Modern Control

ECE 463 - Modern Control

Jake Glower - Lecture #0

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Feedback

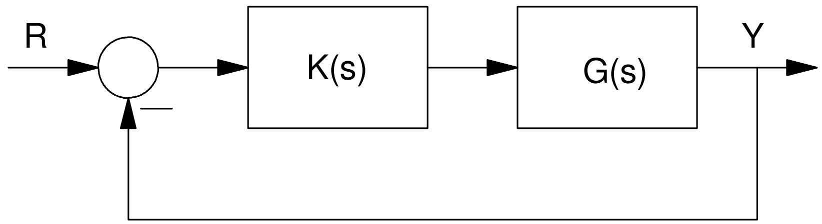
Feedback is very useful

- Instead of specifying the input, you specify the desired output
- It can improve the response of a system
- It can turn an unstable system into a stable system (closed-loop)

ECE 461: Classic Control

One solution: Use output feedback

- G(s) is the plant
- K(s) is a pre-filter, making a poorly behaving system behave like a well-behaved system



Tools for finding K(s)

- Root Locus
- Nichols Charts
- Bode Plots

Sometimes, classic control works very well

- Controlling DC motors
- Controlling temperature
- Controlling water flow

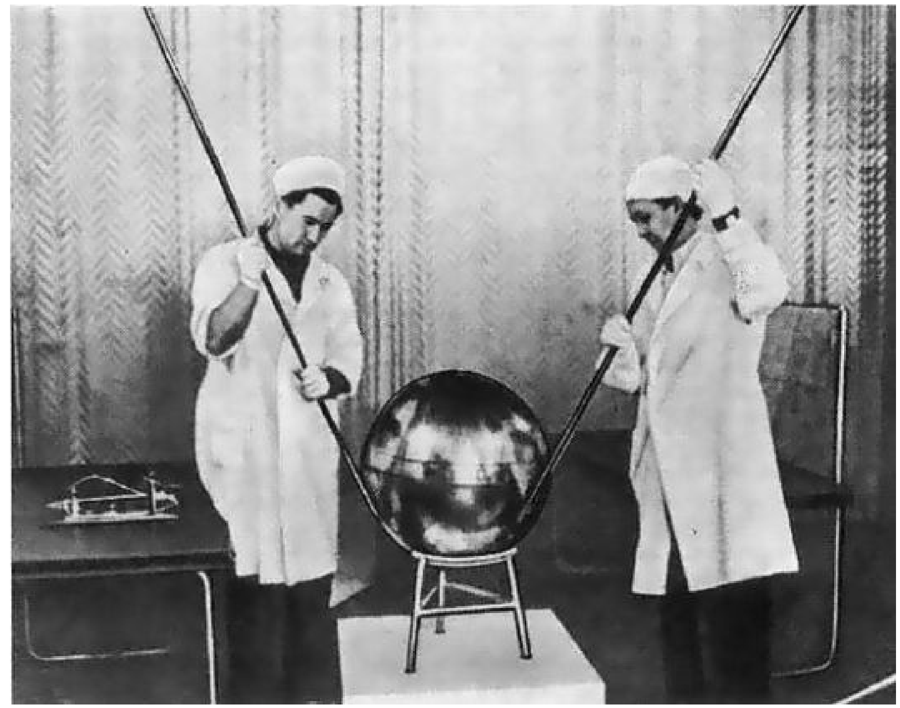
Sometimes, Classic Control doesn't

- Rockets
- Systems with multiple unstable poles

Sputnik & Modern Control

https://theinvisibleagent.files.wordpress.com/2011/09/sputnik3.jpg

While NASA was struggling to get a rocket into space, the Soviet Union placed a satellite into orbit in 1957



This was a major shock to the U.S.

- Russia was ahead of the U.S.

This led to researchers looking at Soviet technical journals to learn how the Soviets stabilize their rockets

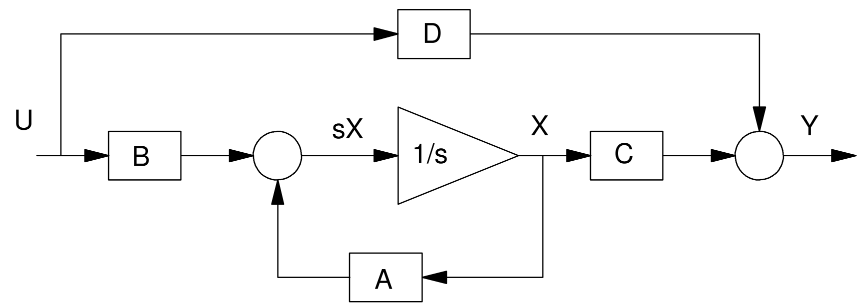
- The techniques used is termed "Modern Control"

Modern Control (ECE 463/663)

- A matrix approach to designing feedback control systems

Let X define the energy in the system

- X = [position, velocity, ...]
- potential energy, kinetic energy, ...



If you specify how the energy moves around, you've specified the dynamics

$$sX = AX + BU$$

$$Y = CD + DU$$

Modern vs. Classical Control

The transfer function from U to Y is

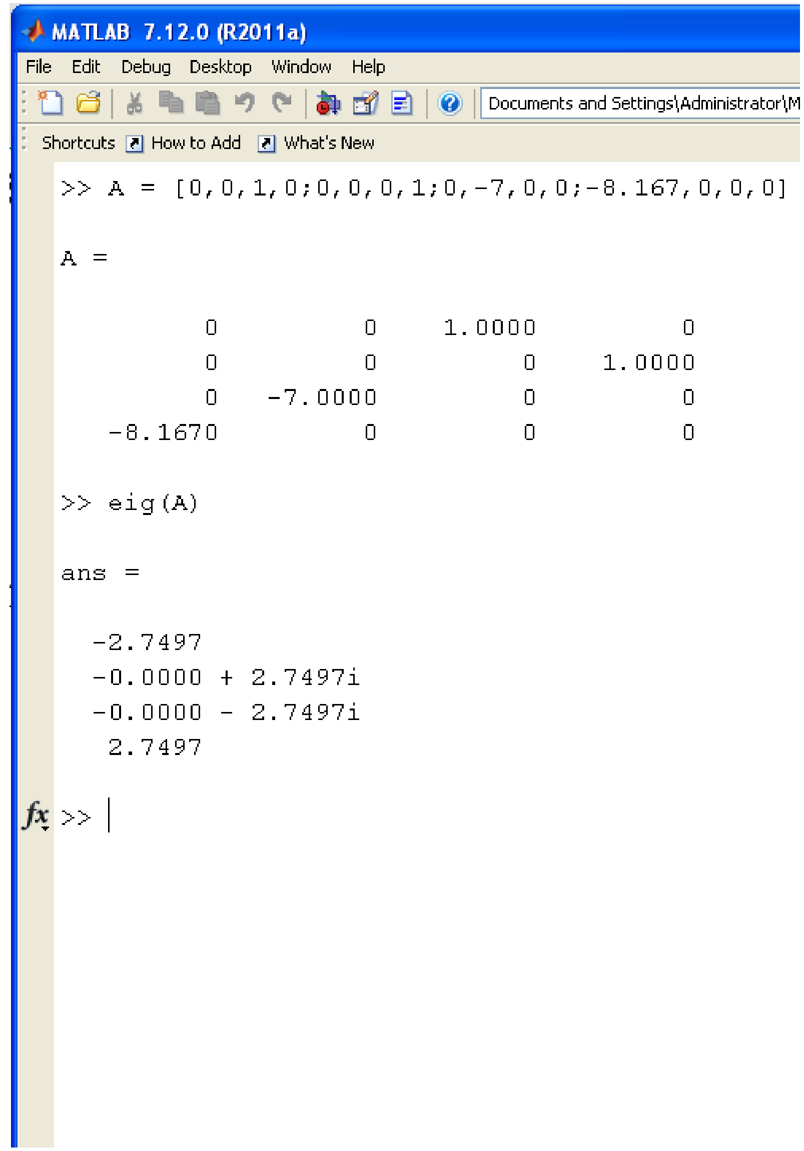
$$Y = \left(C(sI - A)^{-1} B + D \right) U$$

$$G(s) = C(sI - A)^{-1} B + D$$

The eigenvalues of A are all important

- Eigenvalues = Poles
- Negative definite = stable
- Positive real = unstable
- Complex = oscillatory

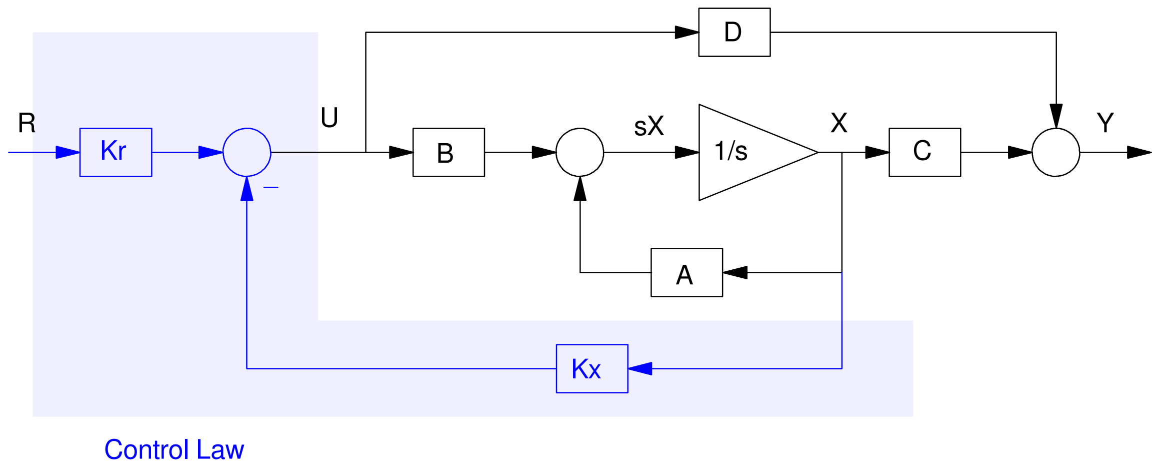
If you can shift the eigenvalues of A, you can change how the system behaves



Full State Feedback

A is an NxN matrix

- It has N eigenvalues
- It has N poles



If you define U as

$$U = -K_x X + K_r R$$

the dynamics become

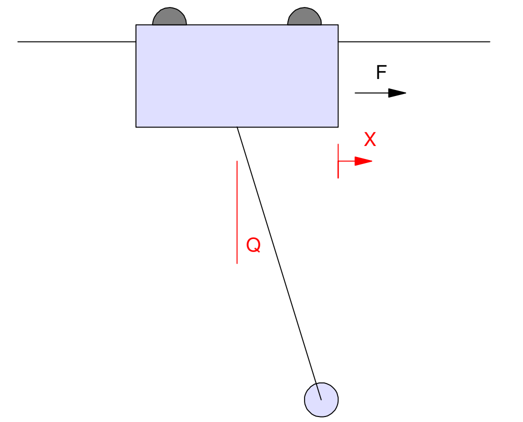
$$sX = (A - BK_x)X + BK_r R$$

$$Y = CX + DU$$

With Kx, you have N degrees of freedom to place the N closed-loop eigenvalues.

How do you determine the system dynamics?

- Week 1-5 (Test #1)
- LaGrangian formulation

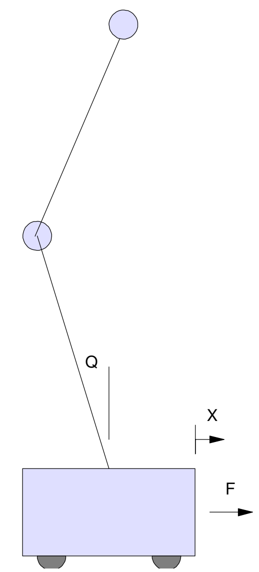


Define the energy in the system

$$L = KE - PE$$

Define how the energy moves about the system

Linearize to determine the state-space model

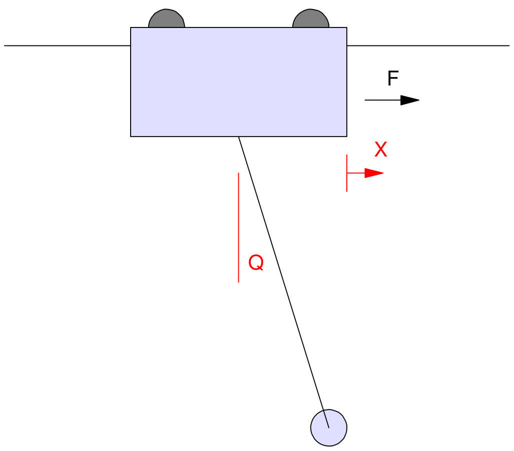


Works for both linear and nonlinear systems

Systems we look at in ECE 463

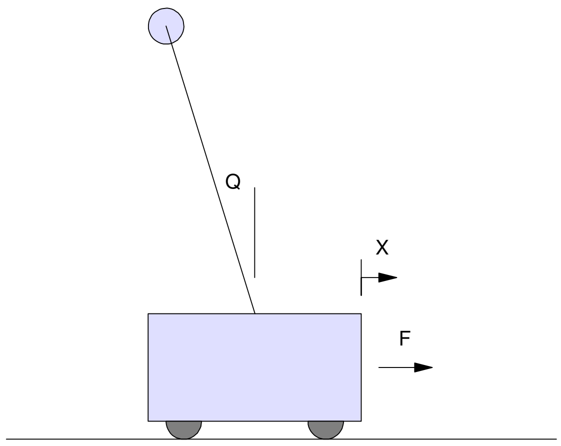
Gantry System

$$x = \left(\frac{2(s+j3)(s-j3)}{s^2(s+j4)(s-j4)} \right) F$$



Cart and Pendulum

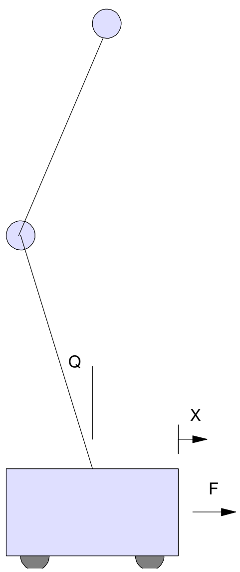
$$x = \left(\frac{2(s+3)(s-3)}{s^2(s+4)(s-4)} \right) F$$



Systems we look at in ECE 463

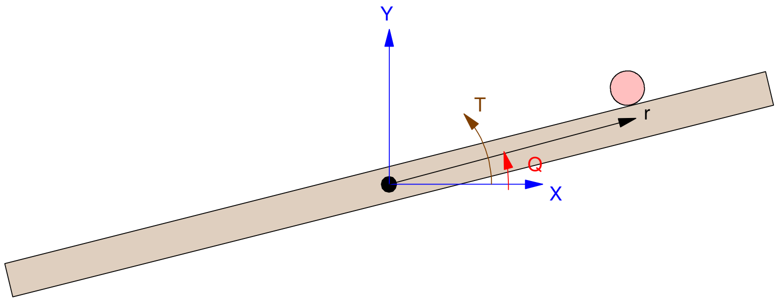
Double Pendulum

$$x = \left(\frac{(s+3.5\pm j1.5)(s-3.5\pm j1.5)}{s^2(s+4.5\pm j1.5)(s-4.5\pm j1.5)} \right) F$$



Ball and Beam

$$r = \left(\frac{10}{(s+3)(s-3)(s+j3)(s-j3)} \right) T$$



Problem: How do you determine Kx?

Pole Placement

- **Week 5-8 & Test #2**

- Bass Gura
- Kx contains N terms
- (A - B Kx) has N eigenvalues
- N equations and N unknowns

Good:

- Complete freedom
- Can place poles anywhere

Bad:

- Complete freedom
- Where *should* you place the poles

Optimal Control

- **Weeks 9-12 & Test #3**

- Based upon Calculus of Variations
- Define a cost function
- J = f(energy) + g($input^2$)
- Determine Kx to minimize J

Good:

- Places all poles in their optimal spot

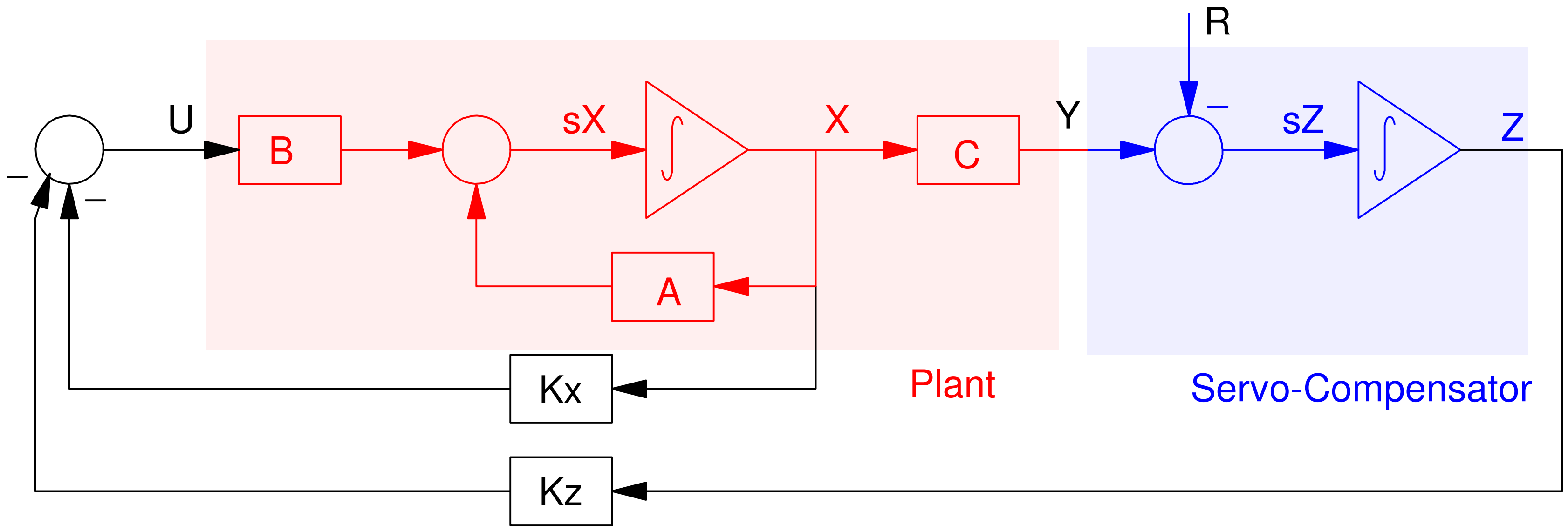
Bad:

- Cost function is arbitrary
- I'd prefer "good control" over "optimal control"
- It's a tool to find Kx

Problem: How to you track a constant set point?

- ECE 461: Make it a type-1 system
- ECE 463: Add a servo compensator

Also allows you to track multiple sine waves

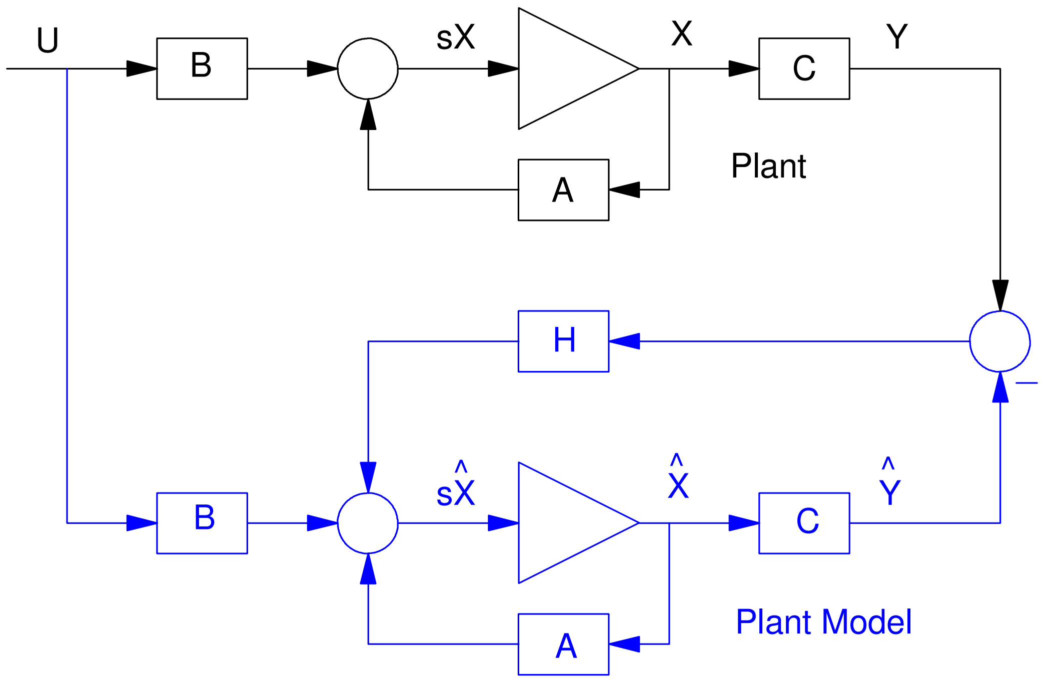


Problem: What if you can't measure *all* of the states?

- Kx contains N terms
- You need to know all N states in X

If you can't measure the states, estimate them from the input and output

- Full-Order Observer



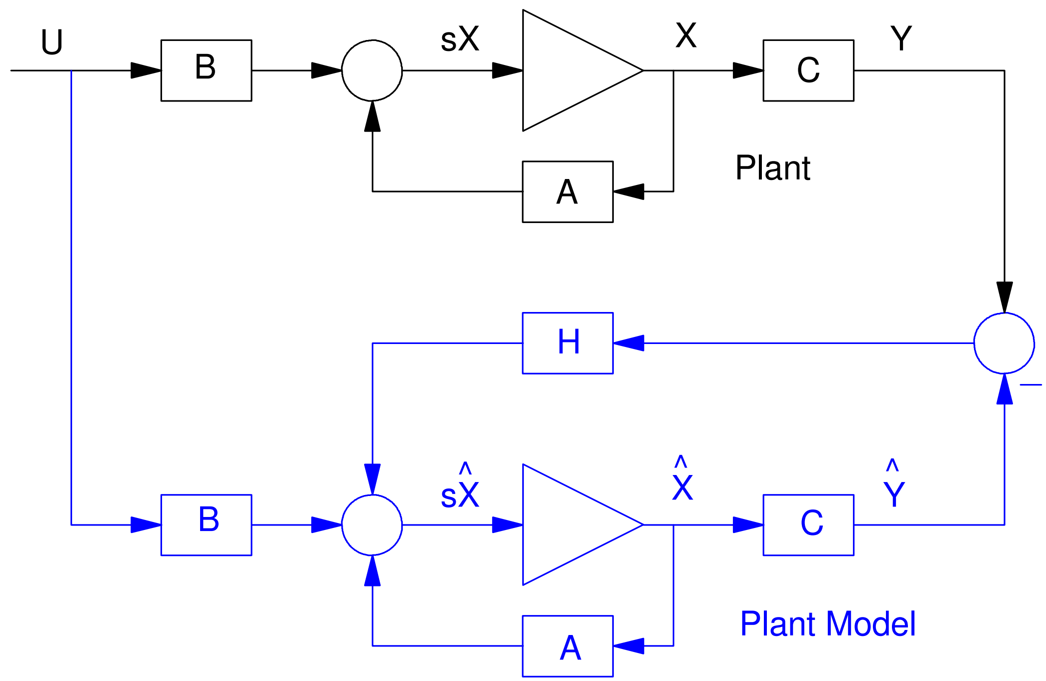
Problem: How do you choose H?

Option #1: Pole Placement

- Bass Gura

Option #2: Optimal Control

- Produces a Kalman Filter
- Optimal H given sensor noise and input noise



ECE 463/663 Web Site

- www.BisonAcademy.com

Syllabus is posted

- Daily lecture topics

All lecture notes are posted

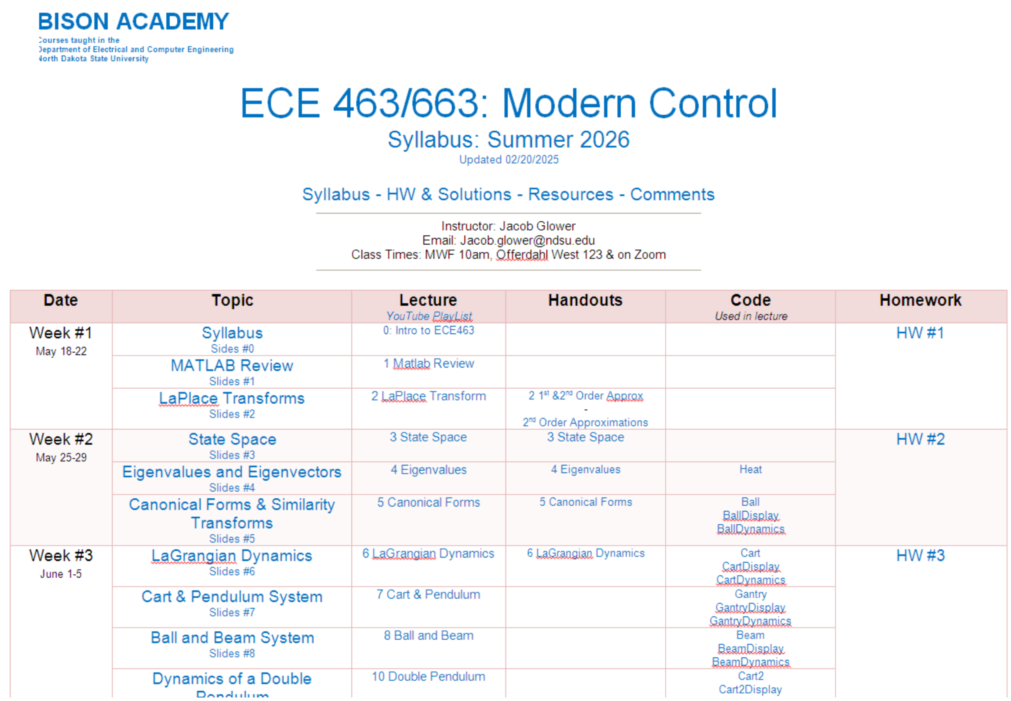
- pdf format

Videos of all lectures are linked

- YouTube videos

Homework Sets are posted

- Solutions posted after due date


BISON ACADEMY
Courses taught in the
Department of Electrical and Computer Engineering
North Dakota State University

ECE 463/663: Modern Control
Syllabus: Summer 2026
Updated 02/20/2025

Syllabus - HW & Solutions - Resources - Comments

Instructor: Jacob Glower
Email: Jacob.glower@ndsu.edu
Class Times: MWF 10am, Offerdahl West 123 & on Zoom

Date	Topic	Lecture *YouTube PlayList*	Handouts	Code *Used in lecture*	Homework
Week #1 May 18-22	Syllabus Sides #0	0: Intro to ECE463			HW #1
	MATLAB Review Slides #1	1 Matlab Review			
	LaPlace Transforms Slides #2	2 LaPlace Transform	2 1st &2nd Order Approx - 2nd Order Approximations		
Week #2 May 25-29	State Space Slides #3	3 State Space	3 State Space		HW #2
	Eigenvalues and Eigenvectors Slides #4	4 Eigenvalues	4 Eigenvalues	Heat	
	Canonical Forms & Similarity Transforms Slides #5	5 Canonical Forms	5 Canonical Forms	Ball BallDisplay BallDynamics	
Week #3 June 1-5	LaGrangian Dynamics Slides #6	6 LaGrangian Dynamics	6 LaGrangian Dynamics	Cart CartDisplay CartDynamics	HW #3
	Cart & Pendulum System Slides #7	7 Cart & Pendulum		Gantry GantryDisplay GantryDynamics	
	Ball and Beam System Slides #8	8 Ball and Beam		Beam BeamDisplay BeamDynamics	
	Dynamics of a Double Pendulum	10 Double Pendulum		Cart2 Cart2Display	

Homework Sets & Solutions

Last few offerings posted

- Serves as a reference if you need help with the homework
- Systems and dynamics are different, but the procedure will be similar

Also has sample tests and solutions

BISON ACADEMY

Courses taught in the
Department of Electrical and Computer Engineering
North Dakota State University

ECE 463/663: Modern Control

Homework and Solutions

Syllabus - HW & Solutions - Resources - Comments

Spring 2019	Spring 2018	Spring 2017	Spring 2016
1: Dominant Poles Solution #1	1: Dominant Poles Solution #1	1: Dominant Poles Solution #1	1: LaPlace Transforms Solution #1
2: State Space Solution #2	2: State Space Solution #2	2: State Space Solution #2	2: Modeling in State Space Solution #2
3: Canonocal Forms Solution #3	3: Canonical Forms Solution #3	3: Canonical Forms Solution #3	3: Canonical Forms Solution #3
4: LaGrangian Dynamics Solution #4	4: LaGrangian Dynamics Solution #4	4: LaGrangian Dynamics Solution #4	4: LaGrangian Dynamics Solution #4
Test #1 Test #1 Solution	Test #1 Test #1 Solution	Test #1	5: Controllability and Observability Solution #5
5: Pole Placement Solution #5	5: Pole Placement Solution #5	5: Pole Placement Solution #5	6: Pole Placement Solution #6
6: Servo Compensators Solution #6	6: Servo Compensators Solution #6	6: Servo Compensators Solution #6	7: Servo Compensators Solution #7
7: Linear Observers Solution #7	7: Observers Solution #7	7: Linear Observers Solution #7	8: Observers Solution #8
Test #2 Solution A : Solution B	Test #2 Test #2 Solution	Test #2 Solutions: A Level - B Level - C Level	9: Calculus of Variation Solution #9
8: Calculus of Variations Solution #8	8: Calculus of Variations Solution #8	8: Calculus of Variations Solution #8	10: LQG Control Solution #10
9: Optimal Control Solution #9	9: Optimal Control Solution #9	9: LQG Control Solution #9	11: Kalman Filters

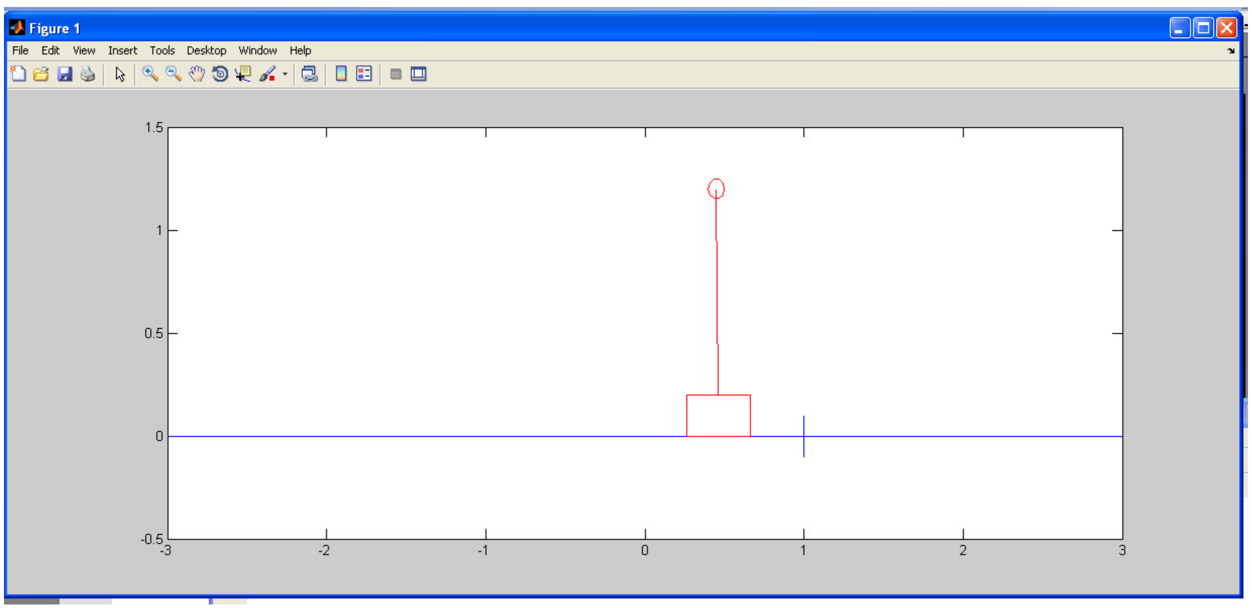
ECE 463: Modern Control

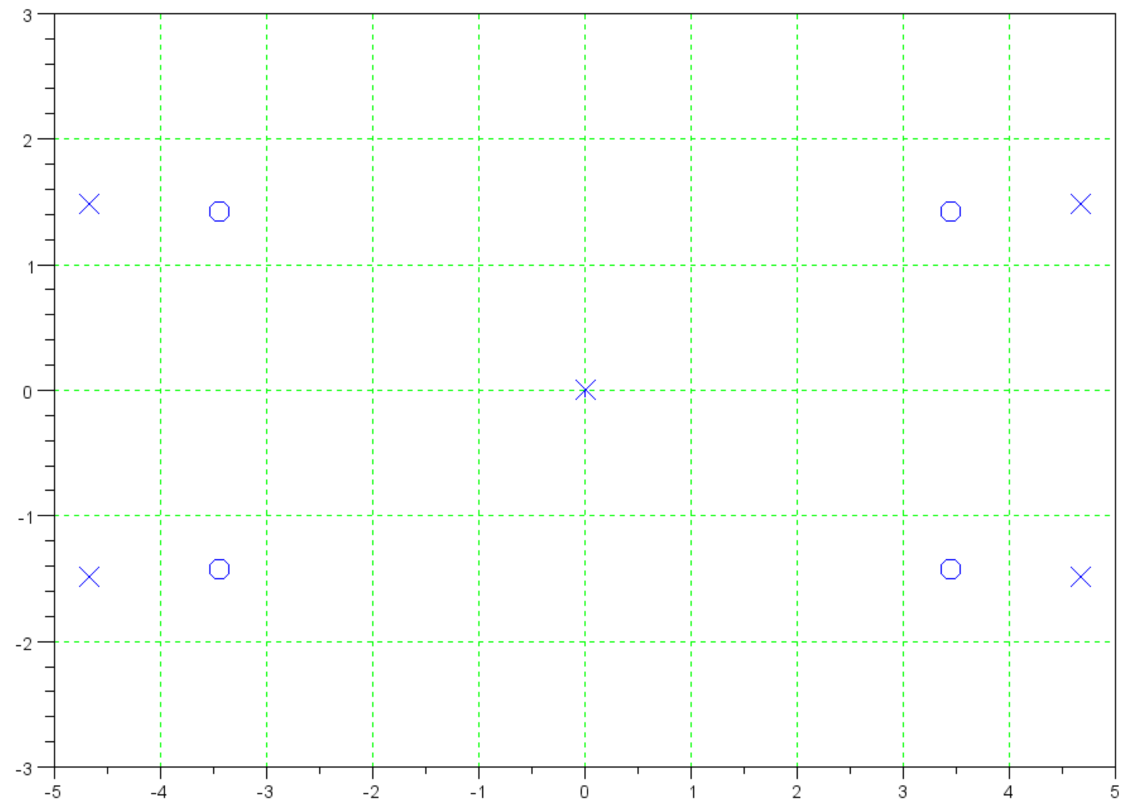
Fun class were we use Matlab extensively

- Simulating nonlinear dynamic systems
- Manipulating NxN matrices

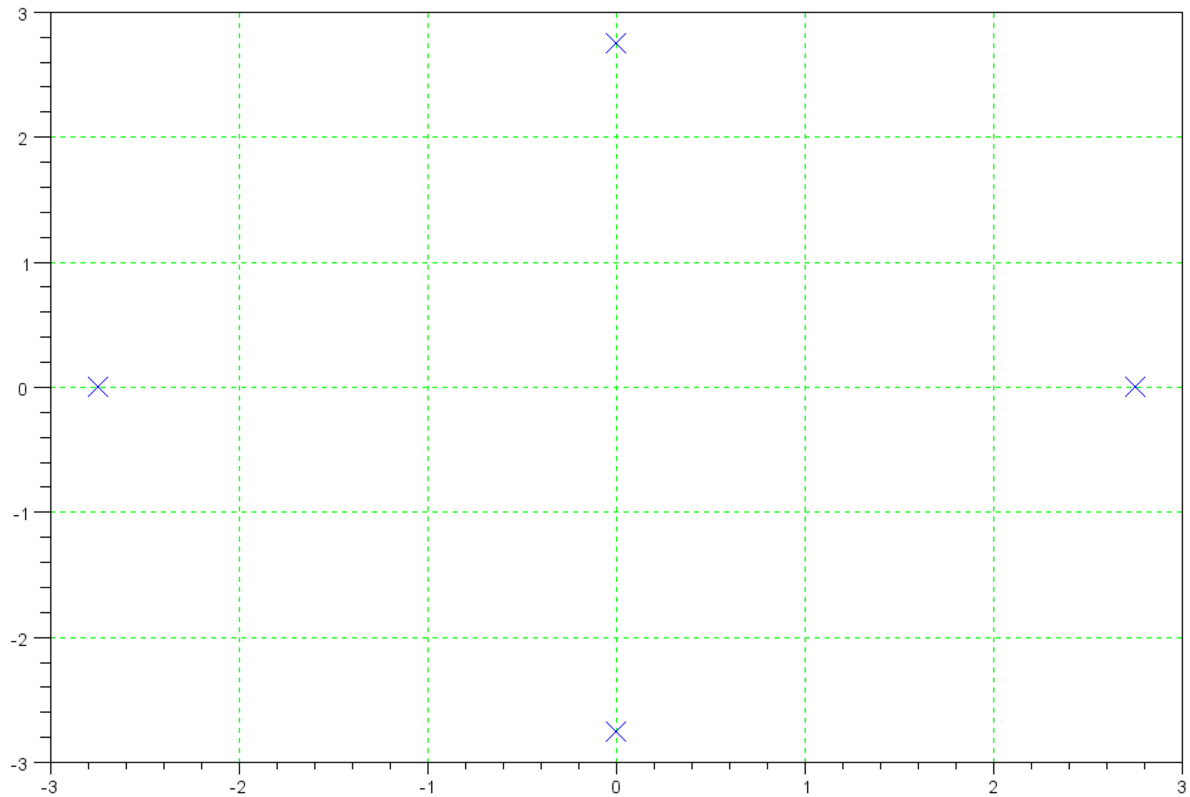
Heavy use of matrix algebra

- We cover what you'll use in this class
- Math 129 is the main class we use





3
2
1
0
-1
-2
-3
-5
-4
-3
-2
-1
0
1
2
3
4
5


3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3