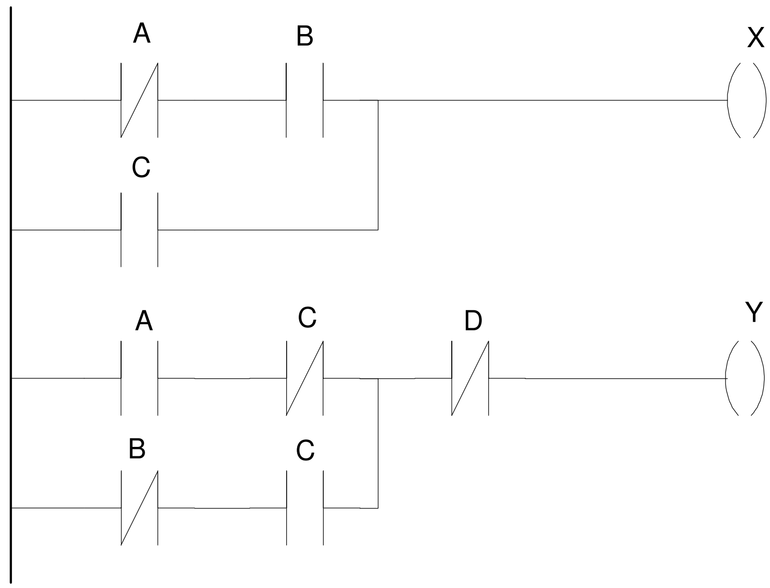
Homework #2: ECE 461 / 661

Intro to Ladder Logic, 1st & 2nd-Order Approximations, Block Diagrams

Ladder Logic

1) Find the logic represented by the following ladder diagram



Answer:

$$X = \overline{A}B + C$$

$$Y = \overline{D}\left(A\overline{C} + \overline{B}C\right)$$

This can also be written using a prime to denote not

$$X = A'B + C$$

$$Y = D'(AC' + B'C)$$

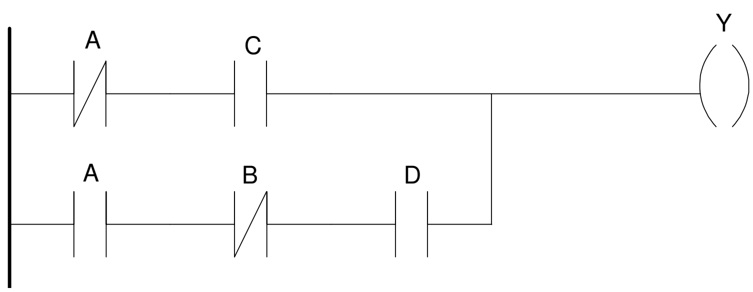
2) Find the logic expression for the following Karnough map.

- Create a ladder diagram to output Y(A, B, C, D)

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	x	1	1
11	0	0	x	0
10	0	1	1	x

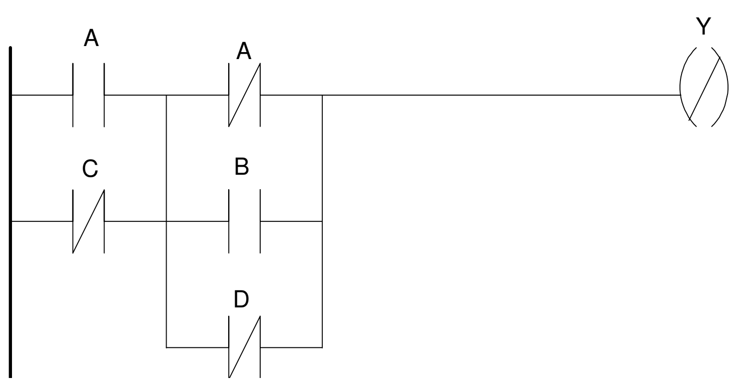
Option 1: Circle the ones

$$Y = \overline{A}C + A\overline{B}D$$



Option #2: Using DeMorgan's law

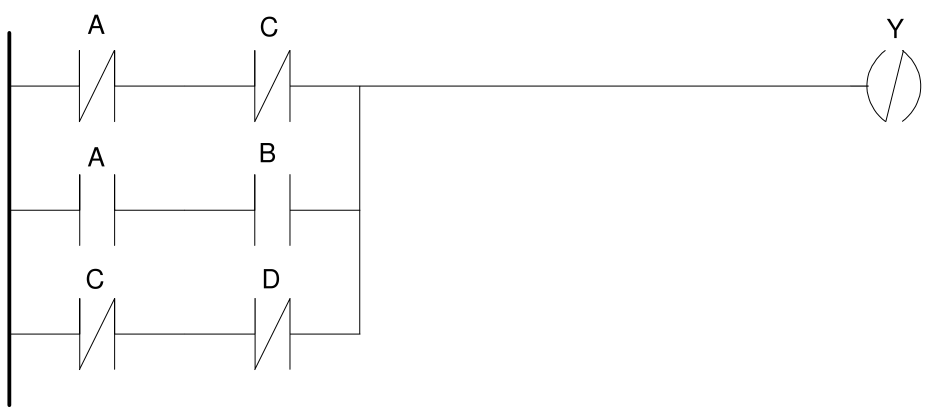
$$\overline{Y} = \left(A + \overline{C}\right)\left(\overline{A} + B + \overline{D}\right)$$



Option 3: Circle the zeros

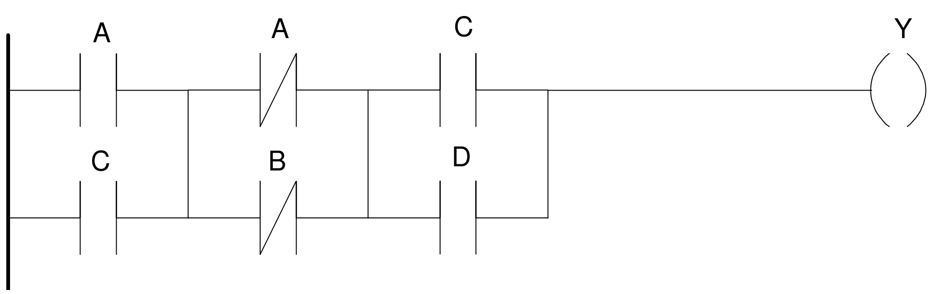
AB \ CD	00	01	11	10
00	0	0	1	1
01	0	x	1	1
11	0	0	x	0
10	0	1	1	x

$$\overline{Y} = \overline{A}\,\overline{C} + AB + \overline{C}\,\overline{D}$$



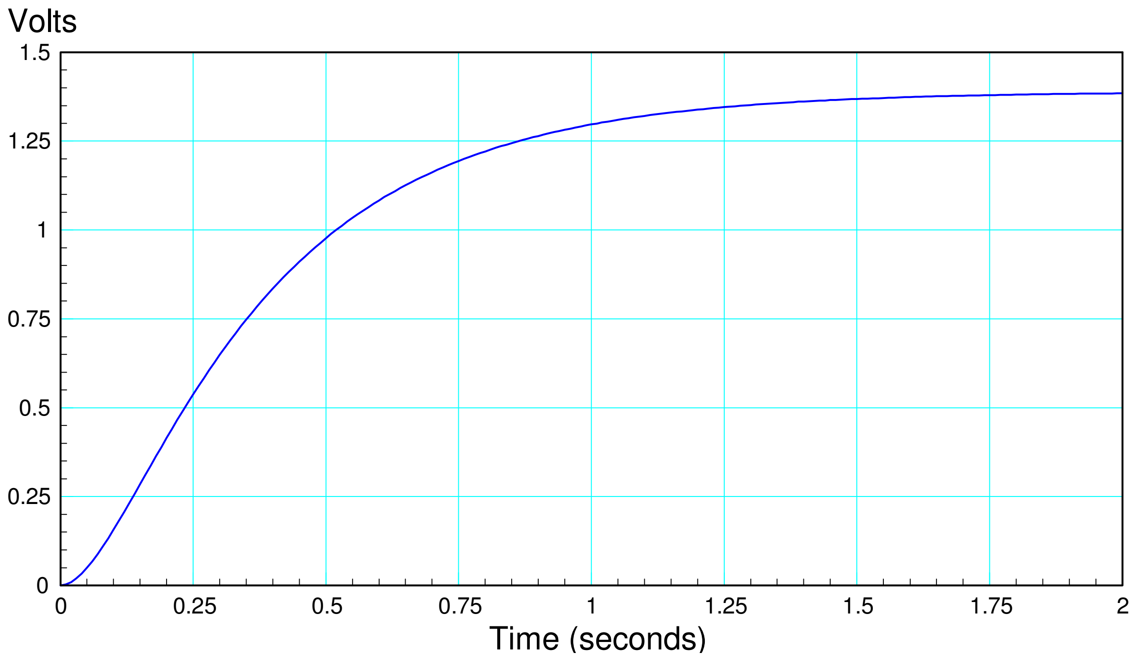
Option #4: Use DeMorgan's law

$$Y = (A + C)\left(\overline{A} + \overline{B}\right)(C + D)$$



1st & 2nd Order Approxmations

3) Find the transfer function, G(s), which has the following step response



This is a first order system (no oscillations), so G(s) is of the form

$$G(s) = \left(\frac{a}{s+b}\right)$$

The DC gain is 1.35

$$\left(\frac{a}{s+b}\right)_{s=0} = \left(\frac{a}{b}\right) = 1.35$$

The 2% settling time is about 1.5 seconds

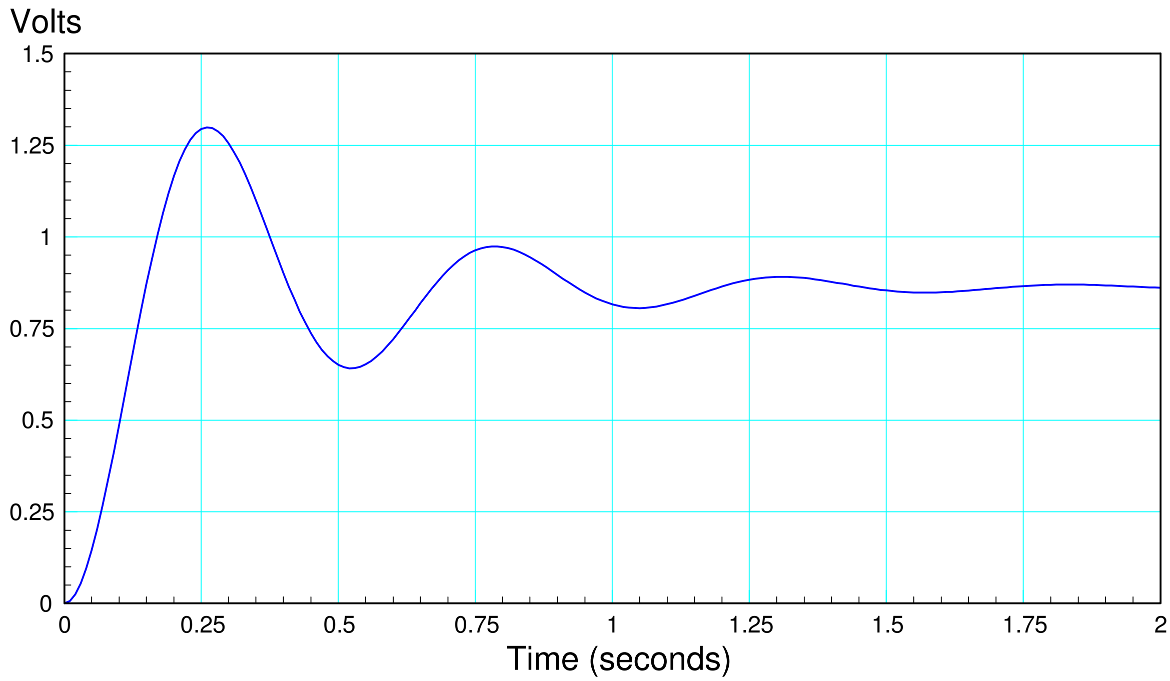
$$\left(\frac{4}{b}\right) = 1.5s$$

$$b = 2.67$$

so

$$\boxed{G(s) \approx \left(\frac{3.60}{s+2.67}\right)}$$

4) Find the transfer function, G(s), which has the following step response



This is a second-order system (oscillations mean complex poles)

$$G(s) = \left(\frac{a}{s^2+bs+c}\right) = \left(\frac{k}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

The 2% settling time is about 1.75 seconds

$$\left(\frac{4}{\sigma}\right) = 1.75$$

$$\sigma = 2.29$$

The frequency of oscillation (in rad/sec) is

$$\omega_d = \left(\frac{2\text{ cycles}}{1.05\text{s}}\right)2\pi = 11.97$$

so

$$G(s) \approx \left(\frac{k}{(s+2.29+j11.97)(s+2.29-j11.97)}\right)$$

Pick 'k' so that the DC gain is 0.85

$$\left(\frac{k}{(s+2.29+j11.97)(s+2.29-j11.97)}\right)_{s=0} = 0.85$$

$$\boxed{G(s) \approx \left(\frac{126}{(s+2.29+j11.97)(s+2.29-j11.97)}\right)}$$

5) For the following 5th-order system

$$Y = \left(\frac{200(s+1)(s+20)}{(s+2+j5)(s+2-j5)(s+20+j10)(s+20-j20)(s+30)} \right) X$$

a) Find a 2nd-order approximation for this system

Keep the two slowest poles

s = -2 + j5

Ignore zeros that are 3-10x faster

Keep the zero at (s+1)

Maybe keep the zero at (s+20)

Match the DC gain

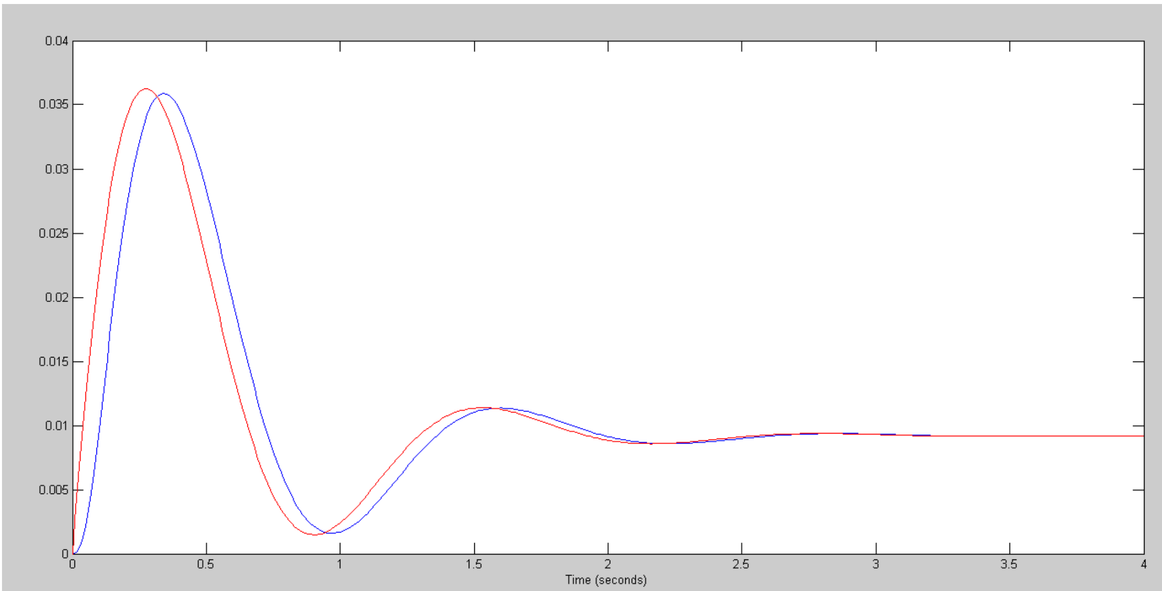
$$Y \approx \left(\frac{0.2667(s+1)}{(s+2+j5)(s+2-j5)} \right) X$$

In Matlab:

```
>> G5 = zpk([-1,-20],[-2+5j,-2-5j,-20+10j,-20-10j,-30],200)

            200 (s+1) (s+20)
----------------------------------------
(s+30) (s^2 + 4s + 29) (s^2 + 40s + 500)

>> DC = evalfr(G5,0)

DC =    0.0092

>> G2 = zpk([-1],[-2+5j,-2-5j],1);
>> k = evalfr(G5,0) / evalfr(G2,0)

k =    0.2667

>> G2 = zpk([-1],[-2+5j,-2-5j],k)

 0.26667 (s+1)
---------------
(s^2 + 4s + 29)
```

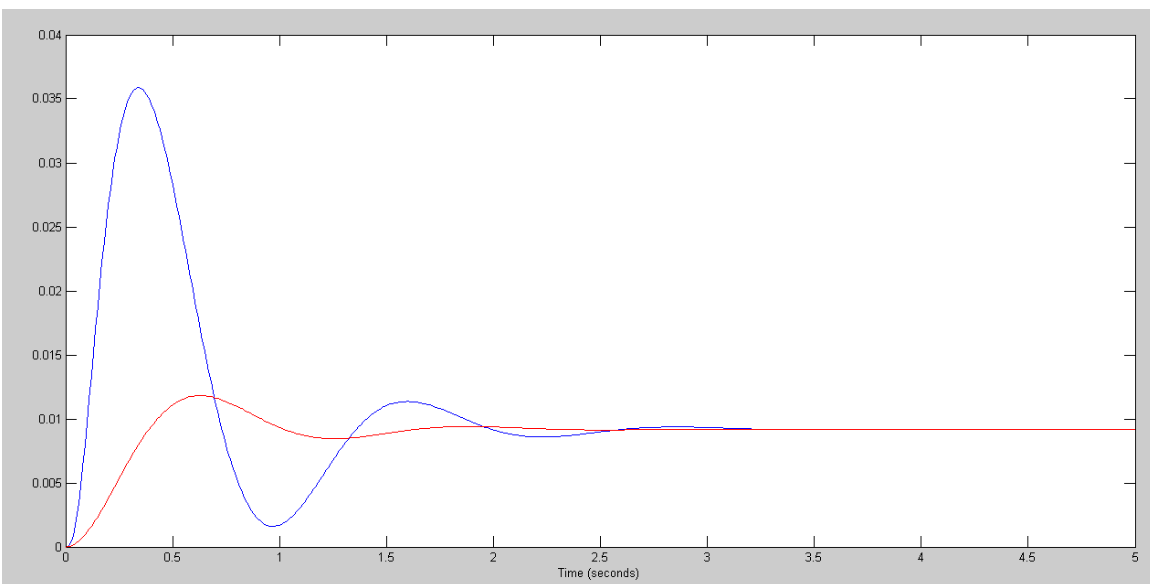
b) Plot the step response of the 5th-order system and the 2nd-order system

```
>> t = [0:0.01:4]';
>> y5 = step(G5,t);
>> y2 = step(G2,t);
>> plot(t,y5,'b',t,y2,'r')
>> xlabel('Time (seconds)')
>>
```



5th-order system (blue) & 2nd-order approximation (red)

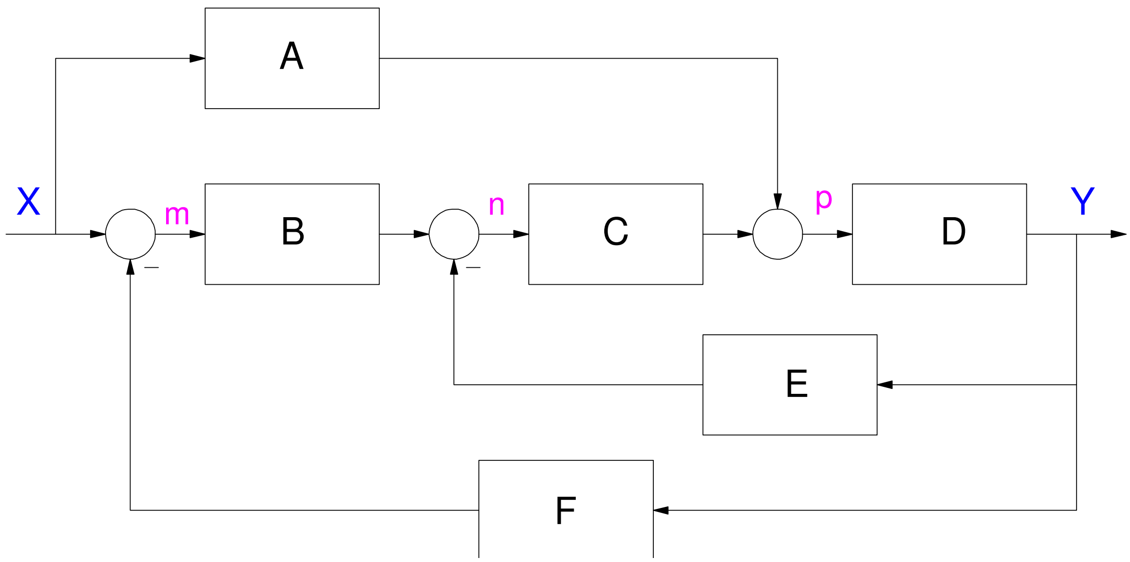
Note: If you leave out the zero, the step responses no longer match



5th-order system (blue) and 2nd-order approximation if you leave out the zero at (s+1) (red)

Block Diagrams

6) Simplify the following block diagram to find the transfer funciton from X to Y



Option #1: Shortcut

$$Y = \left(\frac{\text{gain from X to Y}}{1 + \text{loop gains}}\right) X$$

$$Y = \left(\frac{AD+BCD}{1+CDE+BCDF}\right) X$$

Option #2: Long Way

$$m = X - FY$$

$$n = Bm - EY$$

$$p = AX + Cn$$

$$Y = Dp$$

Substitute and solve

$$n = B(X - FY) - EY = BX - (BF + E)Y$$

$$p = AX + C(BX - (BF + E)Y) = (A + CB)X - (CBF + CE)Y$$

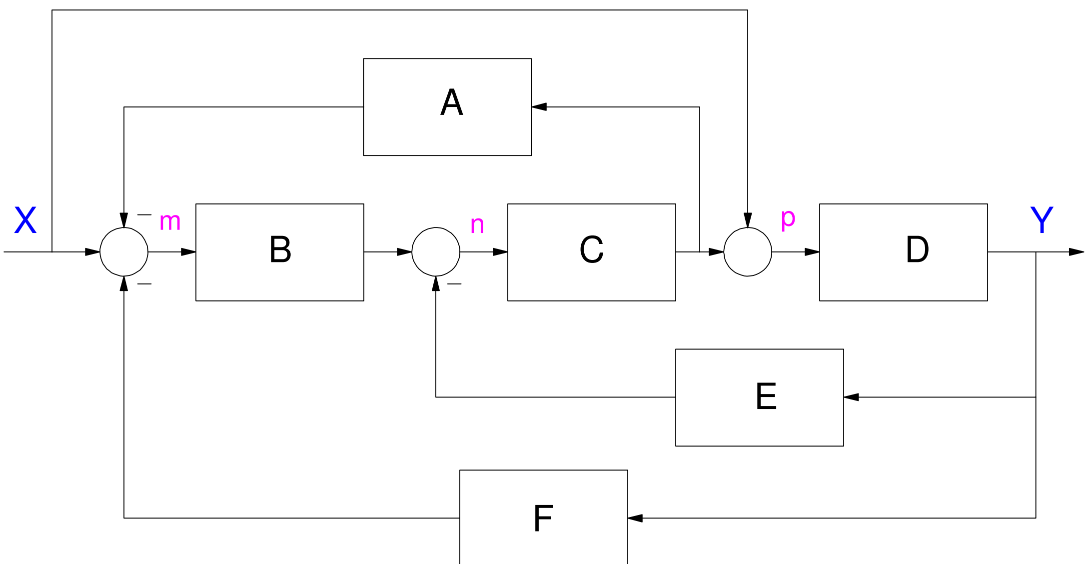
$$Y = D((A + CB)X - (CBF + CE)Y)$$

$$(1 + DCBF + DCE)Y = (DA + DCB)X$$

$$Y = \left(\frac{DA+DCB}{1+DCBF+DCE}\right) X$$

same answer

7) Simplify the following block diagram to find the transfer funciton from X to Y



Shortcut

$$Y = \left(\frac{D+BCD}{1+ABC+CDE+BCDF}\right)X$$

Long Way

$$m = X - ACn - FY$$

$$n = Bm - EY$$

$$p = X + Cn$$

$$Y = Dp$$

Substituting

$$n = B(X - ACn - FY) - EY$$

$$(1 + BAC)n = BX - (BF + E)Y$$

$$p = X + C\left(\frac{BX-(BF+E)Y}{(1+BAC)}\right)$$

$$Y = D\left(X + C\left(\frac{BX-(BF+E)Y}{(1+BAC)}\right)\right)$$

Doing some algebra

$$Y = D\left(X + C\left(\frac{BX-(BF+E)Y}{(1+BAC)}\right)\right)$$

$$(1 + BAC)Y = (1 + BAC)DX + DC(BX - (BF + E)Y)$$

$$(1 + BAC)Y = (D + BACD + DCB)X - (DCBF + DCE)Y$$

$$(1 + BAC + DCBF + DCE)Y = (D + BACD + DCB)X$$

$$Y = \left(\frac{D+BACD+DCB}{1+BAC+DCBF+DCE}\right)X$$

Comparing to what I got with the shortcut

$$Y = \left(\frac{D+BCD}{1+ABC+CDE+BCDF}\right)X$$

the short-cut was close, but it missed one term in the numerator (BACD)