Homework #7: ECE 461/661

Error Constants, Routh Criteria, Skething a Root Locus

Error Constants

1) Determine the error constants and steady-state error for the following systems

G(s)	System Type	Kp	Kv	Error for a unit step input
$\left(\frac{20}{(s+2)(s+10)}\right)$	0	1	0	1/2
$\left(\frac{20}{s(s+2)(s+10)}\right)$	1	infinity n/a	1	0
$\left(\frac{20(s+0.3)}{s^2(s+2)(s+10)}\right)$	2	infinity n/a	infinity	0
$\left(\frac{20}{(s-2)(s+10)}\right)$	0	-1	0	infinity

Routh Criteria

Determine the range of k that results in a negative definite polynomial (i.e. a stable system)

2) $(s-2)(s+10)(s+12)+2k=0$

$s^3+20s^2+76s-240+2k=0$

1	76	0	
20	-240+2k	0	
$\frac{-\begin{vmatrix} 1 & 76 \\ 20 & -240+2k \end{vmatrix}}{20} = 88-0.1k$	$\frac{-\begin{vmatrix} 1 & 0 \\ 20 & 0 \end{vmatrix}}{20} = 0$	0	k < 880
$\frac{-\begin{vmatrix} 20 & 240+2k \\ 64-0.1k & 0 \end{vmatrix}}{64-0.1k} = -240+2k$	0	0	k > 120
0	0	0	

result:

120 < k < 880

3) $(s+2)(s+5)(s+10)(s+12)+2k=0$

$s^4+29s^3+284s^2+1060s+1200+2k=0$

1	284	1200+2k	
29	1,060	0	
247.4833 (a)	1200 + 2k (d)	0	
919.3645 - 0.2344k (b)	0	0	k < 3922
1200 + 2k (c)	0	0	k > -600
0	0	0	

ans **-600 < k < 3922**

(a) $\dfrac{-\begin{vmatrix} 1 & 284 \\ 29 & 1060 \end{vmatrix}}{29} = 247.45$

(b) $\dfrac{-\begin{vmatrix} 29 & 1060 \\ 247.45 & 1200+2k \end{vmatrix}}{247.45} = 919.35 - 0.2344k$

(c) $\dfrac{-\begin{vmatrix} 247.45 & 1200+2k \\ 919.35-0.2344k & 0 \end{vmatrix}}{919.35-0.2344k} = 1200 + 2k$

(d) $\dfrac{-\begin{vmatrix} 1 & 1200+2k \\ 29 & 0 \end{vmatrix}}{29} = 1200 + 2k$

Sketching a Root Locus

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), and asymptotes

4) $(s-2)(s+10)(s+12)+2k=0$

Real Axis Loci: (+2, -10), (-12, -infinity)

Breakaway Point: s = -2.295

$$\frac{d}{ds}\left(\frac{(s-2)(s+10)(s+12)}{2}\right)=0$$

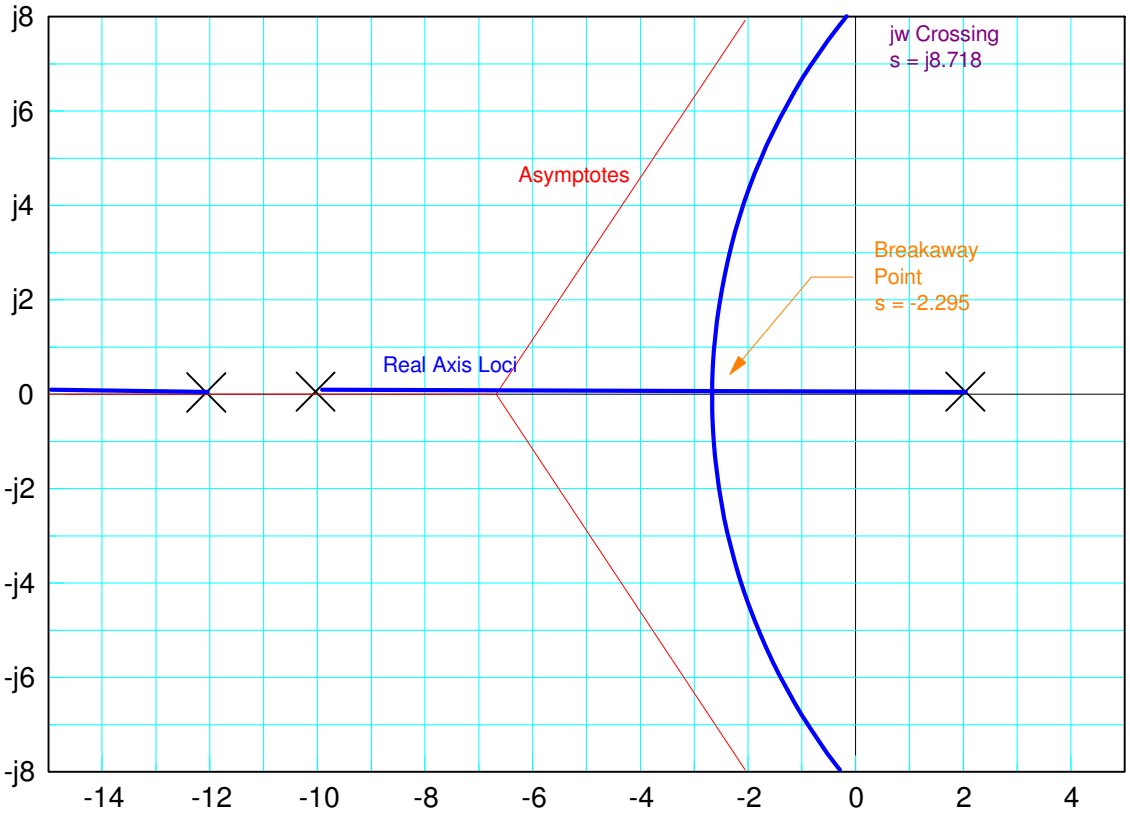
search along the line (x + j0.1) until the angles add to 180 degres

jw Crossing: s = j8.718

search along the line (0 + jx) until the angles add to 180 degrees

Asymptotes:

- 3 asymptotes
- Asymptote Intersect: $\left(\frac{+2-10-12}{3}\right)=-6.667$
- Asymptote Angle: 180 degrees, +/- 60 degrees



5) $(s+2)(s+5)(s+10)(s+12)+2k=0$

Real Axis Loci: (-2, -5), (-10, -12)

Breakaway Points: s = -11.135, s = -3.125

- $\frac{d}{ds}\left(\frac{(s+2)(s+5)(s+10)(s+12)}{2}\right)=0$
- Search along the line (x + j0.1) until the angle of G(s) is 180 degrees

jw Crossing: s = j6.046

- search along the line (0 + jx) until the angle of G(s) is 180 degrees

Asymptotes

- 4 asymptotes
- +/- 45 degrees, +/- 135 degrees
- Intersect: $=\left(\frac{-2-5-10-12}{4}\right)=-7.25$