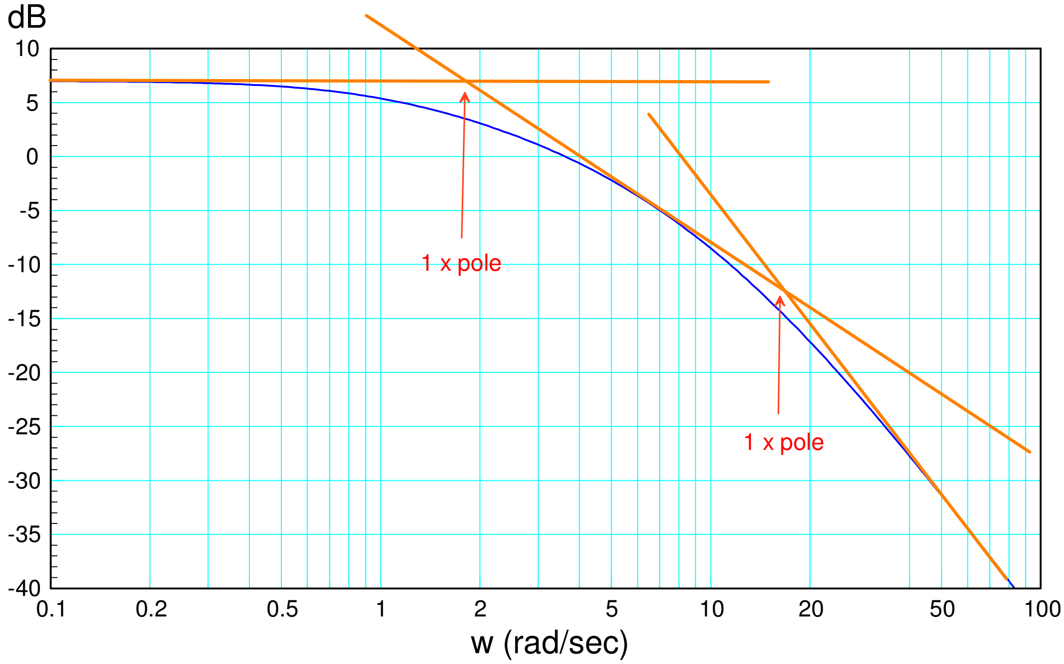
Homework #12: ECE 461/661

Bode Plots. Nichols charts and gain & lead compensation

Bode Plots

1) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw in the asymptotes at multiples of 20dB / decade (shown in orange)

Step 2: Each corner is a pole (gain drops) or a zero (gain increases). The number of poles is the number of 20dB/decade changes

- pole at 1.8 rad/sec
- pole at 17 rad/sec

$$G(s) \approx \left(\frac{k}{(s+1.8)(s+17)}\right)$$

Step 3: Determine k by matching the gain at a frequency

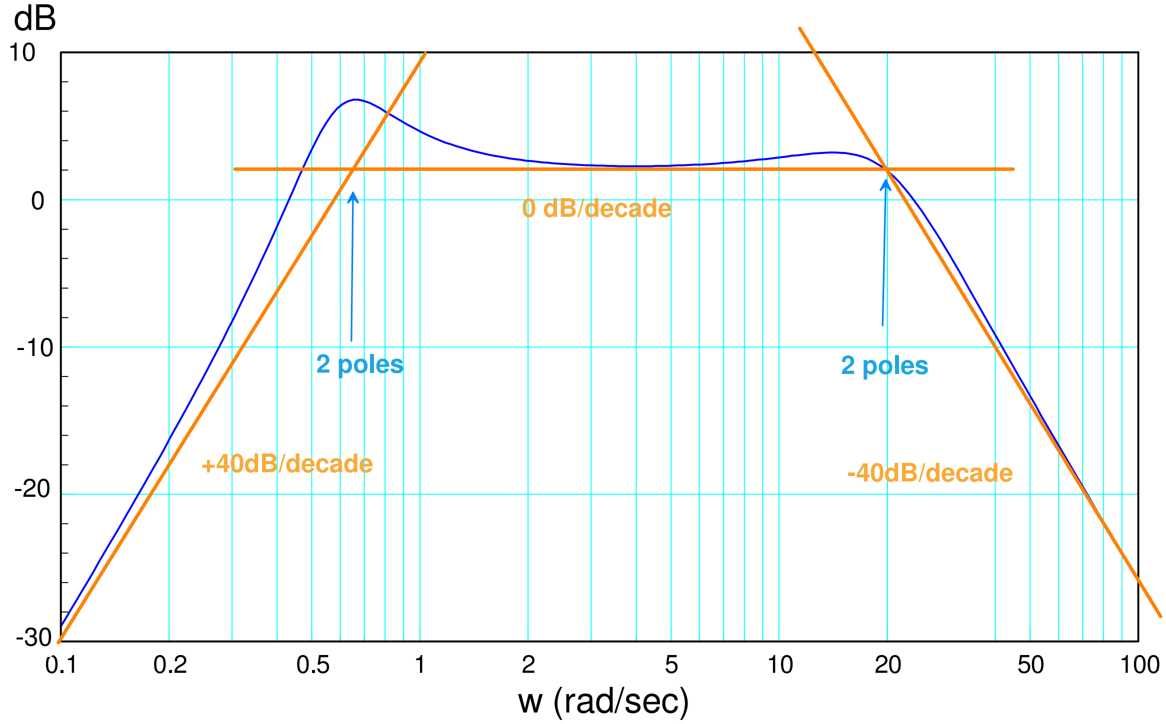
At 0.1 rad/sec, the gain should be +7dB = 2.239

$$\left(\frac{k}{(s+1.8)(s+17)}\right)_{s=j0.1} = 2.239$$

$$k = 68.50$$

$$\boxed{G(s) \approx \left(\frac{68.50}{(s+1.8)(s+17)}\right)}$$

2) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw in the asymptotes (shown in orange)

Step 2: Determine the poles and zeros (magnitude = corner frequency)

- 2 zeros left of 0.1 (assume at s = 0)
- 2 poles at 0.63 rad/sec
- 2 poles at 20 rad/sec

Step 3: Determine the angle from the gain at the corner

pole at 0.63 rad/sec

gain at corner = +4dB above the corner = 1.585

$$\frac{1}{2\zeta} = 1.585$$

$$\zeta = 0.315 \qquad (71.6^0)$$

pole at 20 rad/sec

gain at corner = +0dB = 1

$$\frac{1}{2\zeta} = 1.00$$

$$\zeta = 0.50 \qquad (60^0)$$

Add a gain, k, so that at 5 rad/sec, the gain is +2dB

$$G(s) = \left(\frac{503 \cdot s^2}{\left(s+0.63\angle\pm 71.6^0\right)\left(s+20\angle\pm 60^0\right)} \right)$$

Nichols Charts

3) The gain vs. frequency of a system is measured

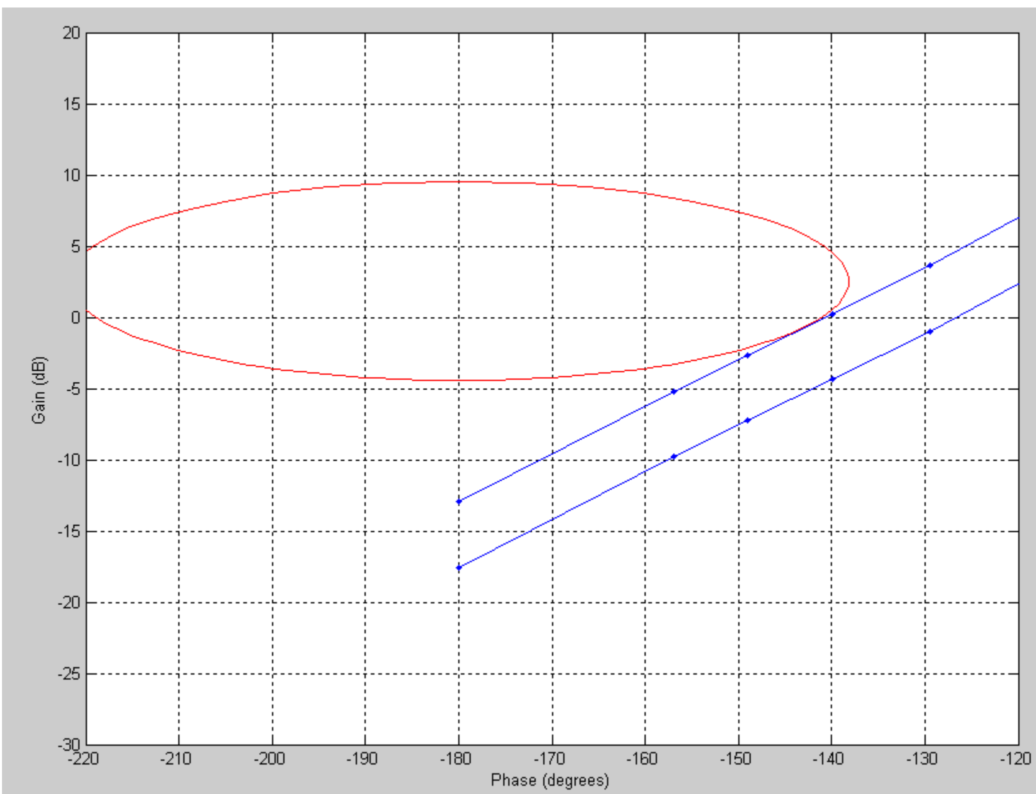
w (rad/sec)	2	3	4	5	6	10
Gain (dB)	3.29	-0.97	-4.36	-7.25	-9.81	-17.56
Phase (deg)	-117.51	-129.49	-139.97	-149.04	-156.89	-180

Using this data

- Transfer it to a Nichols chart
- Determine the maximum gain that results in a stable system **ans = k = +17.56 dB**
- Determine the gain, k, that results in a maximum closed-loop gain of Mm = 1.5 **ans: k = 1.70**

In Matlab:

```
>> GdB = [3.29,-0.97,-4.36,-7.25,-9.81,-17.56]'
>> Pdeg = [-117.51,-129.49,-139.97,-149.04,-156.89,-180]'

>>G = 10.^(GdB/20) .* exp(j*Pdeg*pi/180)

>> Nichols2(G,1.5);
>> Nichols2(G*[1,1.5],1.5);
>> Nichols2(G*[1,1.7],1.5);
```



Gain Compensation

Problem 4 & 5) Assume

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

4) Design a gain compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Determine the frequency which has a 50 degree phase margin (-130 degrees of 50 degrees from unstable)

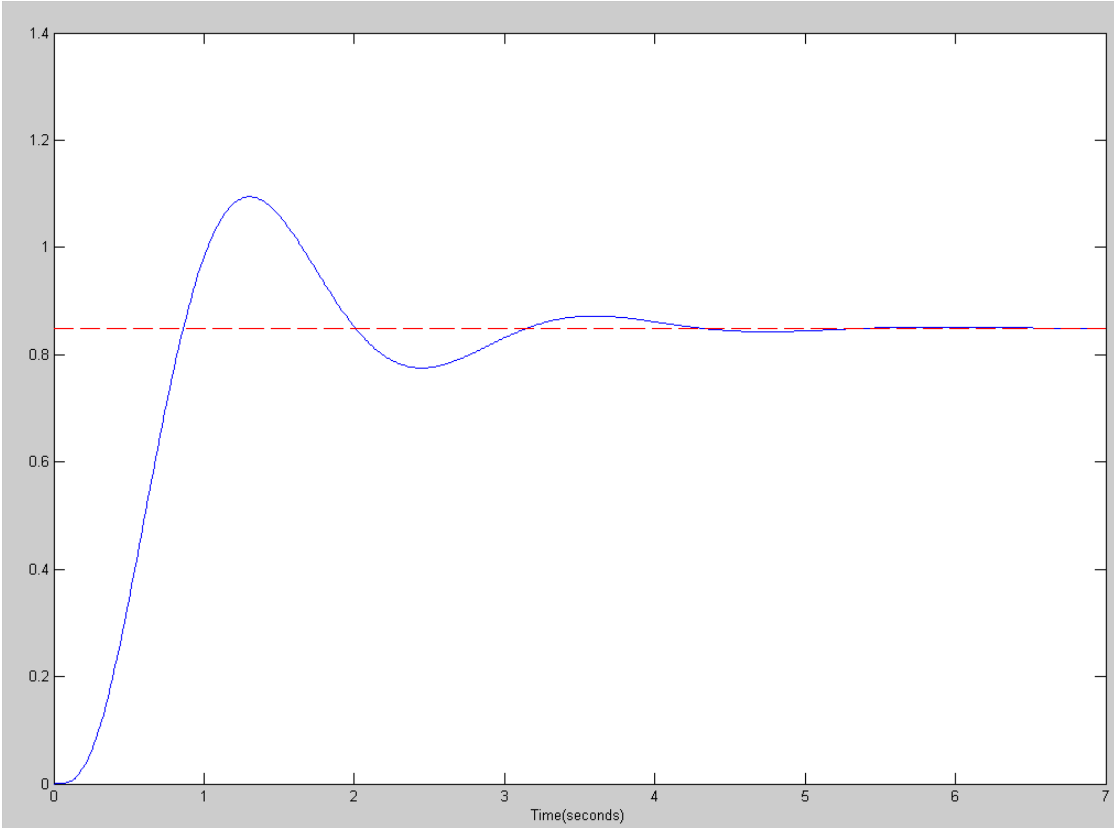
$$G(j2.124) = 0.125\angle -130^0$$

Pick 'k' to make the gain equal to 1.00 at this frequency

$$k = \frac{1}{0.125} = 8.017$$

Check the step response in Matlab

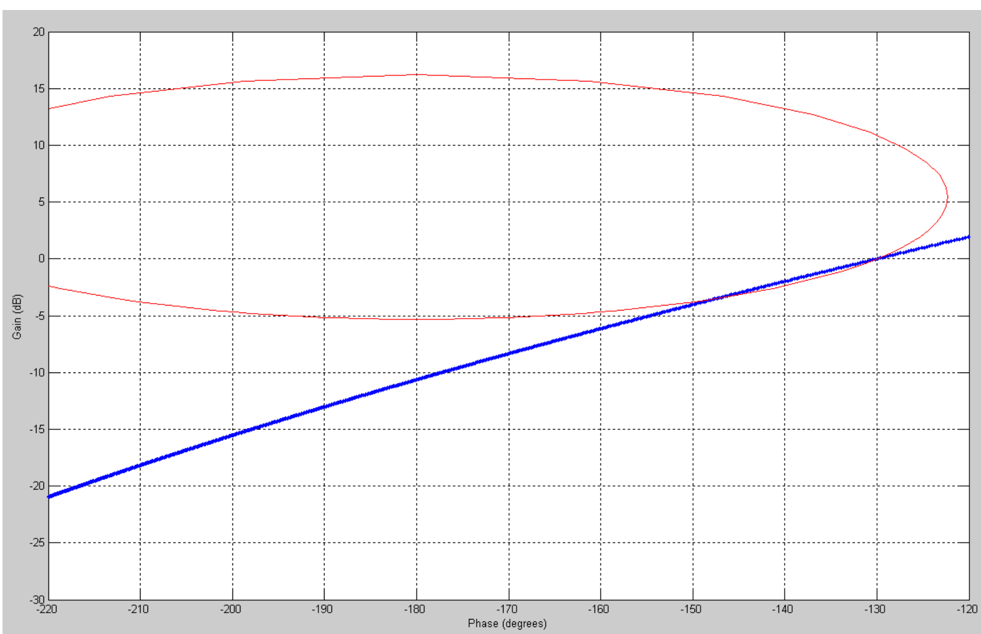
```
>> G = zpk([],[-0.47,-3.40,-9,-16.77],170)

                170
--------------------------------
(s+0.47) (s+3.4) (s+9) (s+16.77)

>> k = 8.017

k =    8.0170

>> Gcl = minreal(G*k / (1+G*k));
>> eig(Gcl)

  -1.0415 + 2.7437i
  -1.0415 - 2.7437i
 -11.8805
 -15.6765

>> t = [0:0.01:7]';
>> y = step(Gcl,t);
>> DC = evalfr(Gcl,0)

DC =    0.8496

>> OS = max(y) / DC

OS =    1.2880

>> plot(t,y,'b',t,0*t+DC,'r--')
>> xlabel('Time(seconds)');
```



Sidelight: Designing for a phase margin is *almost* the same as designing for a resonance of

$$M_m = \left| \frac{1\angle{-130^0}}{1+1\angle{-130^0}} \right| = 1.1831$$

You actually intersect this M-circle at (0dB, -130 degrees)



Lead Compensation

5) Design a lead compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Let

$$K(s) = k\left(\frac{s+3.40}{s+34}\right)$$

$$GK = \left(\frac{170k}{(s+0.47)(s+9.00)(s+16.77)(s+34)}\right)$$

Determine the frequency where the angle of GK is -130 degrees (for a 50 degree phase margin)

$$GK(j4.223) = 0.00679\angle -130^0$$

Pick 'k' to make the gain one at this frequency

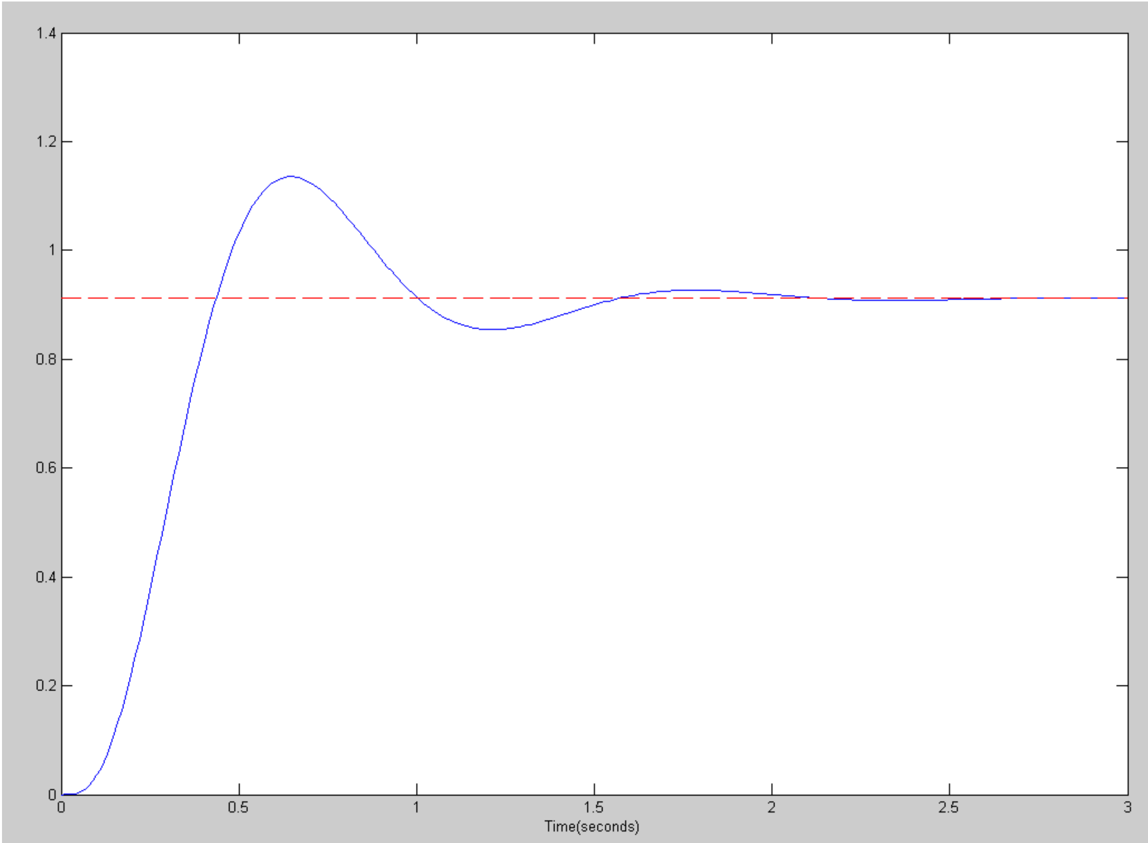
$$k = \frac{1}{0.00679} = 147.223$$

and

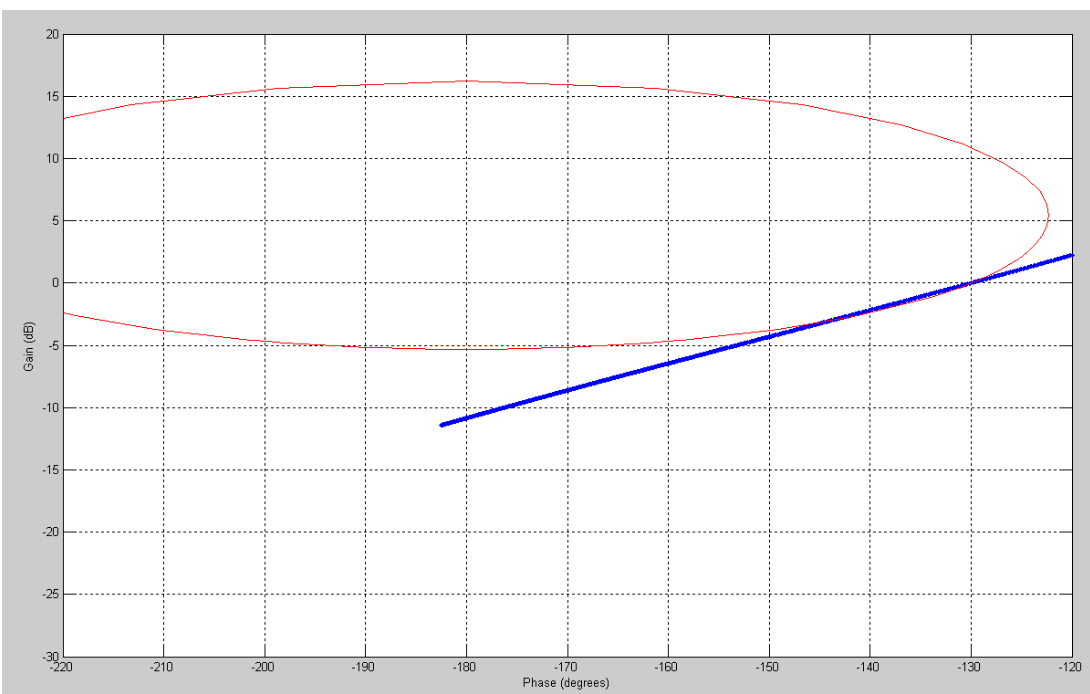
$$K(s) = 147.223\left(\frac{s+3.40}{s+34}\right)$$

Checking the step response in Matlab

```
>> G = zpk([],[-0.47,-3.40,-9,-16.77],170)

               170
--------------------------------
(s+0.47) (s+3.4) (s+9) (s+16.77)

>> K = zpk(-3.40,-34,147.223)

147.223 (s+3.4)
---------------
    (s+34)

>> Gcl = minreal(G*K / (1+G*K));
>> eig(Gcl)

 -23.8705
 -31.6018
  -2.3839 + 5.5401i
  -2.3839 - 5.5401i

>> DC = evalfr(Gcl,0)

DC =    0.9121

>> OS = max(y) / DC

OS =    1.1998

>> t = [0:0.01:7]';
>> y = step(Gcl,t);
>> plot(t,y,'b',t,0*t+DC,'r--')
>> xlabel('Time(seconds)');
>>
```



Again, K(s) causes G*K to have a 50 degree phase margin. This is *almost* the same as designing for Mm=1.183



Problem 6 & 7) Assume a 500ms delay is added

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) e^{-0.5s}$$

6) Design a gain compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Find the frequency where the phase of G(s) is -130 degrees

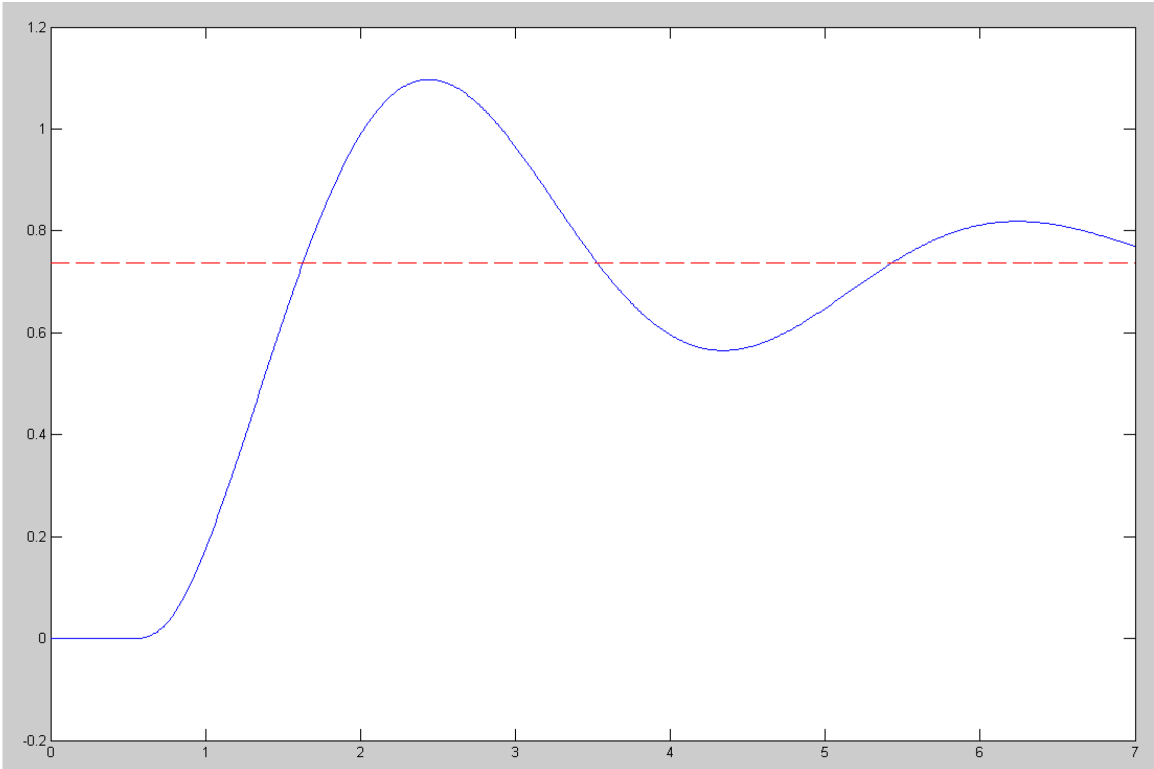
$$G(j1.141) = 0.252 \angle -130^0$$

Pick 'k' to make the gain 1.00 at this frequency

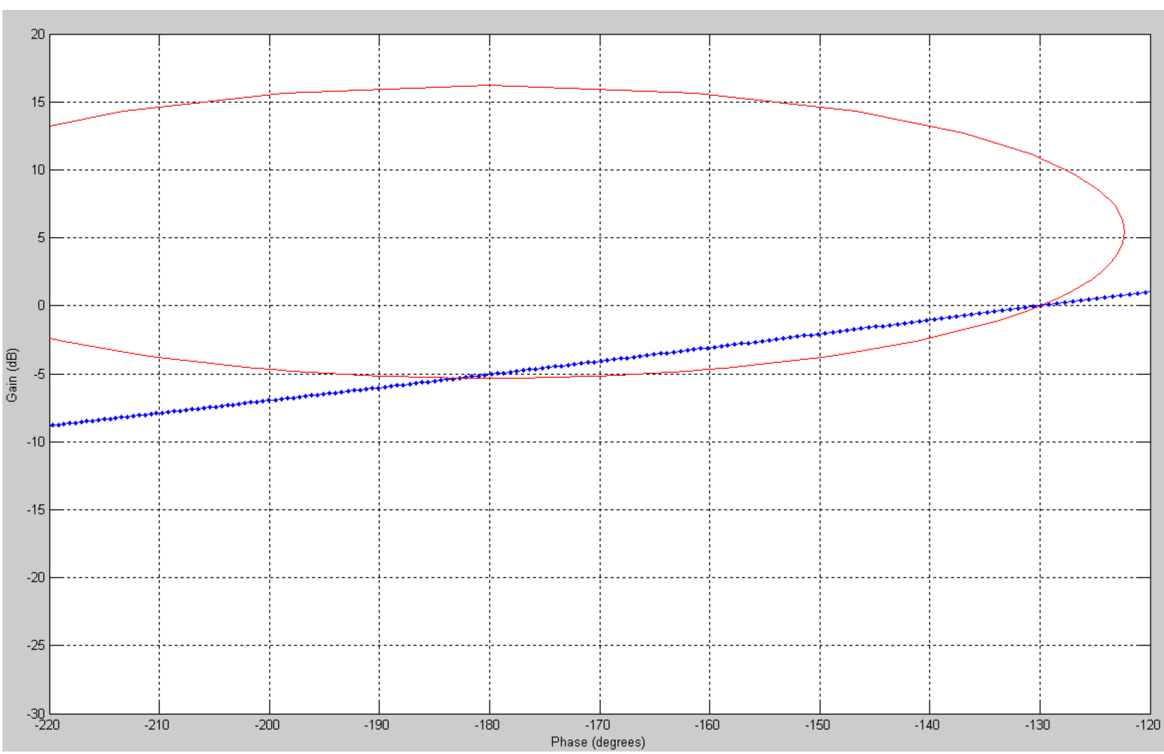
$$k = \frac{1}{0.252} = 3.9710$$

In Matlab

```
>> G = zpk([],[-0.47,-3.40,-9,-16.77],170)

                 170
 --------------------------------
 (s+0.47) (s+3.4) (s+9) (s+16.77)

>> [num,den] = pade(0.5,6);
>> Delay = tf(num,den);
>> K = 3.9710;
>>
>> Gcl = minreal(G*Delay*K / (1 + G*Delay*K));
>> t = [0:0.01:7]';
>> y = step(Gcl,t);
>> DC = evalfr(Gcl,0)

DC =    0.7368

>> plot(t,y,'b',t,0*t+DC,'r--')
>> OS = max(y) / DC

OS =    1.4887
```



Note: The overshoot is a bit higher here. The phase margin is still 50 degrees. However, on a Nichols chart, you can see that the resonance is higher than Mm = 1.1831



Nichols Chart of G*K*Delay

7) Design a lead compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Let

$$K(s) = k\left(\frac{s+3.4}{s+34}\right)$$

$$GK = \left(\frac{170}{(s+0.47)(s+9.00)(s+16.77)(s+34)}\right) e^{-0.5s}$$

Find the frequency where the phase shift is -130 degrees

$$GK(j1.448) = 0.0214\angle -130^0$$

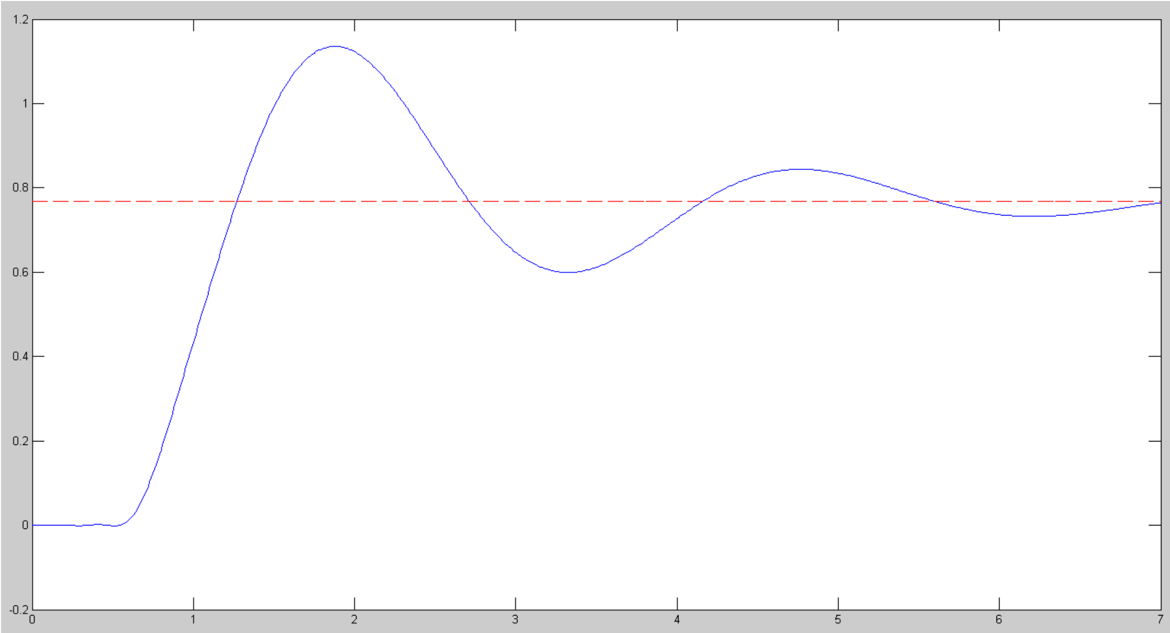
Pick k to make the gain 1.00 at this frequency

$$k = \frac{1}{0.0214} = 46.753$$

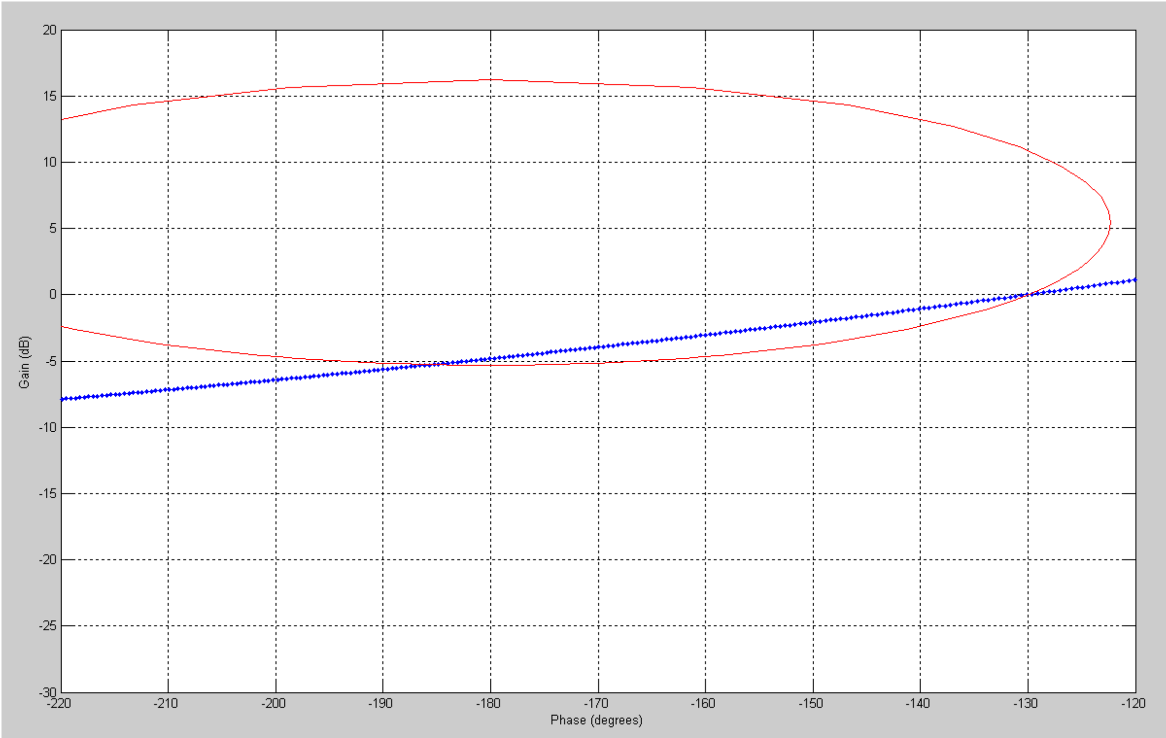
$$K(s) = 46.753\left(\frac{s+3.4}{s+34}\right)$$

Checking the response in Matlab

```
>> G = zpk([],[-0.47,-3.40,-9,-16.77],170);
>> [num,den] = pade(0.5,6);
>> Delay = tf(num,den);
>> K = zpk(-3.4,-34,46.753)

46.753 (s+3.4)
--------------
    (s+34)

>> Gcl = minreal(G*Delay*K / (1 + G*Delay*K));
>> t = [0:0.01:7]';
>> y = step(Gcl,t);
>> DC = evalfr(Gcl,0)
DC =    0.7672

>> OS = max(y) / DC
OS =    1.4799

>> plot(t,y,'b',t,0*t+DC,'r--')
```



Note: Once again, the overshoot seems high. This is due to G*K*Delay getting closer to -1 than it should be - more easily seen on a Nichols chart



Nichols chart of G*K*Delay