ECE 211: Circuits I

Topics

- Ohms Law & Kirchoff's Laws
- Resistor Circuits
- Voltage Division
- CircuitLab
- HP42 Calculator (Free42)

Introduction

Circuits I is the first course you take in ECE which covers the analysis of electrical circuits. In the first half of Circuits I, we cover DC analysis circuits with

- Voltage sources,
- Current sources, and
- Resistors.

For example, a typical circuit would be:

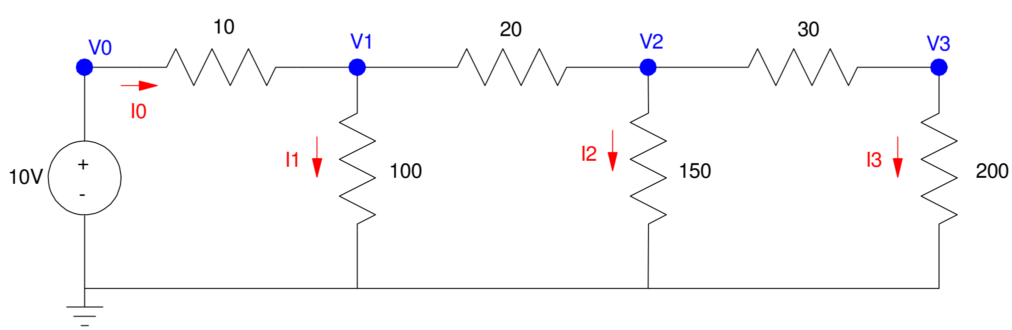

Figure 1: Typical problem in Circuits I: Determine the voltages and currents

Before we start, a couple of definitions:

Protons: Part of the nucleus of an atom. Protons do not move and are bound in place by chemical bonds.

Electrons: Part particle, part wave. Electrons are negatively charged particles which are locked in place in insulators and free to move in conductors. Note that current was discovered before electrons. The definition of positive current flow was somewhat arbitrary at that point. They got it wrong. (The direction of current flow is opposite of the direction of electron flow.)

Current: The flow of electrons. $1A = 1.602 * 10^{19}$ electrons per second.

Voltage: The force that causes current to flow, or, the energy released when current flows. 1 Amp of current flowing across 1 Volt produces 1 Joule.

Resistance: The resistance to current flow.

$V = I R$

1 Volt = 1 Amp flowing across 1 Ohm

An analogy which is often used is water flowing downhill.

- The pressure (or distance) the water flows downhill corresponds to voltage
- The actual flow of water corresponds to current
- The pipe which limits the flow of the water is resistance.

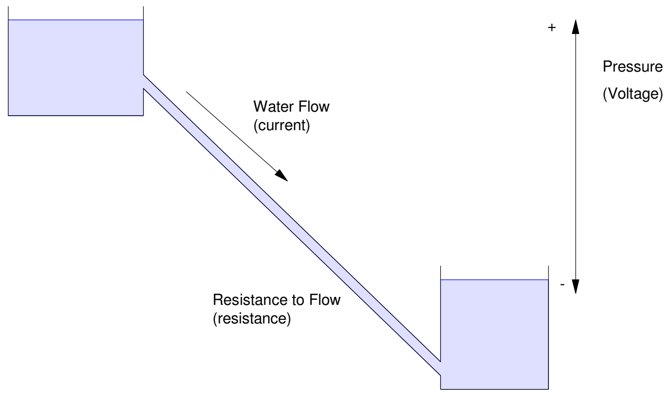

Figure 2: Water pressure and flow analogy for voltage and current

Note that for current to flow, there must be an inlet and an outlet (water in equals water out). The same holds for current.

Electrons (current) can neither be created nor destroyed. They can only be pushed around.

If you have a wire with only one end connected to a circuit, you know right away that the current is zero. The voltage can be anything - just the current is zero.

Circuit Elements:

The basic circuit elements used in Circuits I are voltage sources, current sources, and resistors.

Voltage Source: A voltage source is similar to a battery: it provides a constant voltage across its terminals. To give an idea what voltage is

Voltage	Example	Enough voltage to...
1.5V	AA battery	Turn on a single LED
5.0V	USB Battery	Power a computer
13.2V	Car battery	Turn a motor (starter)
120V	Wall Outlet	Power a computer, hair dryer
13,200V	Residential transmission lines	Power a residential subdivision Kill a person
345,000V	Long distance transmission lines	Power a small city Create an arc 11cm long

Table 1: Voltages and what they can do

The circuit symbols used for voltage sources are as follows. The middle symbol is a reference to the galvanic cells used by Faraday back in the 1800's for science demonstrations.

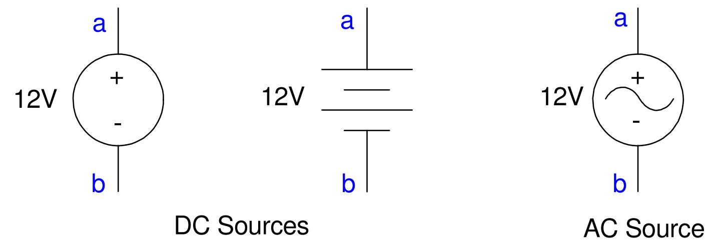

Figure 3: Symbols for a voltage source

Current Sources: A current sources sets the current flowing in a circuit. These are used for

- LED lights: The current determines the light output. A constant current sources results in a constant light output.
- DC Motors: Current is torque. A constant current source provides a constant torque (i.e. constant acceleration for an electric vehicle.)

Examples of currents and what they mean are:

Current	Diameter of wire to carry.	Example
1uA 0.000 001A	0.000 7mm	Sleep mode for a TV remote
1mA 0.001A	0.021 mm	Power a single LED (dim)
1A	0.72mm	Charging a cell phone
10A	2.3mm	Hair dryer
100A	7.3mm	Welding

Table 2: Currents and what they can do

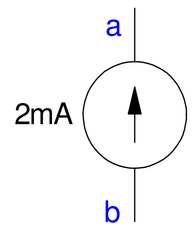

Figure 4: Symbol for a current source

Resistors: Resistors turn current into heat. The voltage across a resistor and the power it dissipates are

$$V = IR \quad \text{Volts}$$

$$P = VI = I^2 R \quad \text{Watts}$$

All materials have an innate resistance. For example, copper has a resistivity of

$$\rho = 1.7 \cdot 10^{-8}\ \Omega m$$

If you have a copper wire that's 100m long and 1mm diameter, it will have a resistance of

$$R = \frac{\rho L}{A} = \frac{\left(1.7 \cdot 10^{-8}\ \Omega m\right)(100m)}{\pi \cdot (0.0005m)^2} = 6.8\Omega$$

Resistors used in circuits are typically made by winding a long piece of wire around a ceramic core. By varying the thickness of the wire and the length used, you can set the resistance from 0.1 Ohm to 10M Ohms.

Resistors are inexpensive (less than $0.01 each) and available in 10% increments (100, 110, 120, 130, ...).

Color coding is used to signify the value of a resistor:

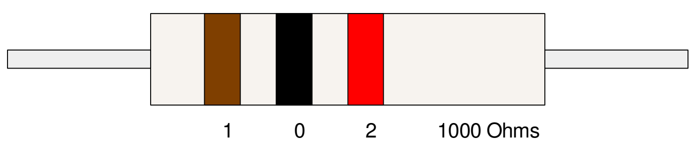

Figure 4: 3-band resistor code

0	1	2	3	4	5	6	7	8	9
black	brown	red	orange	yellow	green	blue	violet	grey	white

Table 3: Color code for resistors

Silver	Gold	Red	Brown	Green
+/- 10%	+/- 5%	+/- 2%	+/- 1%	+/- 0.5%

Table 4: Tolerance band for resistors

Circuit Analysis: CircuitLab

One way to analyze a circuit is to use a circuit simulator. CircuitLab is a *very* friendly simulator that all students at NDSU have access to.

- When you register, use your NDSU email address
- Otherwise, it's $24/year to use

It's basically drag and drop. For example, to build the circuit on page #1, drag and drop to create

Step 1: Drag and drop the circuit elements.

- R rotates the element

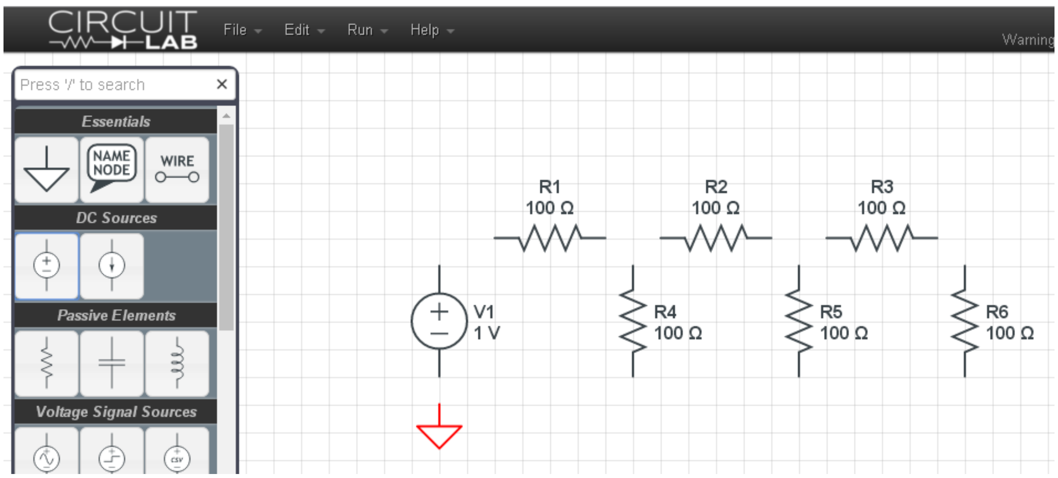

Figure 5: Start of a circuit in CircuitLab

Step 2: Connect the elements together and set the values

- Double click on an element to change it
- Add Node Name for voltages you want to look at

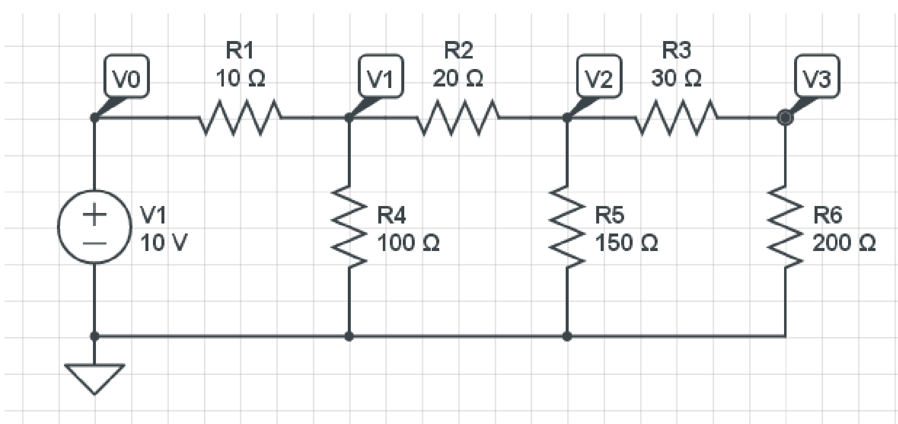

Figure 6: Circuit with elements connected and values set

Step 3: Run a DC simulation

- Simulate
- DC
- + Add Expression (choose V0, V1, V2, V3)
- Run DC Solver

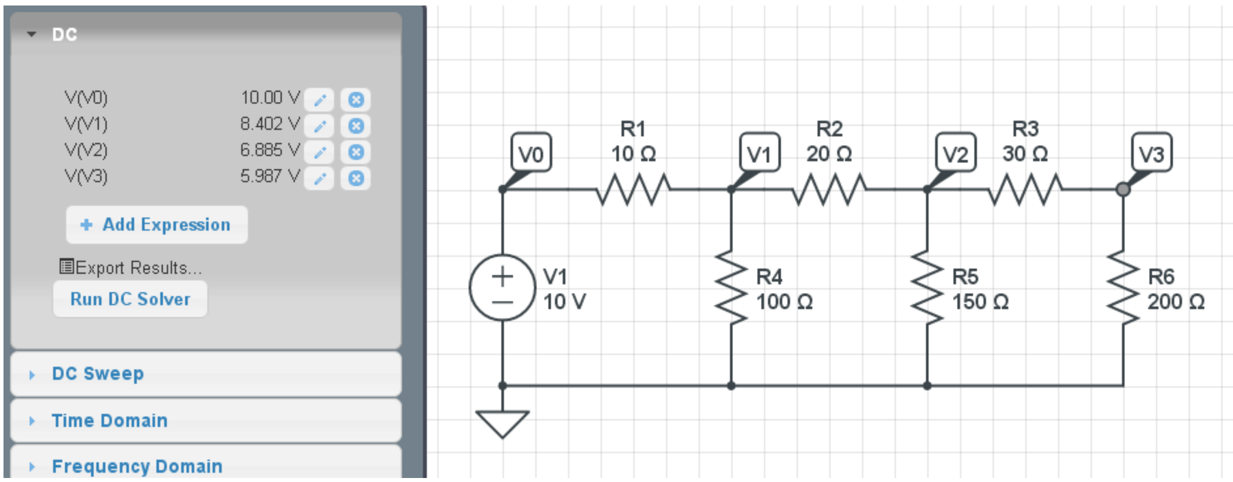

Figure 7: Solving for the voltages in CircuitLab

If you want to know the currents

- Click on the side of the resistor the current goes into (left side of R1, top of R4)
- Run DC Solver

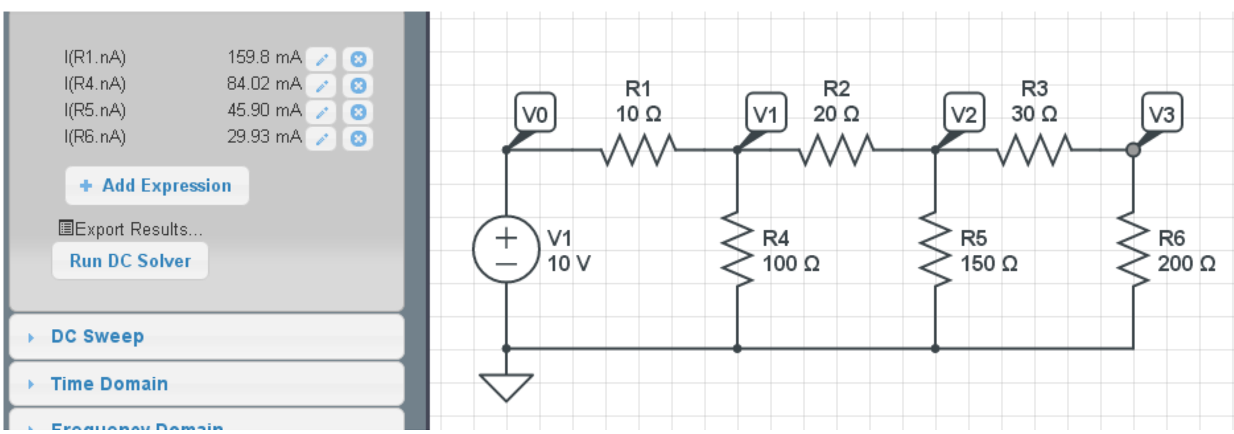

Figure 8: Solving for currents in CircuitLab

Circuit Analysis

You can also solve for the voltages and currents by hand. This is typically the way you do it on tests and homework sets.

There are several techniques for circuit analysis. The techniques covered in this class are

- Conservation of Voltage: Kirchoff's Voltage Law,
- Conservation of Current: Kirchoff's Current Law,
- Simplification of Resistor Circuits, and
- Voltage Division

Before starting, a couple of definitions:

Nodes:

A node is indicated by a solid line. It represents a wire (with a resistance of zero ideally). The voltage at any point on the wire is the same. The following circuit, for example, has four voltage nodes including ground.

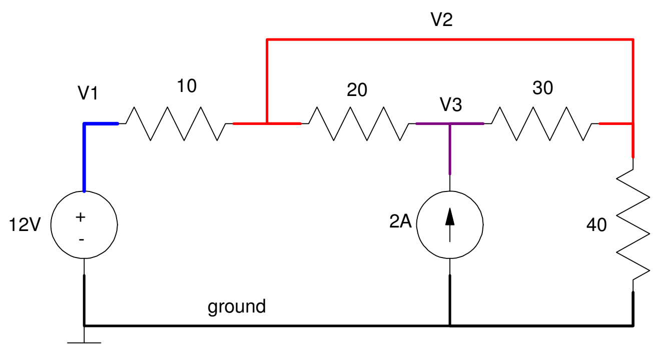

Figure 9: Voltage nodes are sections of a circuit which are shorted togther (and likewise have the same votlage)

Ground:

Ground is a voltage node which all other voltages are referenced to. Usually, ground is the same as earth ground - it doesn't have to be, however. Birds sitting on a high-voltage line, for example, treat the 13kV line as their ground reference.

The symbol for ground is one of the following. If both are used in a circuit, it means that there are two reference voltages - which may be different. This can be important: if a person on the ground uses earth as his/her ground reference while a bird on the high-voltage line uses 13kV as her ground reference, everyone is OK as long as you keep the two grounds separate. If you connect the two, however, the bird goes poof.

Figure 10: Symbols for ground. If both are used in a circuit, they may be different.

Conservation of Voltage: Kirchoff's Voltage Law

If you measure the voltages around any closed path and wind up where you started, the sum of the voltages must be zero.

- i.e. If you short the leads of a volt meter together, you'll measure 0V.

For example, determine the unknown voltages (V1..V6) for the following circuit.

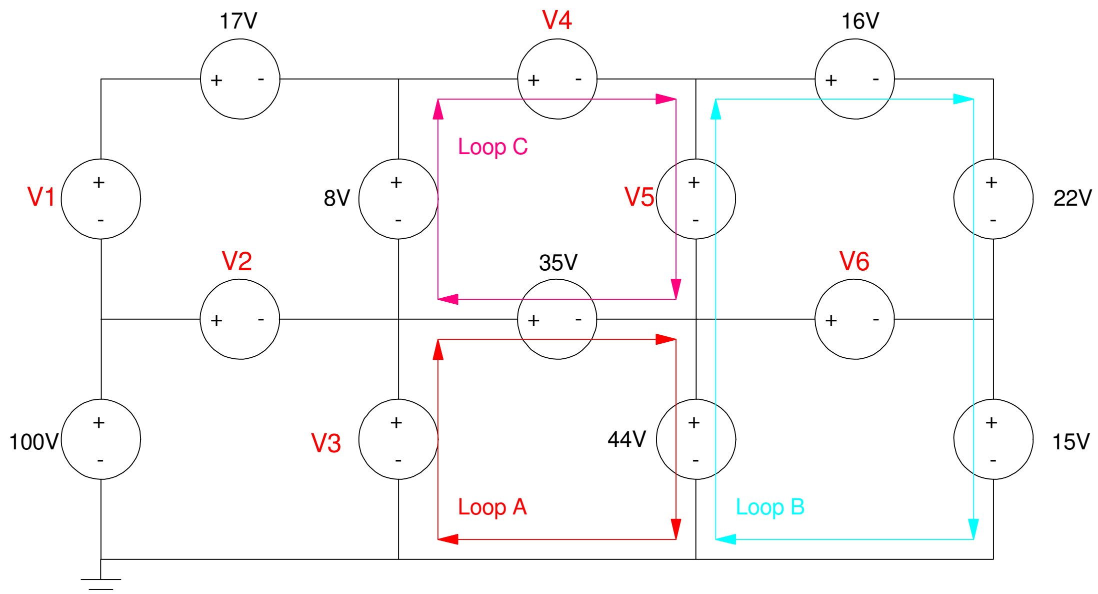

Figure 11: Conservation of Voltage: The sum of the voltages around any closed path must be zero.

Using Kirchoff's Voltage Law,

- Sum the voltages around a closed path
- To be consistent, add if you hit the + sign first, subtract if you hit the - sign first.
- Look for paths with one unknown.

Loop A:

- $-V3 + 35 + 44 = 0$
- $V3 = 79V$

Loop B:

- $-44 - V5 + 16 + 22 + 15 = 0$
- $V5 = 9V$

Loop C:

- $-8 + V4 + V5 - 35 = 0$
- $-8 + V4 + 9 - 35 = 0$
- $V4 = 34V$

Conservation of Current: Kirchoff's Current Law

The sum of the currents leaving a node must be zero

- The sum of the currents going into a node must be zero
- Current in = Current Out

Kirchoff's current law simply states you cannot create matter. Current is the flow of electrons, a physical entity. If you cannot create matter, you cannot create electrons (i.e. current in must match with the current out of a node).

For example, determine the unknown currents for the following circuit:

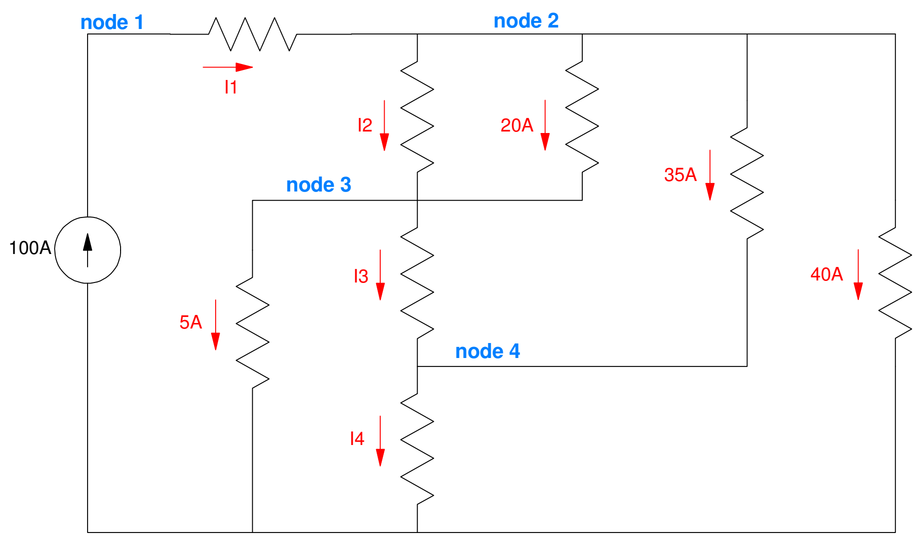

Figure 12: Find the unknown currents using conservation of current

At Node #1, current in = current out

$$100 = I1$$

At Node #2, current in = current out

$$I1 = I2 + 20 + 35 + 40$$

$$I2 = 5A$$

At Node #3, current in = current out

$$I2 + 20 = 5 + I3$$

$$I3 = 20A$$

At Node #4, current in = current out

$$I3 + 35 = I4$$

$$I4 = 55A$$

Resistor Circuits

The symbol for a resistor is as follows. The passive-sign convention defines the current going from the + voltage to the - voltage:

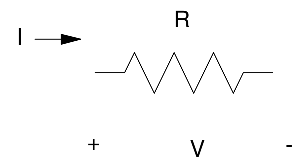

Figure 13: Symbol for a resistor: Note that the current goes into the + terminal.

With this notation, the voltage-current relationship is

$$V = IR$$ Volts

This also leads to the equations

$$I = \frac{V}{R}$$ Amps

$$R = \frac{V}{I}$$ Ohms

One way to remember this is with the following figure:

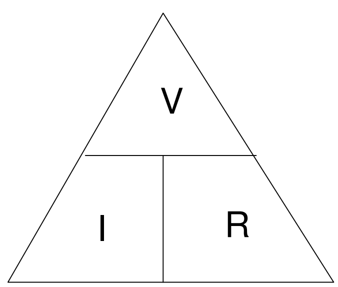

Figure 14: Using the passive sign-convention, V = IR, I = V/R, R = V/I

Given a circuit made up of resistors, you can simplify them into a net resistance using some tricks:

Resistors in Series: Resistors in series add.

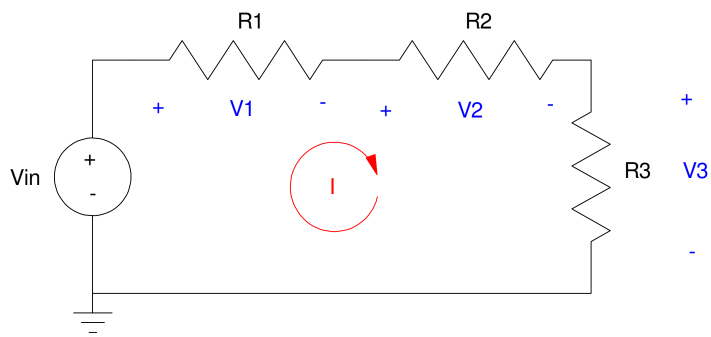

Figure 15: Resistors in series add: Rnet = R1 + R2 + R3

The total voltage is

$$V = V_1 + V_2 + V_3$$

From V = IR

$$V = IR_1 + IR_2 + IR_3$$

$$V = I(R_1 + R_2 + R_3)$$

The effective resistance is

$$R_{net} = R_1 + R_2 + R_3$$

Resistors in Parallel: These add as the sum of the inverses, inverted.

Example

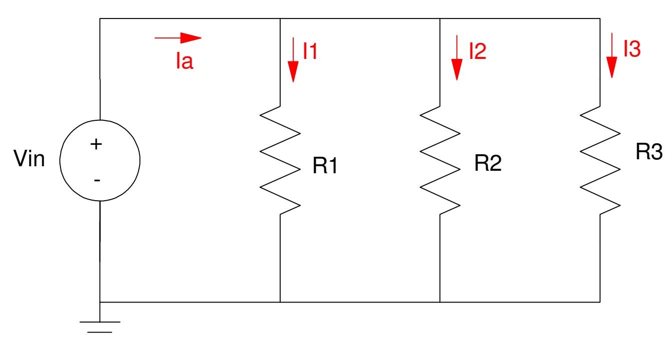

Figure 16: Resistors in parallel add as the sum of the inverses, inverted: $R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_3}\right)^{-1}$

Current In = Current Out

$$I_a = I_1 + I_2 + I_3$$

From V = IR

$$I_a = \left(\frac{V_{in}}{R_1}\right) + \left(\frac{V_{in}}{R_2}\right) + \left(\frac{V_{in}}{R_3}\right)$$

$$I_a = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_{in}$$

$$V_{in} = I_a \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

Voltage Division: The voltage V3 is

$$I = \left(\frac{V_{in}}{R_1+R_2+R_3}\right)$$

$$V_3 = IR_3$$

$$V_3 = \left(\frac{R_3}{R_1+R_2+R_3}\right) V_{in}$$

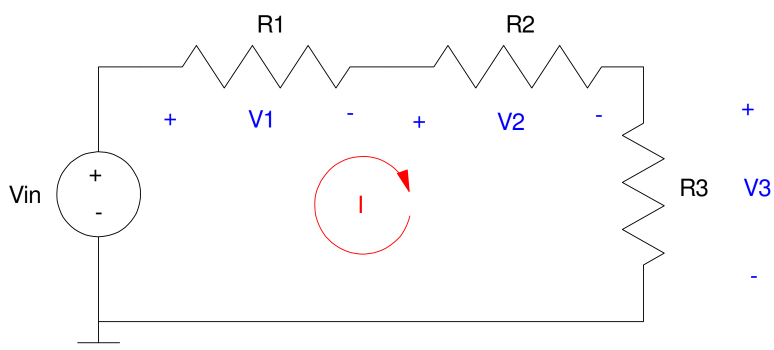

Figure 17: Voltage division: find Vx as a funciton of Vin

The voltage across a resistor for resistors in series is

$$V_R = \left(\frac{\text{The resistance you're measuring across}}{\text{The total resistance}}\right) \cdot \text{The input voltage}$$

With these tools, you can simplify various resistor circuits.

HP42 Calculator (Free42)

When you take EE 206 Circuits I and thereafter, you are going to need a calculator which can

- Add, subtract, multiply, and divide real numbers,
- Add subtract, multiply, and divide complex numbers,
- Can solve f(x) = 0,
- And can do this quickly and accurately.

Your grade on midterms depends upon this.

Two calculators that I highly recommend are the HP35s and HP42. Both are worth about ten points on a midterm in my experience. Essentially, HP is a company dedicated to and staffed by electrical and computer enginineers. Not surprisingly, their calculators work really well on ECE-type problems. They *do* have a learning curve, though.

First off, where to buy an HP calculator. For this class, I recommend a free app on your cell-phone: Free42. This app emulates the best calculator ever made in my opinion, and is 100% free. Donations to the founder are of course accepted - but not required.

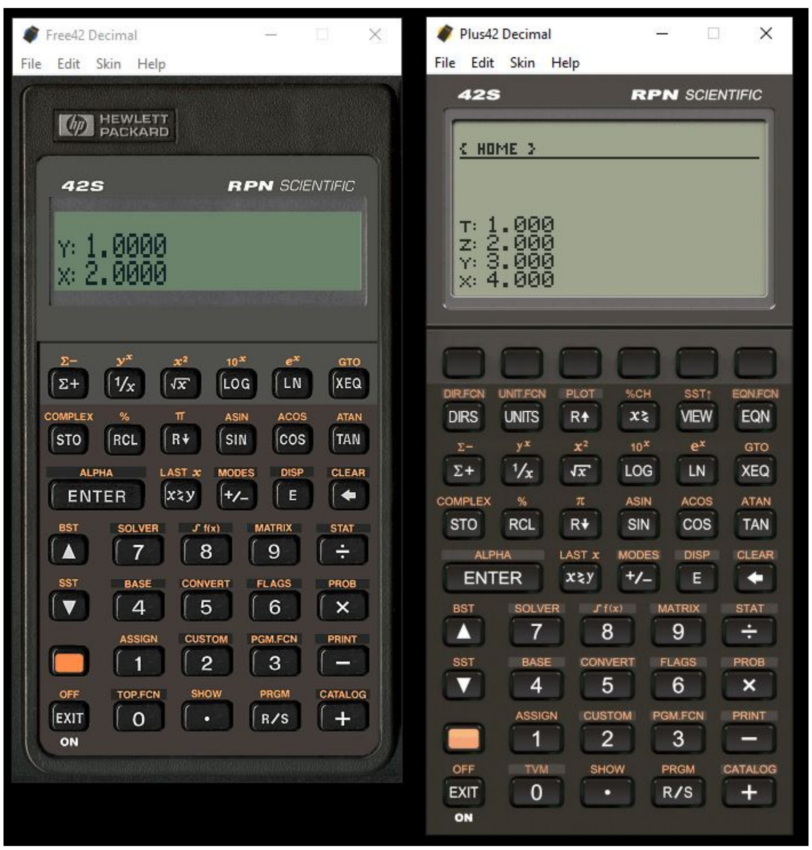

Figure 18: Free42 (left) and Plus42 (Right)
Both are the same, except the Plus42 shows all four elements on the stack.

Next, how to operate an HP42. Videos work better in showing how to use an HP, but here's an attempt using text.

HP calculators are a little different: they use reverse polish notation (RPN) instead of algebraic notation like everyone else. RPN takes time to get use to, but once you do, it's *really* fast. Plus, it works equally well with real numbers as well as complex numbers (coming soon).

The first thing to note is that HP's operate with a thing called the stack. This is four registers (numbers) called X, Y, Z, and T:

- T *top of the stack*
- Z
- Y
- X *bottom of the stack*

When you type in a number, it is placed in the X register.

When you press enter, the data is pushed onto the stack:

- T receives data from Z
- Z receives data from Y
- Y receives data from X
- X remains unchanged, but is replaced by a new number if you press a number

For example, to initialize the stack with {1, 2, 3, 4} as show in the previous picture, type in

```
1
enter
2
enter
3
enter
4
```

Figure 19: What the stack looks like after three enter commands

The key X<>Y swaps the contents of the X and Y register

The key R↓ rotates the stack down (rolls the stack so each element shifts one position

- X receives data from Y
- Y receives data from Z
- Z receives data from T
- T receives data from X

Addition, subtraction, multiplication, and division operate on the X and Y register:

- + X becomes Y + X
- - X becomes Y - X
- * X becomes Y * X
- / X becomes Y / X

The other registers shift down one

- T remains unchanged
- Z receives data from T
- Y receives data from Z
- X is the result of the operation f(X, Y)

With this, you can solve several problems.

Example 1: Solve for Y

$$Y = \left(\frac{(2+3)(7+6)}{(8+9)}\right)$$

Solution: Press the keys in the following order (other sequences also work)

```
2          x = 2
enter      x = 2       y = 2
3          x = 3       y = 2
+          x = 5
7          x = 7       y = 5
enter      x = 7       y = 7       z = 5
6          x = 6       y = 7       z = 5
+          x = 13      y = 5
*          x = 65
8          x = 8       y = 65
enter      x = 8       y = 8       z = 65
9          x = 9       y = 8       z = 65
+          x = 17      y = 65
/          x = 3.824
```

Note that the beauty of RPN notation is you don't need to use parentheses: you just start in the middle and work your way out. With the stack, you can save previous operations (like (2+3)(7+6) when computing the denominator value).

Example 2: Find the resistance and voltage at T = 20C

$$R = 1000 \cdot \exp\left(\frac{3905}{T+273} - \frac{3905}{298}\right)\Omega$$

$$V = \left(\frac{R}{R+1000}\right)10V$$

Solution: Start in the middle and work your way out

```
20          x = 20
enter       x = 20      y = 20
273         x = 273     y = 20
+           x = 293
3905        x = 3905    y = 293
X<>Y        x = 293     y = 3905
/           x = 13.33
3905        x = 3905    y = 13.33
enter       x = 3905    y = 3905    z = 13.33
298         x = 298     y = 3905    z = 13.33
/           x = 13.10   y = 13.33
-           x = 0.22
e^x         x = 1.25
1000        x = 1000    y = 1.25
*           x = 1250.06             R = 1250.06 Ohms

enter       x = 1250    y = 1250
enter       x = 1250    y = 1250    z = 1250
1000        x = 1000    y = 1250    z = 1250
+           x = 2250    z = 1250
/           x = 0.556
10          x = 10      y = 0.556
*           x = 5.557               V = 5.557 Volts
```

In the following resistor circuits, we'll do more examples with an HP calculator.

Note: Problems presented in this class are similar to what you'll see on tests in Circuits I and Circuits II. Whatever calculator you choose to use, make sure it can solve these types of problems. Your grade in Circuits I and II will depend upon it.

Resistor Circuit Examples

Example 1: Find the resistance from a to b

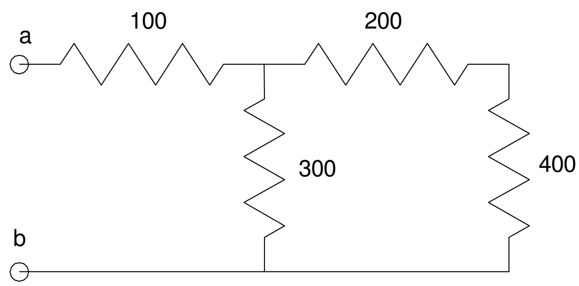

Figure 20: Find the resistance between a and b (Rab)

Solution:

The 200 and 400 Ohm resistor are in series

$$200 + 400 = 600$$

The 600 (net) and 300 are in parallel

$$300 \,||\, 600 = \left(\frac{1}{300} + \frac{1}{600}\right)^{-1} = 200$$

200 (net) and 100 are in series

$$200 + 100 = 300$$

The net resistance is 300 Ohms.

HP42 Solution:

```
400          x = 400
enter        x = 400      y = 400
200          x = 200      y = 400
+            x = 600
1/X          x = 0.0017
300          x = 300      y = 0.0017
1/X          x = 0.0033   y = 0.0017
+            x = 0.0050
1/X          x = 200.00
100          x = 100      y = 200.00
+            x = 300.00
```

CircuitLab Solution:

In CircuitLab you can check your answer:

- Build the circuit
- Apply a V test voltage
- Measure the current. From V = IR, you should get 1/300A

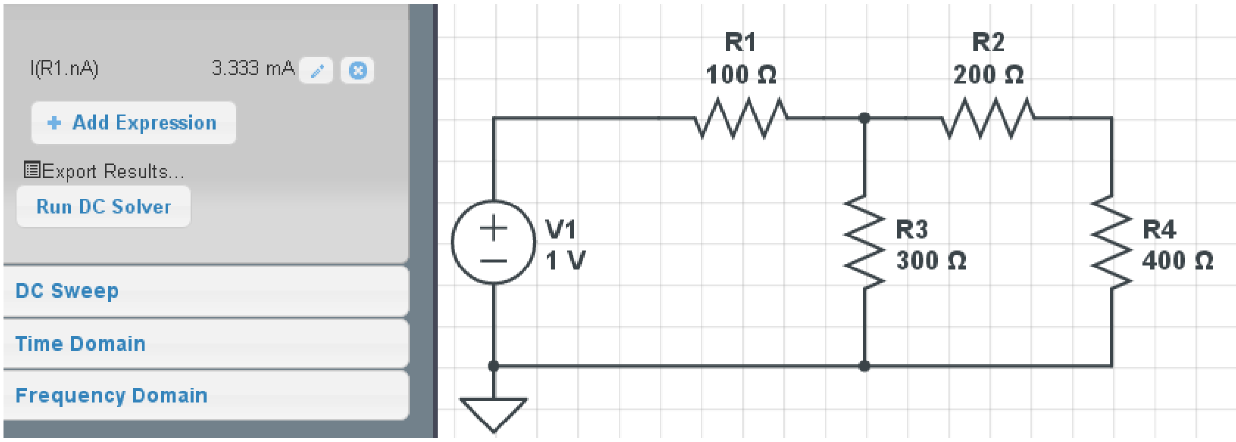

Figure 21: Finding the resistance using CircuiLab

Example 2: Find the net resistance Rab

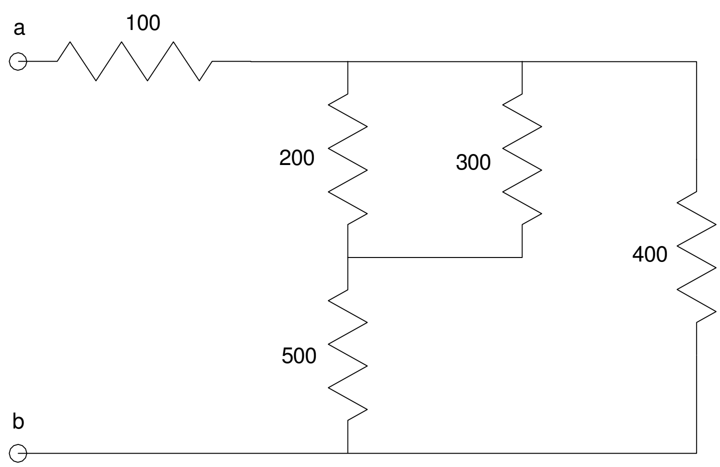

Figure 22: Example 2: Find the resistance between a and b (Rab)

Solution: The 200 and 300 are in parallel

200 || 300 = 120 Ohms

This is in series with 500

120 + 500 = 620

This is in parallel with 400

620 || 400 = 243.137

This is in series with 100

243.137 + 100 = 343.137

The net resistance is 343.137 Ohms

HP42 Solution:

```
200            x = 200
1/X            x = 0.0050
300            x = 300      y = 0.0050
1/X            x = 0.0033   y = 0.0050
+              x = 0.0083
1/X            x = 120.00                R3 || R6 = 120.00 Ohms
500            x = 500      y = 120
+              x = 620
1/X            x = 0.0016
400            x = 400      y = 0.0016
1/X            x = 0.025    y = 0.0016
+              x = 0.0041
1/X            x = 243.14
100            x = 100      y = 243.14
+              x = 343.14                The net resistance is 343.14 Ohms
```

CircuitLab Solution:

Build the circuit in CircuitLab.

- Apply a 1V test voltage
- Determine the current draw
- Calculate the resistance as R = V / I

$$R = \left(\frac{1.000V}{2.914mA}\right) = 343.17\Omega$$

(There is some rounding error in the answer)

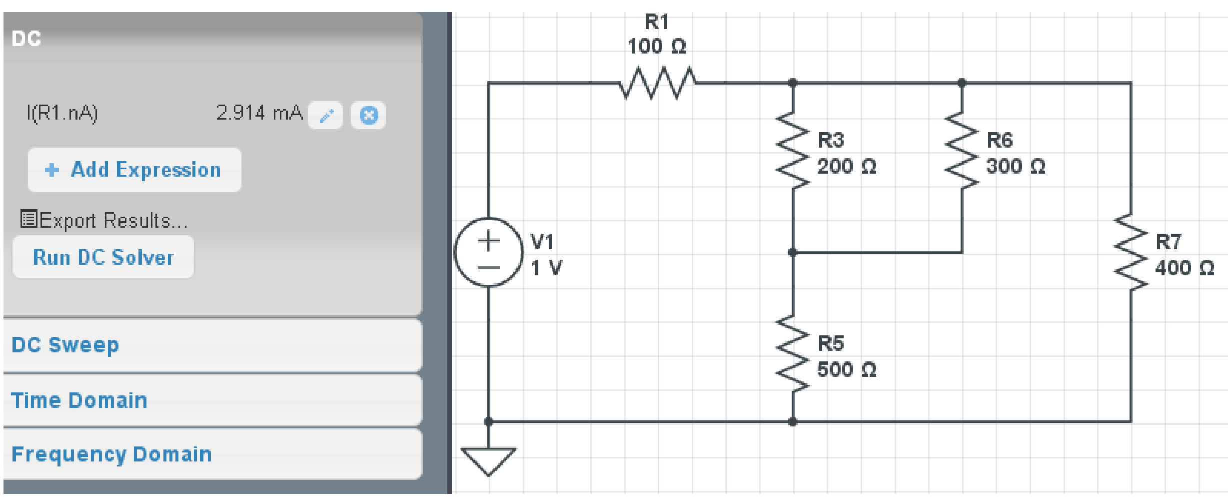

Figure 23: Finding the resistance using CircuitLab

Circuit Analysis via Voltage Division

Using voltage division, determine V1, V2, and V3

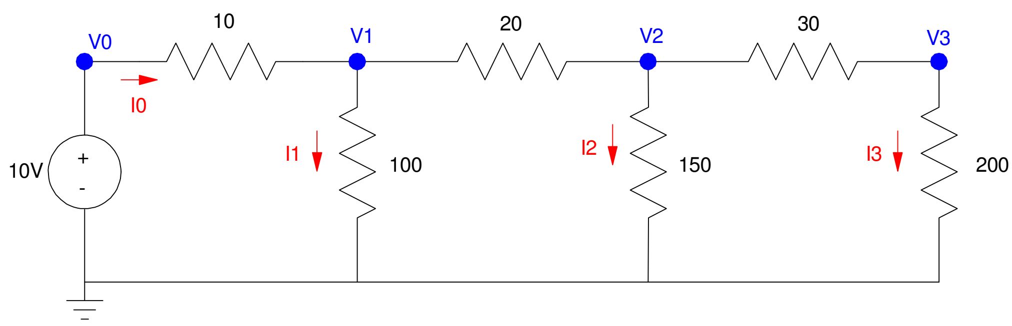

Figure 24: Given the above circuit, find the voltages

Solution: Determine the resistance at V1 looking right:

200 + 30 = 230 Ohms

230 || 150 = 90.79 Ohms

90.79 + 200 = 110.79 Ohms

110.79 || 100 = 52.56 Ohms

The resistance to ground at V1 is 52.56 Ohms. By voltage division

$$V_1 = \left(\frac{52.56}{52.56+10}\right) V_0$$

$$V_1 = 8.4015V$$

Now that we know V1, find V2. From the above analysis, the resistance to ground at V2 looking right is

200 + 30 = 230 Ohms

230 || 150 = 90.79 Ohms

By voltage division,

$$V_2 = \left(\frac{90.79}{90.79+20}\right) V_1$$

$$V_2 = 6.8849V$$

By voltage division

$$V_3 = \left(\frac{200}{200+30}\right) V_3$$

$$V_3 = 5.9868V$$

HP42 Solution:

	x	y	z	t
200	200			
enter	200	200		
30	30	200		
+	230			
1/X	0.0043			
150	150	0.0043		
1/X	0.0067	0.0042		
+	0.0110			
1/X	90.7895			
STO 02				
20	20	90.7895		
+	110.7895			
1/X	0.0090			
100	100	0.0090		
1/X	0.0100	0.0090		
+	0.0190			
1/X	52.5593			
STO 01				
enter	52.5593	52.5593		
enter	52.5593	52.5593	52.5593	
10	10	52.5593	52.5593	
+	62.5593	52.5593		
/	0.8402			
10	10	0.8402		
*	**8.4015**		***V1 = 8.4015 Ohms***	
RCL 02	90.7895	8.4015		
RCL 02	90.7895	90.7895	8.4015	
20	20	90.7895	90.7895	8.4015
+	110.7895	90.7895	8.4015	
/	0.8195	8.4015		
*	**6.8849**		***V2 = 6.8849V***	
200	200	6.8849		
enter	200	200	6.8849	
enter	200	200	200	6.8849
30	30	200	200	6.8849
+	230	200	6.8849	
/	0.8696	6.8849		
*	**5.9868**		***V3 = 5.9868V***	

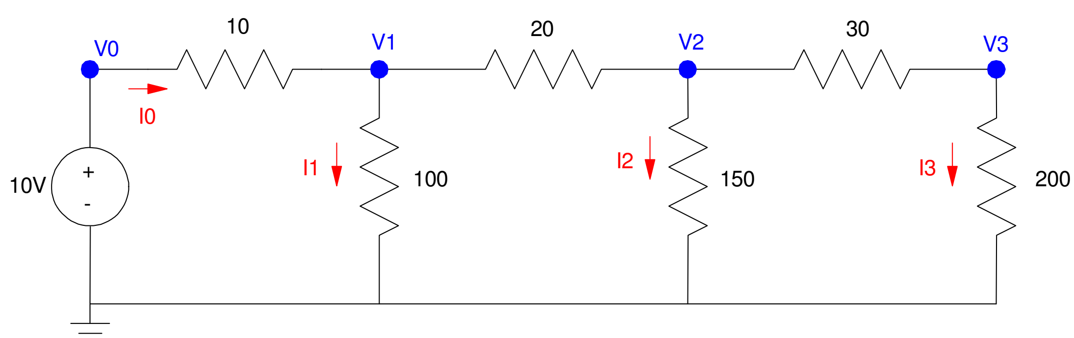

Figure 25: Circuit used for the previous calculations

CircuitLab Solution

- Built the circuit in CircuitLab
- Solve using the DC solver

These match the voltages you get with CircuitLab:

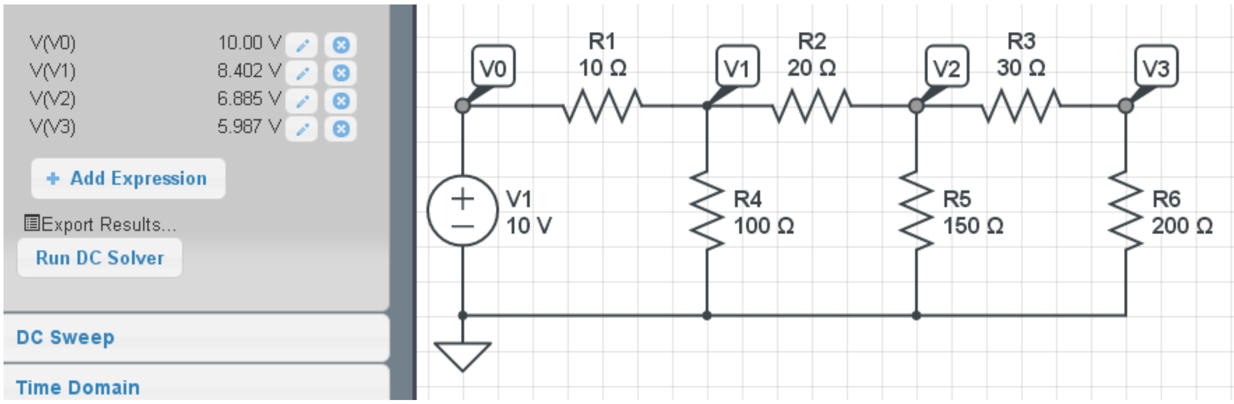

Figure 26: CircuitLab can be used to verify your calculations

Summary

- The sum of the voltages around any closed path must be zero (Kirchoff's voltage law)
- The sum of the current coming out of a node must be zero (Kirchoff's current law)
- Resistors in series add
- Resistors in parallel add as the sum of the inverses, inverted
- This allows you to find the net resistance of resistor networks.
- With voltage division, you can find the voltages for some circuits.

and...

- HP calculators work well when solving these types of problems