omework #1: ECE 461 / 661

LaPlace Transforms

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Problems 1-3) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{100(s+2)}{(s+1)(s+5)(s+20)}\right)X$$

1) What is the differential equation relating X and Y?

Multiply out the polynomials

```
>> poly([-1,-5,-20])

ans =      1     26    125    100
```

$$Y = \left(\frac{100s+200}{s^3+26s^2+125s+100}\right)X$$

Cross multiply

$$(s^3 + 26s^2 + 125s + 100)Y = (100s + 200)X$$

Note that 'sY' means 'the derivative of y'

$$\boxed{y''' + 26y'' + 125y' + 100y = 100x' + 200x}$$

2) Determine y(t) assuming

$$x(t) = 3\cos(2t) + 5\sin(2t)$$

Use phasors

$$s = j2$$

$$X = 3 - j5$$

(in phasor notation, cosine is real, -sine is complex)

$$Y = \left(\frac{100(s+2)}{(s+1)(s+5)(s+20)}\right)X$$

Substitute

$$Y = \left(\frac{100(s+2)}{(s+1)(s+5)(s+20)}\right)_{s=j2} \cdot (3 - j5)$$

```
>> s = 2j;
>> X = 3 - 5j

X =   3.0000 - 5.0000i

>> Y = 100*(s+2) / ( (s+1)*(s+5)*(s+20) ) * X

Y =  -1.7617 - 6.5825i
```

which means

$$\boxed{y(t) = -1.7617\cos(2t) + 6.5825\sin(2t)}$$

3) Determine y(t) assuming x(t) is a unit step input

$$x(t) = u(t)$$

This is a LaPlace transform problem

$$Y = \left(\frac{100(s+2)}{(s+1)(s+5)(s+20)}\right)X$$

$$X(s) = \frac{1}{s}$$

so

$$Y = \left(\frac{100(s+2)}{(s+1)(s+5)(s+20)}\right)\left(\frac{1}{s}\right)$$

Do a partial fraction expansion

$$Y = (\tfrac{a}{s}) + \left(\frac{b}{s+1}\right) + \left(\frac{c}{s+5}\right) + \left(\frac{d}{s+20}\right)$$

```
>> s = 0;
>> a = 100*(s+2) / ( (s+1)*(s+5)*(s+20) )

a =     2

>> s = -1;
>> b = 100*(s+2) / ( (s)*(s+5)*(s+20) )

b =   -1.3158

>> s = -5;
>> c = 100*(s+2) / ( (s)*(s+1)*(s+20) )

c =    -1

>> s = -20;
>> d = 100*(s+2) / ( (s)*(s+1)*(s+5) )

d =    0.3158
```

$$Y = \left(\frac{2}{s}\right) + \left(\frac{-1.3158}{s+1}\right) + \left(\frac{-1}{s+5}\right) + \left(\frac{0.3158}{s+20}\right)$$

Take the inverse LaPlace transform

$$\boxed{y(t) = (2 - 1.3158e^{-t} - e^{-5t} + 0.3158e^{-20t})u(t)}$$

Problems 4-6) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{400}{(s+1+j4)(s+1-j4)(s+20)}\right)X$$

4) What is the differential equation relating X and Y?

Multiply out the polynomials

```
>> poly([-1-4j,-1+4j,-20])

ans =      1     22     57    340
```

$$Y = \left(\frac{400}{s^3+22s^2+57s+340}\right)X$$

Cross multiply

$$(s^3 + 22s^2 + 57s + 340)Y = (400)X$$

Note that 'sY' means 'the derivative of y(t)')

$$\boxed{y''' + 22y'' + 57y' + 340y = 400x}$$

5) Determine y(t) assuming

$$x(t) = 3\cos(2t) + 5\sin(2t)$$

This is a phasor problem

$$s = j2$$

$$X = 3 - j5$$

$$Y = \left(\frac{400}{(s+1+j4)(s+1-j4)(s+20)}\right) X$$

Substituting

$$Y = \left(\frac{400}{(s+1+j4)(s+1-j4)(s+20)}\right)_{s=j2} \cdot (3 - j5)$$

```
>> X = 3 - 5j

X =   3.0000 - 5.0000i

>> s = 2j

s =        0 + 2.0000i

>> Y = 400 / ( (s+1+4j)*(s+1-4j)*(s+20) ) * X

Y =   1.2095 - 8.4453i
```

meaning

$$\boxed{y(t) = 1.2095\cos(2t) + 8.4453\sin(2t)}$$

6) Determine y(t) assuming x(t) is a unit step input

$$x(t) = u(t)$$

This is a LaPlace transform problem

$$Y = \left(\frac{400}{(s+1+j4)(s+1-j4)(s+20)}\right) X$$

$$X(s) = \left(\frac{1}{s}\right)$$

giving

$$Y = \left(\frac{400}{(s+1+j4)(s+1-j4)(s+20)}\right)\left(\frac{1}{s}\right)$$

Doing a partial fraction expansion

$$Y = \left(\frac{a}{s}\right) + \left(\frac{b}{s+1+j4}\right) + \left(\frac{c}{s+1-j4}\right) + \left(\frac{d}{s+20}\right)$$

```
>> s = 0;
>> a = 400 / ( (s+1+4j)*(s+1-4j)*(s+20) )

a =    1.1765

>> s = -1 - 4j;
>> b = 400 / ( (s)*(s+1-4j)*(s+20) )

b =  -0.5617 - 0.2731i

>> s = -1 + 4j;
>> c = 400 / ( (s)*(s+1+4j)*(s+20) )

c =  -0.5617 + 0.2731i

>> s = -20;
>> d = 400 / ( (s)*(s+1+4j)*(s+1-4j) )

d =   -0.0531

>>
```

$$Y = \left(\frac{1.1765}{s}\right) + \left(\frac{-0.5617-0.2731}{s+1+j4}\right) + \left(\frac{-0.5617+0.2731}{s+1-j4}\right) + \left(\frac{-0.0531}{s+20}\right)$$

Polar form is actually easier to use

$$Y = \left(\frac{1.1765}{s}\right) + \left(\frac{0.6246\angle 154.07^0}{s+1+j4}\right) + \left(\frac{0.6246\angle -154.07^0}{s+1-j4}\right) + \left(\frac{-0.0531}{s+20}\right)$$

Take the inverse LaPlace transform

$$\boxed{y(t) = (1.1765 + 1.2491 e^{-t} \cos(4t - 154.07^0) - 0.0531 e^{-20t}) u(t)}$$