

ECE 463/663 Handout #5

Canonical Forms

Express the following in state space form two different ways:

$$Y = \left(\frac{2(s+3)}{(s+1)(s+5)} \right) U = \left(\frac{2s+6}{s^2+6s+5} \right) U$$

Solution

$$Y = \left(\frac{2(s+3)}{(s+1)(s+5)} \right) U = \left(\frac{2s+6}{s^2+6s+5} \right) U$$

Controller Canonical Form

$$sX = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 6 & 2 \end{bmatrix} X$$

Observer Canonical Form

- Transpose controller canonical form

$$sX = \begin{bmatrix} 0 & -5 \\ 1 & -6 \end{bmatrix} X + \begin{bmatrix} 6 \\ 2 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} X$$

Cascade Form

$$Y = \left(\frac{a}{s+1} \right) + \left(\frac{b}{(s+1)(s+5)} \right) = \left(\frac{a(s+5)}{(s+1)(s+5)} \right) + \left(\frac{b}{(s+1)(s+5)} \right)$$

$$a = 2$$

$$b = -4$$

$$sX = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 2 & -4 \end{bmatrix} X$$

Jordan form

$$Y = \left(\frac{2(s+3)}{(s+1)(s+5)} \right) U = \left(\left(\frac{1}{s+1} \right) + \left(\frac{1}{s+5} \right) \right) U$$

$$sX = \begin{bmatrix} -1 & 0 \\ 0 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 & 1 \end{bmatrix} X$$