

Homework #11: ECE 461/661

Digital PID Control.

PID Control

Assume $T = 0.5$ seconds:

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

1) Design a digital I controller

$$K(s) = k \left(\frac{z}{z-1} \right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with $K(z)*G(s)$)

Use the alternate method: Plant + Zero-Order Hold + $K(z) = -1$

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot k \left(\frac{z}{z-1} \right) \right)_{s=(-1+j2)a} = 1 \angle 180^\circ$$

Search along the 20% overshoot damping line until the angle is 180 degrees

(time passes)

$$s = -0.1879 + j0.3758$$

$$z = 0.8943 + j0.1700$$

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot k \left(\frac{z}{z-1} \right) \right)_s = 3.6439k \angle 180^\circ$$

meaning

$$k = \frac{1}{3.6439} = 0.2744$$

and

$$K(z) = 0.2744 \left(\frac{z}{z-1} \right)$$

Matlab Simulation (kind of tricky)

- Set the integration step size to 10ms (50 steps per 500ms sampling time)
- Find the state-space model for G(s)
- Convert to discrete-time with sampling rate dt as

$$X(k+1) = (\exp(A \cdot dt)) \cdot X(k) + (B \cdot dt) \cdot U(k)$$

```
Gs = zpk([], [-0.47, -3.4, -9, -16.77], 170);
dt = 0.01;
T = 0.5;

Gs_ABCD = ss(Gs);

A = Gs_ABCD.A;
B = Gs_ABCD.B;
C = Gs_ABCD.C;
D = Gs_ABCD.D;

t = 0;

Data = [];
X = zeros(4,1);

Y = 0;
U = 0;
R = 1;

while(t < 30)

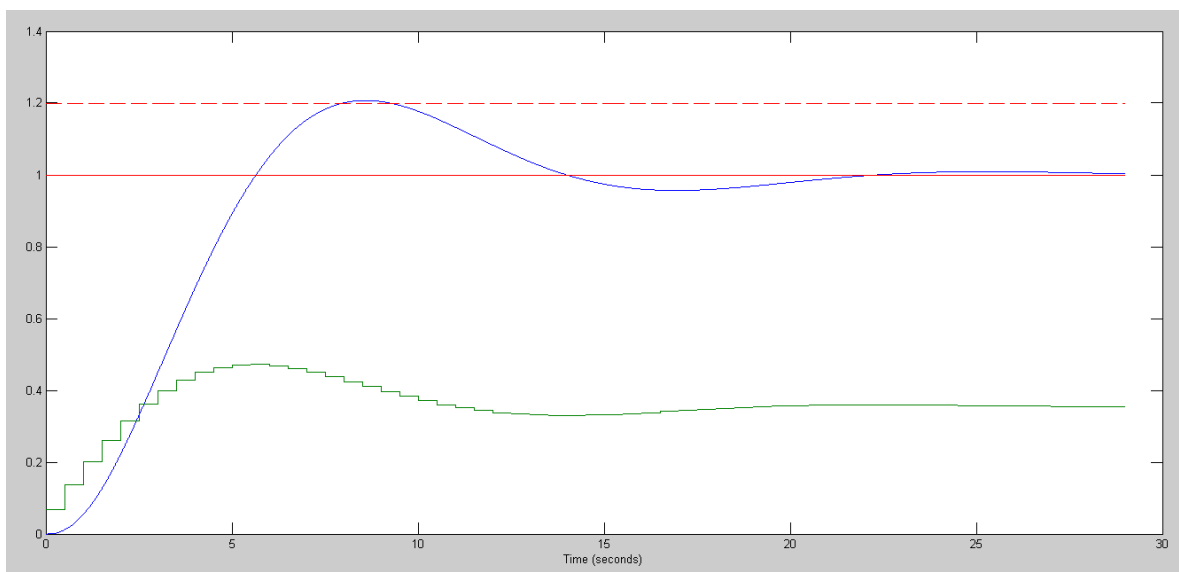
    if(mod(t,T) < dt)
        E0 = R-Y;
        U = U + 0.2744*E0;
    end

    dX = A*X + B*U;
    Y = C*X + D*U;

    X = X + dX * dt;
    t = t + dt;

    Data = [Data ; [t, Y, 0.5*U, R]];
end

t = Data(:,1);
plot(t, Data(:,2:4), t, 0*t+1, 'r--', t, 0*t+1.2, 'r--')
xlabel('Time (seconds)')
```



2) Assume $T = 0.5$ seconds and

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a digital PI controller

$$K(s) = k \left(\frac{z-a}{z-1} \right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with $K(z)*G(s)$)

Pick 'a' to cancel the pole at $s = -0.47$

$$s = -0.47$$

$$z = e^{sT} = e^{(-0.47)(0.5)} = 0.7906$$

Again, use the alternate approach

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot k \left(\frac{z-0.7906}{z-1} \right) \right)_{s=(-1+j2)a} = 1 \angle 180^\circ$$

Search along the damping line until the angle is 180 degrees

$$s = -0.7242 + j1.4483$$

$$z = e^{sT} = 0.5215 + j0.4613$$

To find k, make the gain at this point equal to 1.0000

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot k \left(\frac{z-0.7906}{z-1} \right) \right)_s = 0.2703k \angle 180^\circ$$

$$k = \frac{1}{0.2703} = 3.6994$$

and

$$K(z) = 3.6994 \left(\frac{z-0.7906}{z-1} \right)$$

Matlab Code

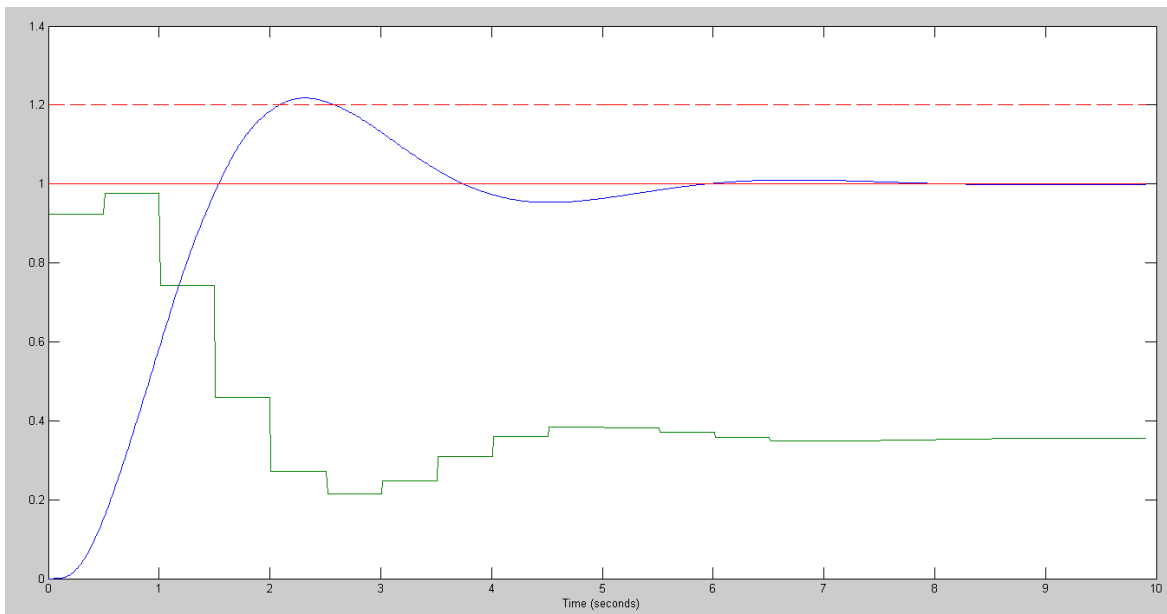
```
while(t < 9.9)
    if(mod(t,T) < dt)
        E1 = E0;
        E0 = R-Y;
        U = U + 3.6944*(E0 - 0.7906*E1);
    end

    dX = A*X + B*U;
    Y = C*X + D*U;

    X = X + dX * dt;
    t = t + dt;

    Data = [Data ; [t, Y, 0.25*U, R]];
end

t = Data(:,1);
plot(t, Data(:,2:4), t, 0*t+1,'r--',t, 0*t+1.2,'r--')
xlabel('Time (seconds)')
```



3) Assume $T = 0.5$ seconds and

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a digital PID controller

$$K(s) = k \left(\frac{(z-a)(z-b)}{z(z-1)} \right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with $K(z)*G(s)$)

Pick the two zeros to cancel the two slowest poles

$$s = -0.47 \quad z = e^{sT} = 0.7906$$

$$s = -3.40 \quad z = e^{sT} = 0.1827$$

meaning $K(z)$ is of the form

$$K(z) = k \left(\frac{(z-0.7906)(z-0.1827)}{z(z-1)} \right)$$

Again, use the approach where you search along the damping line until the angle is 180 degrees

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot k \left(\frac{(z-0.7906)(z-0.1827)}{z(z-1)} \right) \right)_{s=(-1+j2)a} = 1 \angle 180^\circ$$

Searching results in

$$s = -0.8892 + j1.7783$$

$$z = e^{sT} = 0.4039 + j0.4978$$

To make the gain equal to -1.0000 at this point

$$k = 5.0936$$

and

$$K(z) = 5.0936 \left(\frac{(z-0.7906)(z-0.1827)}{z(z-1)} \right)$$

or

$$K(z) = 5.0936 \left(\frac{(z^2 - 0.9633z + 0.1444)}{z(z-1)} \right)$$

In Matlab

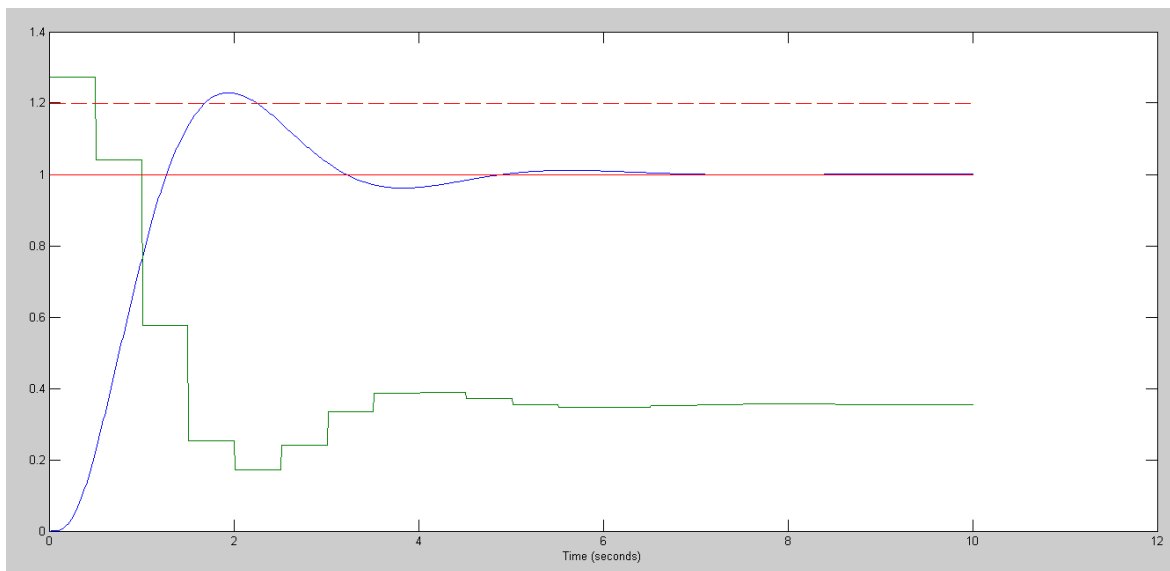
```
while(t < 10)
    if(mod(t,T) < dt)
        E2 = E1;
        E1 = E0;
        E0 = R-Y;
        U = U + 5.0936*(E0 - 0.9633*E1 + 0.1444*E2);
    end

    dX = A*X + B*U;
    Y = C*X + D*U;

    X = X + dX * dt;
    t = t + dt;

    Data = [Data ; [t, Y, 0.25*U, R]];
end

t = Data(:,1);
plot(t, Data(:,2:4), t, 0*t+1,'r--',t, 0*t+1.2,'r--')
xlabel('Time (seconds)')
```



Meeting Design Specs

4) Assume a sampling rate of $T = 0.5$ seconds and

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a digital controller that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 10 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with $K(z)*G(s)$)

Translation:

- Make it a type-1 system

Place the closed-loop dominant pole at

- $s = -0.4 + j0.8$
- $z = 0.7541 + j0.3188$

Assume $K(z)$ is of the form

$$K(z) = k \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-a)} \right)$$

Pick 'a' so that $s = -0.4 + j0.8$ is on the root locus (the phase is 180 degrees)

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot k \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-a)} \right) \right)_{s=-0.4+j0.8} = 1 \angle 180^\circ$$

Analyze what we know:

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)} \right) \right)_{s=-0.4+j0.8} = 0.2776 \angle -121.45^\circ$$

meaning, to make the angle 180 degrees

$$\angle(z-a) = 58.5475^\circ$$

$$a = 0.7541 - \left(\frac{0.3188}{\tan(58.5475^\circ)} \right) = 0.5591$$

Find k so that the gain is one

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.25s} \cdot \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-0.5591)} \right) \right)_{s=-0.4+j0.8} = 0.7828 \angle 180^\circ$$

$$k = \frac{1}{0.7828} = 1.3463$$

and

$$K(z) = 1.3463 \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-0.5591)} \right)$$

$$K(z) = 1.3464 \left(\frac{z^2 - 1.9733z + 0.1444}{z^2 - 1.5591z + 0.5591} \right)$$

In Matlab

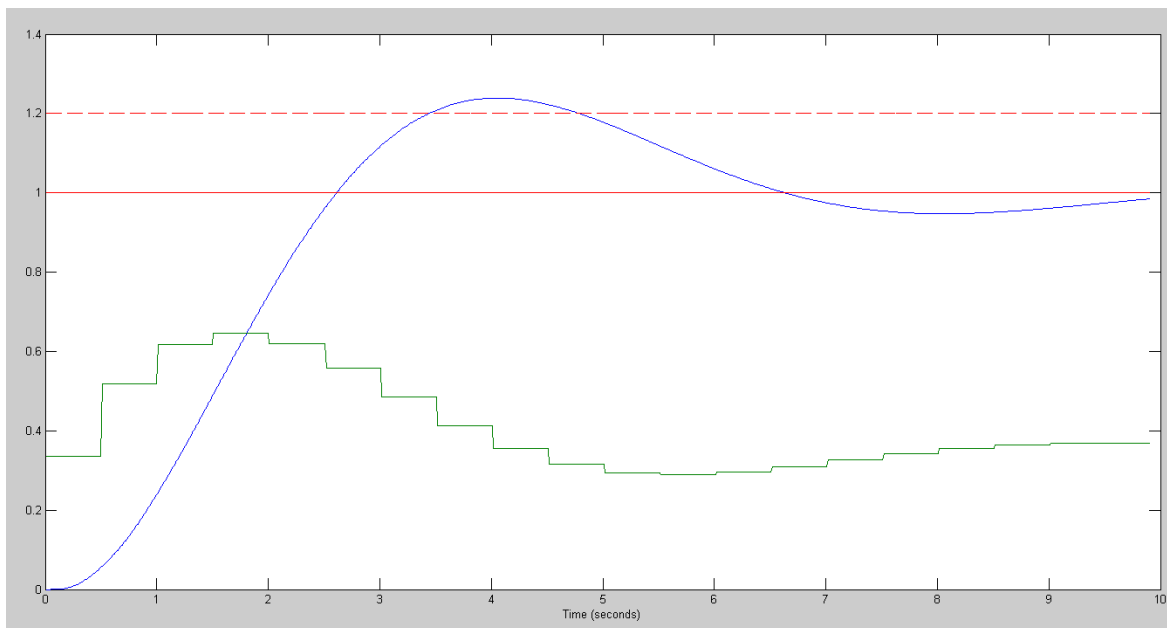
```
dt = 0.01;
T = 0.5;
while(t < 9.9)
    if(mod(t,T) < dt)
        E2 = E1;
        E1 = E0;
        E0 = R-Y;
        U2 = U1;
        U1 = U0;
        U0 = 1.5591*U1 - 0.5591*U2 + 1.3463*(E0 - 0.9633*E1 + 0.1444*E2);
    end

    dX = A*X + B*U0;
    Y = C*X + D*U0;

    X = X + dX * dt;
    t = t + dt;

    Data = [Data ; [t, Y, 0.25*U0, R]];
end

t = Data(:,1);
plot(t, Data(:,2:4), t, 0*t+1, 'r--', t, 0*t+1.2, 'r--')
xlabel('Time (seconds)')
```



5) Assume

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a digital controller with $T = 0.1$ second that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 10 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with $K(z)*G(s)$)

Note: Changing the sampling rate is a big deal: it means a complete redesign of $K(z)$

With $T = 0.1$

- $s = -0.4 + j0.8$
- $z = 0.9577 + j0.0768$

Same procedure but with $T = 0.1$

$$K(z) = k \left(\frac{(z-0.9541)(z-0.7113)}{(z-1)(z-a)} \right)$$

Analyze what we know:

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.9541)(z-0.7113)}{(z-1)} \right) \right)_{s=-0.4+j0.8} = 0.1112 \angle -124.566^\circ$$

meaning

$$\angle(z-a) = 55.4332^\circ$$

$$a = 0.9577 - \left(\frac{0.0768}{\tan(55.4332^\circ)} \right) = 0.9048$$

Analyzing what we now know

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.9541)(z-0.7113)}{(z-1)(z-0.9048)} \right) \right)_{s=-0.4+j0.8} = 1.1927 \angle 180^\circ$$

so

$$k = \frac{1}{1.1927} = 0.8385$$

and

$$K(z) = 0.8385 \left(\frac{(z-0.9541)(z-0.7113)}{(z-1)(z-0.9048)} \right)$$

$$K(z) = 0.8385 \left(\frac{z^2 - 1.6654z + 0.6787}{z^2 - 1.9048z + 0.9048} \right)$$

```

dt = 0.01;
T = 0.1;

while(t < 9.9)
    if(mod(t,T) < dt)
        E2 = E1;
        E1 = E0;
        E0 = R-Y;
        U2 = U1;
        U1 = U0;
        U0 = 1.9048*U1 - 0.9048*U2 + 0.8385*(E0 - 1.6654*E1 + 0.6787*E2);
    end

    dX = A*X + B*U0;
    Y = C*X + D*U0;

    X = X + dX * dt;
    t = t + dt;

    Data = [Data ; [t, Y, 0.25*U0, R]];
end

t = Data(:,1);
plot(t, Data(:,2:4), t, 0*t+1, 'r--', t, 0*t+1.2, 'r--')
xlabel('Time (seconds)')

```

