Homework #9: ECE 461/661

PID, Meeting Specs, Delays.

I Compensation

1) Design an I compensator, , which results in 20% overshoot for a step input.

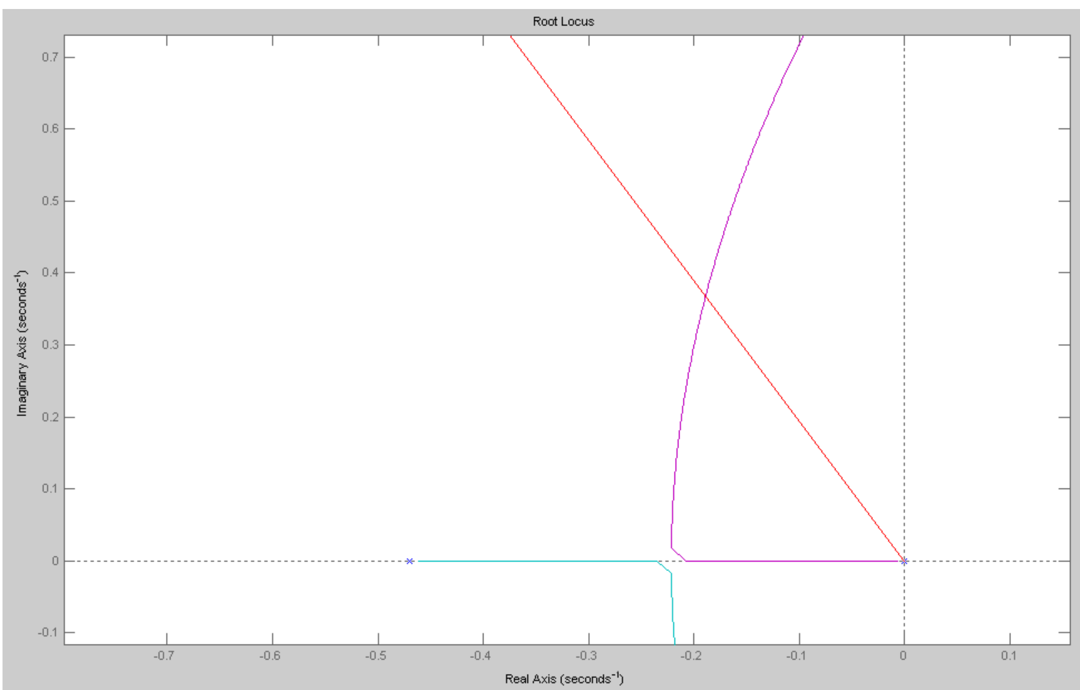
$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

$$K(s) = \left(\frac{k}{s}\right)$$

$$GK = \left(\frac{170k}{s(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

Sketch the root locus

```
>> GK = zpk([],[0,-0.47,-3.4,-9.00,-16.77],170);
>> k = logspace(-2,2,1000)';
>> rlocus(GK,k);
>> hold on
>> plot([0,-3],[0,1.9517*3],'r')
```



Zoom in to find the crossing point

```
>> s = -0.1888 + j0.3685

>> evalfr(GK,s)

ans =  -1.8736 - 0.0001i

>> k = 1/abs(ans)

k =    0.5337
```

For this K(s), determine

The closed-loop dominant pole(s)

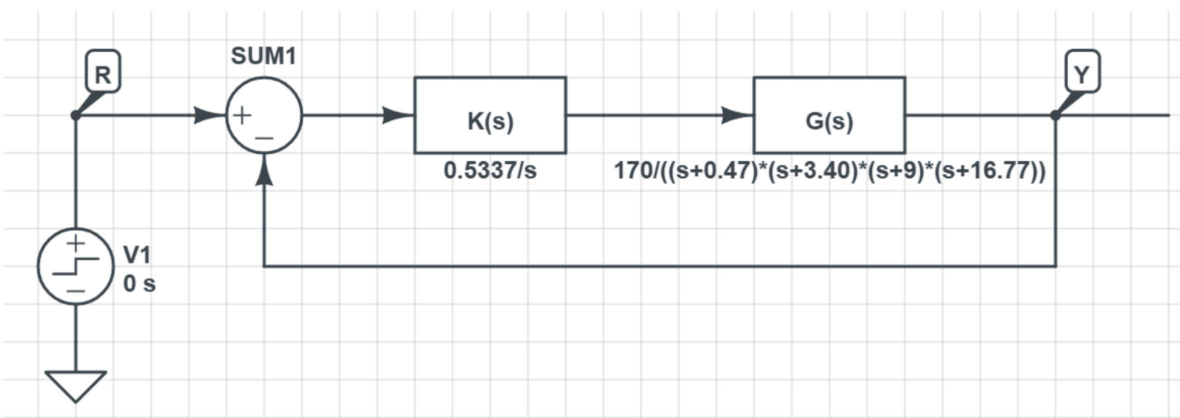
```
>> GKcl = minreal( (GK*k) / (1 + GK*k) );
>> eig(GKcl)

  -0.1888 + 0.3685i
  -0.1888 - 0.3685i
  -3.5165
  -8.9726
 -16.7732
```

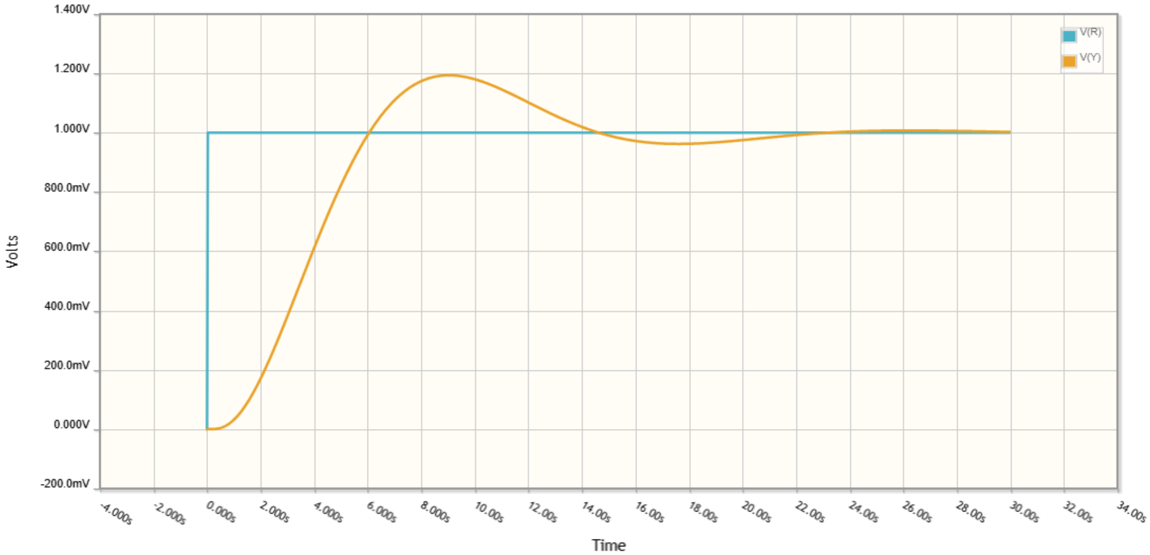
The 2% settling time, the error constant, Kp, and the steady-state error for a step input.

```
>> Ts = 4/0.1888

Ts =   21.1864

>> Kp = evalfr(GK,0) * k

Kp =   Inf

>> Estep = 1 / (Kp + 1)

Estep =     0
```

Check your design in Matlab or Simulink or CircuitLab

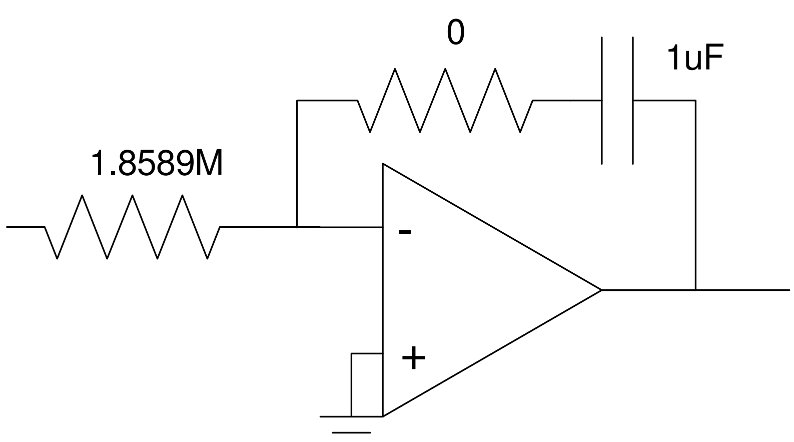
Using CircuitLab:





Give an op-amp circuit to implement K(s)

$$K(s) = \left(\frac{0.5377}{s}\right)$$



PI Compensation

2) Design a PI compensator which results in 20% overshoot for a step input.

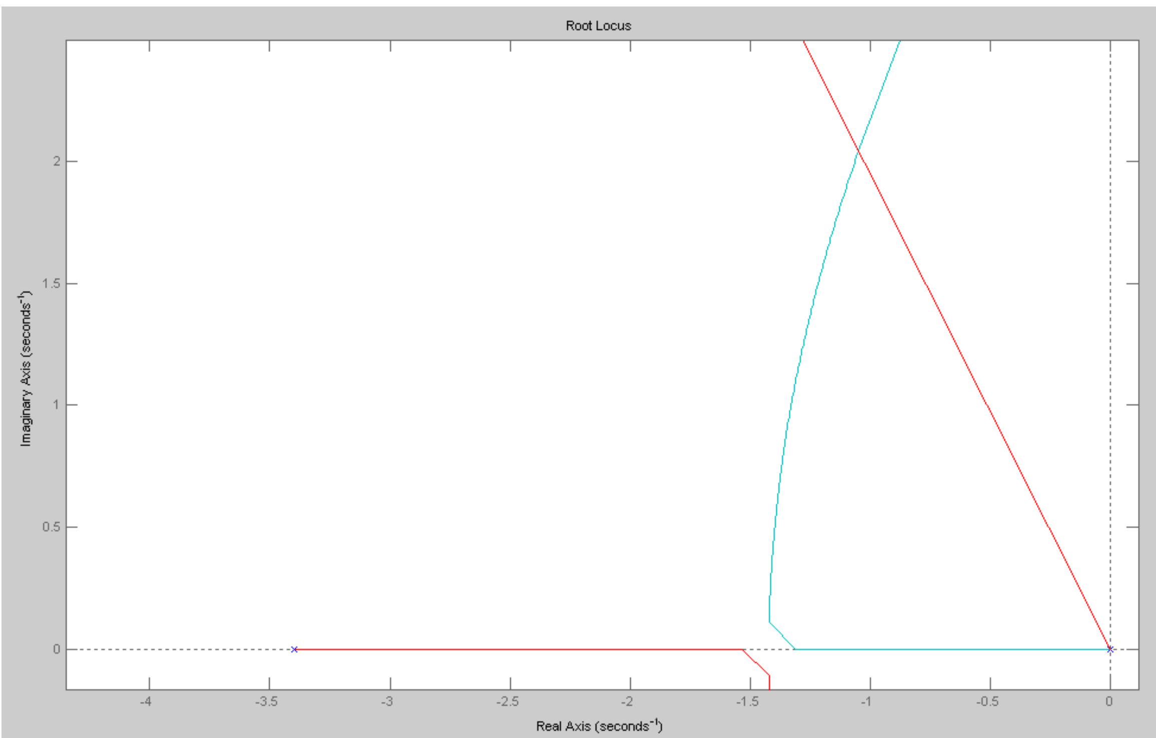
$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

$$K(s) = k\left(\frac{s+0.47}{s}\right)$$

$$GK = \left(\frac{170k}{s(s+3.40)(s+9.00)(s+16.77)}\right)$$

Sketch the root locus along with the damping line

```
>> GK = zpk([],[0,-3.4,-9.00,-16.77],170);
>> k = logspace(-2,2,1000)';
>> rlocus(GK,k);
>> hold on
>> plot([0,-3],[0,1.9517*3],'r')
```



For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

```
>> s = -1.0484 + j*2.0462;
>> evalfr(GK,s)

ans =  -0.1822 + 0.0000i
```

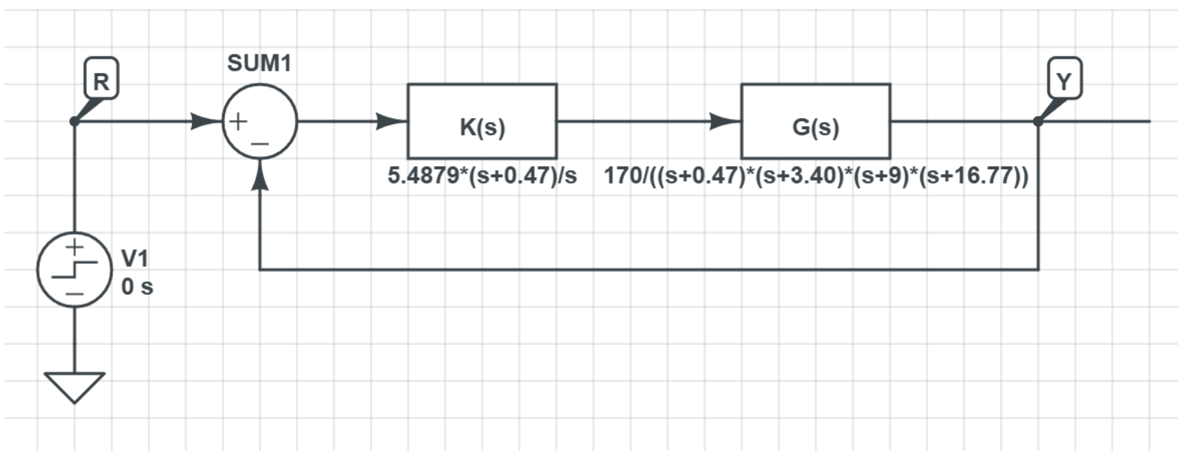
```
>> k = 1/abs(ans)

k =     5.4879

>> GKcl = minreal( (GK*k) / (1 + GK*k) );
>> eig(GKcl)

  -1.0483 + 2.0462i
  -1.0483 - 2.0462i
 -10.9400
 -16.1334

>> Ts = 4 / 1.0483

Ts =     3.8157
```

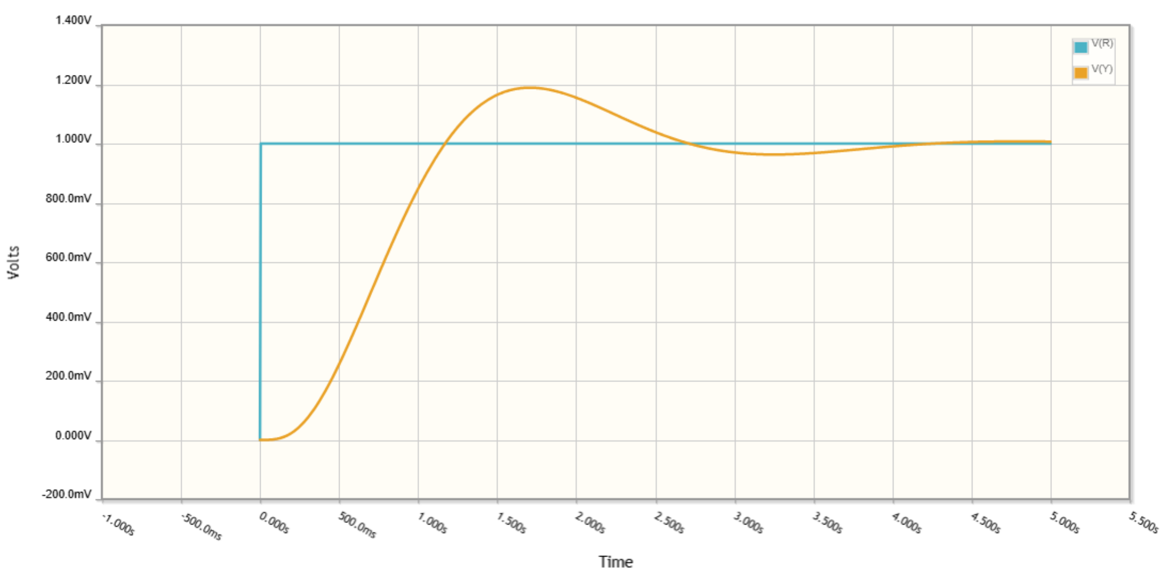
Type-1 system

- No error for a step input

Check your design in Matlab or Simulink or CircuitLab

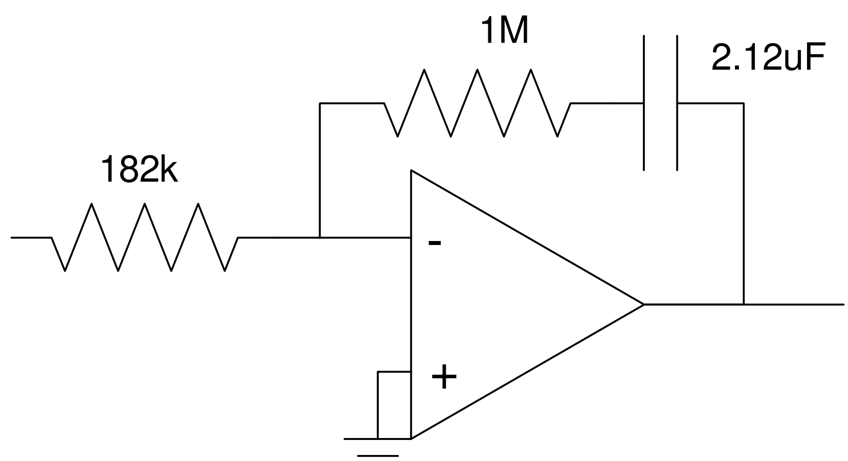
In CircuitLab





Give an op-amp circuit to implement K(s)

$$K(s) = 5.4879\left(\frac{s+0.47}{s}\right)$$



>

Meeting Design Specs

3) Assume

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Translation:

- Make it a type-1 system
  - *no error for a step input*
- Place the closed-loop dominant pole at s = -2 + j4

Let K(s) be of the form

$$K(s) = k\left(\frac{(s+0.47)(s+3.40}{s(s+a)} \right)$$

so that

$$GK = \left(\frac{170k}{s(s+a)(s+9.00)(s+16.77)} \right)$$

Pick 'a' so that s = -2 + j4 is on the root locus

- The angle of GK(s) = 180 degrees

Solving for what we know:

$$\left(\frac{170}{s(s+9.00)(s+16.77)} \right)_{s=-2+j4} = 0.3080\angle -161.463^0$$

For the angles to add up to 180 degrees

$$\angle(s+a) = 18.537^0$$

$$a = \frac{4}{\tan\left(18.537^0\right)} + 2$$

$$a = 13.929$$

meaning

$$K(s) = k\left(\frac{(s+0.47)(s+3.40}{s(s+13.929)} \right)$$

To find 'k', at any point on the root locus, GK = -1

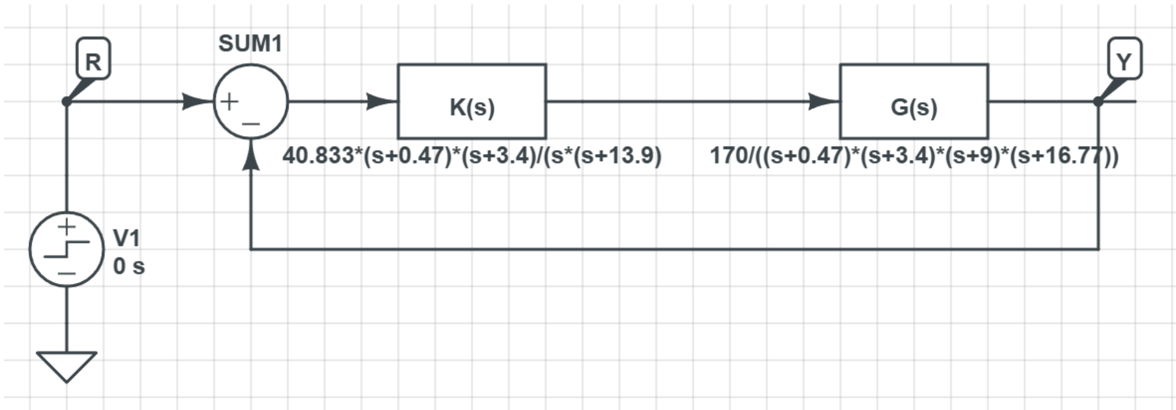
$$GK = \left(\frac{170k}{s(s+9.00)(s+13.929)(s+16.77)} \right)_{s=-2+j4} = 0.024k\angle 180^0$$

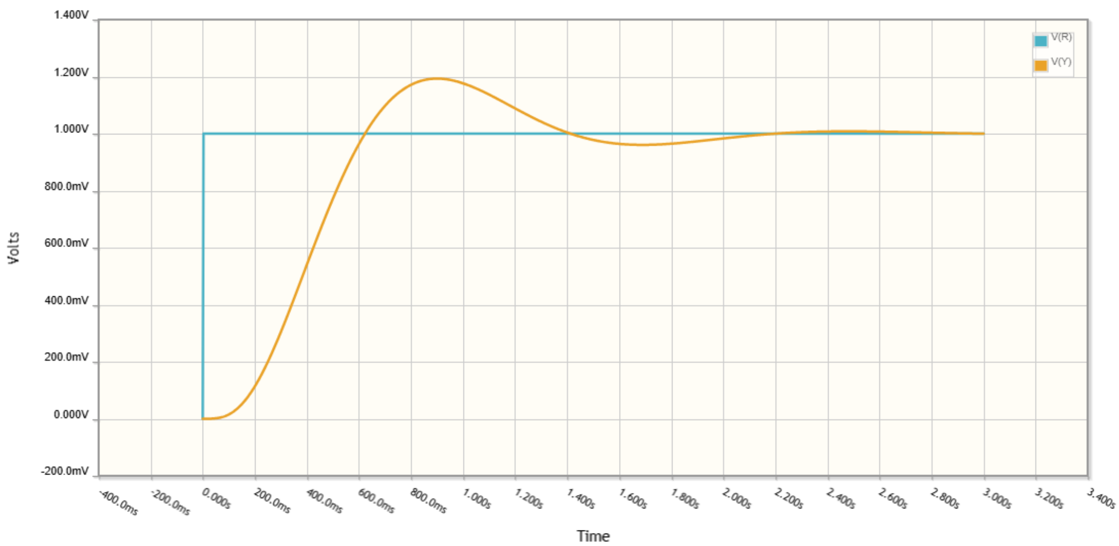
$$k = \frac{1}{0.024} = 40.833$$

meaning

$$\boxed{K(s) = 40.833\left(\frac{(s+0.47)(s+3.40}{s(s+13.929)}\right)}$$

Check your design in Matlab or Simulink or CircuitLab





Requirements:

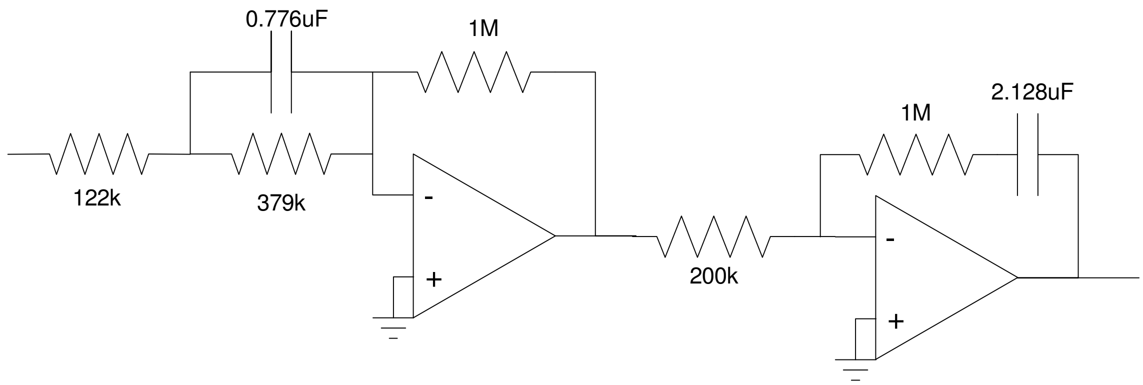
- No error for a step input
  - *check*
- 2% settling time of 2 seconds
  - *check*
- 20% overshoot
  - *check*

Give an op-amp circuit to implement K(s)

$$K(s) = 40.833\left(\frac{(s+0.47)(s+3.40}{s(s+13.929)}\right)$$

Rewrite as

$$K(s) = 8.167\left(\frac{s+3.40}{s+13.929}\right) \cdot 5\left(\frac{s+0.47}{s}\right)$$



Systems with Delays

4) Assume a 100ms delay is added to the system

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)e^{-0.1s}$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Same as before. Let K(s) be of the form

$$K(s) = k\left(\frac{(s+0.47)(s+3.40)}{s(s+a)}\right)$$

resulting in

$$GK = \left(\frac{170k}{s(s+a)(s+9.00)(s+16.77)}\right)e^{-0.1s}$$

Evaluate what we know:

$$\left(\left(\frac{170}{s(s+9.00)(s+16.77)}\right)e^{-0.1s}\right)_{s=-2+j4} = 0.3760\angle+175.618^0$$

There's too much phase shift.

Cancel another pole...

$$K(s) = k\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{170k}{s(s+a)^2(s+16.77)}\right)e^{-0.1s}$$

Evaluate what we know:

$$\left(\left(\frac{170}{s(s+16.77)}\right)e^{-0.1s}\right)_{s=-2+j4} = 3.043\angle-154.637^0$$

meaning

$$\angle(s+a)^2 = 25.363^0$$

$$\angle(s+a) = 12.682^0$$

$$a = \frac{4}{\tan\left(12.682^0\right)} + 2 = 19.776$$

and

$$K(s) = k\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+19.776)^2}\right)$$

At any point on the root locus, GK = -1

$$GK = \left(\left(\frac{170k}{s(s+19.776)^2(s+16.77)} \right) e^{-0.1s} \right)_{s=-2+j4} = 0.009k\angle 180^0$$

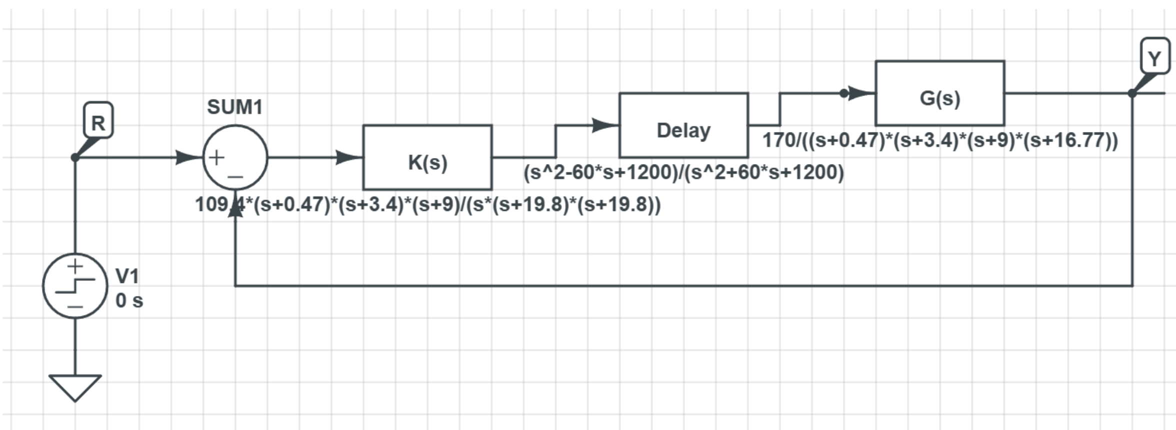
$$k = \frac{1}{0.009} = 109.415$$

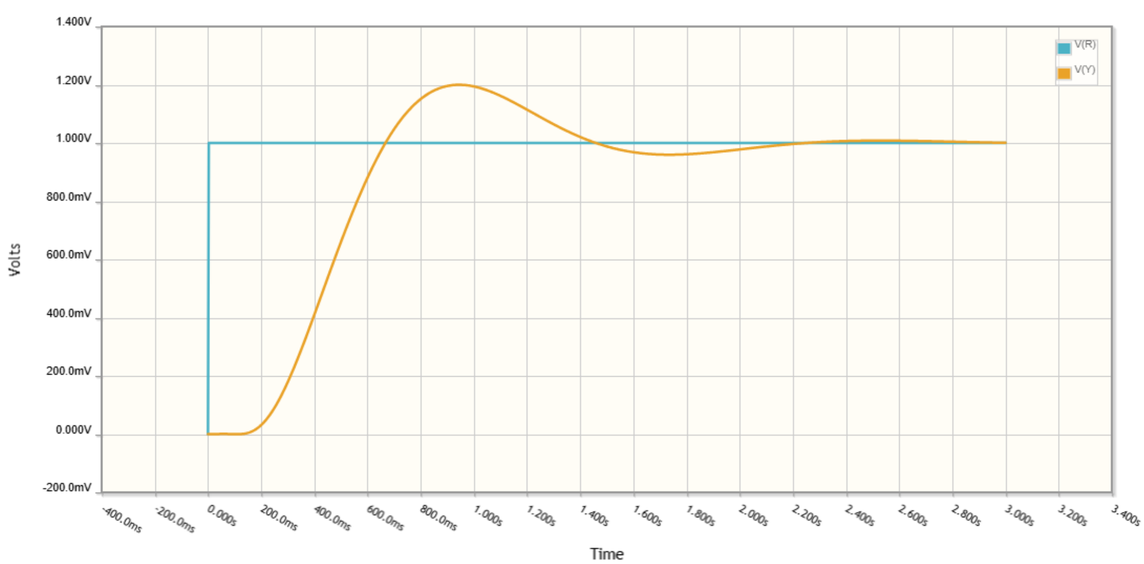
so

$$\boxed{K(s) = 109.415\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+19.776)^2} \right)}$$

Check your design in Matlab or Simulink or CircuitLab

Use a 2nd-order Pade approximation for a delay





Give an op-amp circuit to implement K(s)

$$K(s) = 109.415\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+19.776)^2}\right)$$

Rewrite as

$$K(s) = \left(\frac{s+0.47}{s}\right) \cdot 10\left(\frac{s+3.40}{s+19.776}\right) \cdot 10.94\left(\frac{s+9}{s+19.776}\right)$$