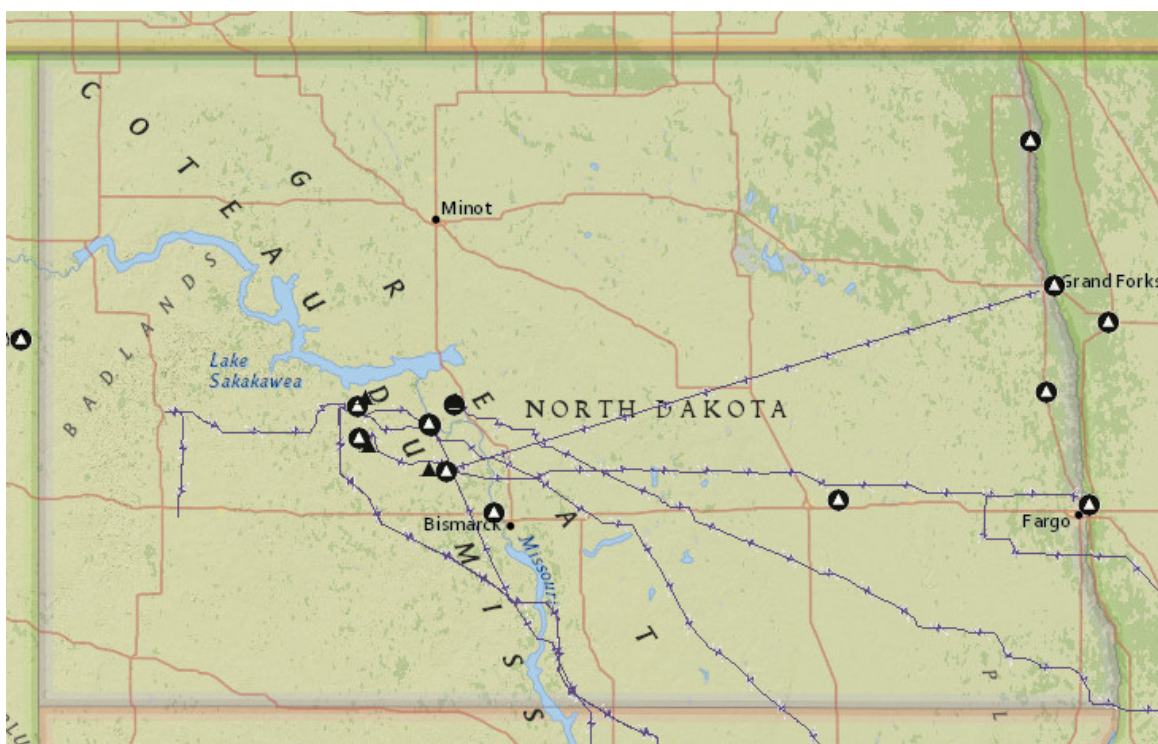


Figure 10: Coal fired power plants and transmission lines (345kV or more) in North Dakota.

Source: <http://www.eia.gov/state/?sid=ND>

