ECE 341 - Homework #13

t-Tests with Two Populations. Summer 2023

Let

- X = 5d10 (the sum of five 10-sided dice) plus 0.5 (X wins on ties)
- Y = 2d4 + 3d6 + 4d8

Monte-Carlo Simulation

1) Run a Monte-Carlo simulation with 100,000 rolls for X and Y. From this, determine the probability that X will win any given game.

In Matlab

```
WIN = 0;
for n=1:1e5
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = sum(d10) + 0.5;
   Y = sum(d4) + sum(d6) + sum(d8);
   if(X > Y)
       WIN = WIN + 1;
   end
end
disp(WIN / 1e5)
  0.2630
```

X has a 26.30% chance of winning any given game

t-Test: Sample Size = 3

2) Take three measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test.

In Matlab

```
X = [];
Y = [];
for n=1:2
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = [X ; sum(d10) + 0.5];
   Y = [Y; sum(d4) + sum(d6) + sum(d8)];
   end
disp([X,Y])
     X     Y
   17.5   33.0
   19.5   27.0
```

Find the mean and standard deviation of X and Y

```
>> Xx = mean(X)

Xx =   18.5000

>> Sx = std(X)

Sx =    1.4142

>> Xy = mean(Y)

Xy =    30

>> Sy = std(Y)

Sy =    4.2426
```

Find the probability that X will win any given game. Create a new variable, W

$$W - X - Y$$

The mean and standard deviation of W are then

```
>> Mw = mean(X) - mean(Y)

Mw =  -11.5000

>> Sw = sqrt(var(X) + var(Y))

Sw =    4.4721
```

Find the t-score for W > 0

```
>> t = Mw / Sw

t =   -2.5715
```

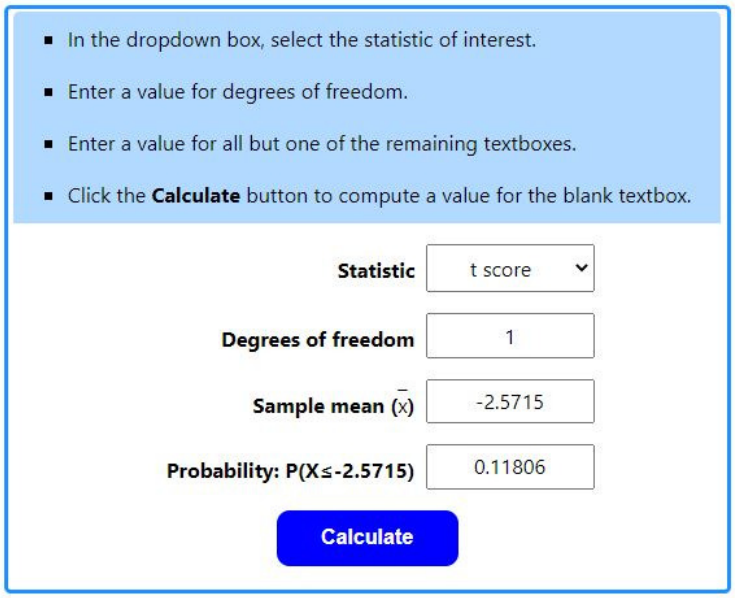
Convert this to a probability using a t-table with 1 degree of freedom (sample size = 2)

p = 0.11806

The probability of X winning any given game is

26.30% **Monte-Carlo with sample size of 100,000**

11.806% **t-test with a sample size of 2**



StatTrem: note: Sample mean is actually the t-score (bug in StatTrek)

t-Test: Sample Size = 6

3) Take six measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test

```
X = [];
Y = [];
for n=1:6
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = [X ; sum(d10) + 0.5];
   Y = [Y; sum(d4) + sum(d6) + sum(d8)];
   end
disp([X,Y])
     X         Y
   32.5000   34.0000
   37.5000   21.0000
   29.5000   30.0000
   28.5000   39.0000
   28.5000   38.0000
   31.5000   36.0000
```

Find the mean and standard deviation of X and Y

```
>> Xx = mean(X)

Xx =   31.3333

>> Sx = std(X)

Sx =    3.4303

>> My = mean(Y)

My =    33

>> Sy = std(Y)

Sy =    6.6933
```

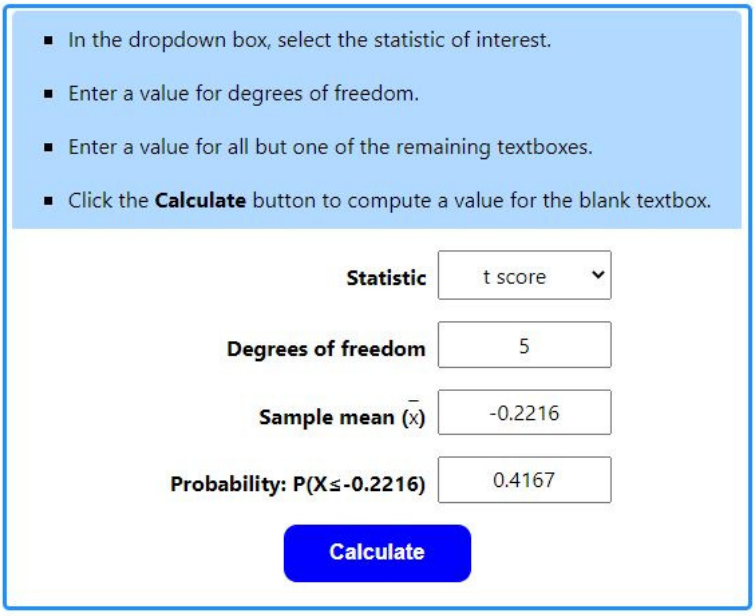
Find the probability that X wins any given game. Create a new variable, W = X - Y

```
>> Mw = mean(X) - mean(Y)

Mw =   -1.6667

>> Sw = sqrt(var(X) + var(Y))

Sw =    7.5211

>> t = Mw / Sw

t =   -0.2216
```

From StatTrek, this corresponds to a probability of 41.67%

The probability of X winning any given game is

26.30%	**Monte-Carlo with sample size of 100,000**
11.806%	**t-test with a sample size of 2**
41.67%	**t-test with a samle size of 6**



StatTrem: note: Sample mean is actually the t-score (bug in StatTrek)

t-Test: Sample Size = 10

4) Take ten measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test

```
X = [];
Y = [];
for n=1:10
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = [X ; sum(d10) + 0.5];
   Y = [Y; sum(d4) + sum(d6) + sum(d8)];
   end
disp([X,Y])
     X          Y
   22.5000   41.0000
   39.5000   32.0000
   17.5000   32.0000
   25.5000   46.0000
   40.5000   31.0000
   17.5000   37.0000
   30.5000   29.0000
   29.5000   24.0000
   27.5000   43.0000
   23.5000   40.0000

Xx = mean(X)
Sx = std(X)
My = mean(Y)
Sy = std(Y)

   Mx =   27.4000
   Sx =    7.9645
   My =   35.5000
   Sy =    6.9801

Mw = mean(X) - mean(Y)
Sw = sqrt(var(X) + var(Y))
t = Xw / Sw

   Mw =   -8.1000
   Sw =   10.5904
   t  =   -0.7648
```

From StatTrek, p = 0.2319

t-Test: Sample Size = 100

5) Take 100 measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test

```
Mx =   27.4400
Sx =    6.3243

My =   32.9000
Sy =    5.9671

Mw =   -5.4600
Sw =    8.6950

t  =   -0.6279
```

From StatTrek, this corresponds to a probability of 26.576%

The probability of X winning any given game is

26.30%	**Monte-Carlo with sample size of 100,000**
11.806%	**t-test with a sample size of 2**
41.67%	**t-test with a samle size of 6**
23.19%	**t-test with a sample size of 10**
26.576%	**t-test with a sample size of 100**

6) If each experiment (die roll) costs $100, how many die rolls would you suggest for determining the odds that X will win any given game?

depends upon how important it is to get the right answer. A sample size of 100 gives essentially the same resutls as a Monte-Carlo simulation with a sample size of 100,000.