ECE 341 - Homework #11

Regression Analysis & Markov Chains - Summer 2023

Regression Analysis

The low temperature in June in Fargo, ND is available at

`http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Low.txt`

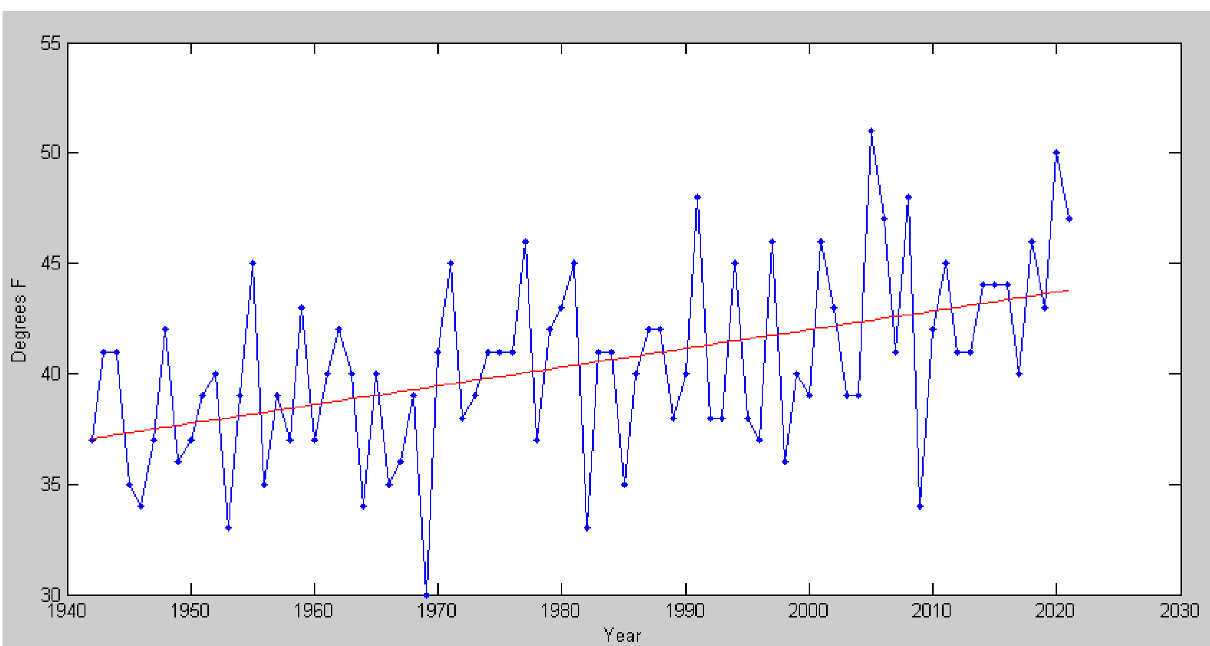
1) Find the least-squares curve fit for this data as

$$T = ay + b$$

where T is the temperature in degrees F and y is the year.

From this curve fit, how much has June in Fargo warmed up since 1942?

```
>> DATA = [ <paste> ];
>> year = DATA(:,1);
>> June = DATA(:,7);
>> plot(year,June,'-.')
>> plot(year,June,'.-')
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*June

a =    0.0849
b = -127.8121

>> A(1)*60
    5.0942
```
`June's low has warmed up +5.09F in the past 60 years in Fargo`
```
>> plot(year,June,'.-',year,B*A,'r')
```



2) Determine the correlation coefficient between

- The low temperature in January and February
  if January was cold, will February be cold?
- The low temperature in January and July.
  if January is cold is July going to be cold?

```
>> Jan = DATA(:,2);
>> Feb = DATA(:,3);
>> Cov = mean(Jan .* Feb) - mean(Jan)*mean(Feb)

Cov =

   19.3135

>> Correlation = Cov / ( std(Jan) * std(Feb) )

Correlation =

    0.2992
```

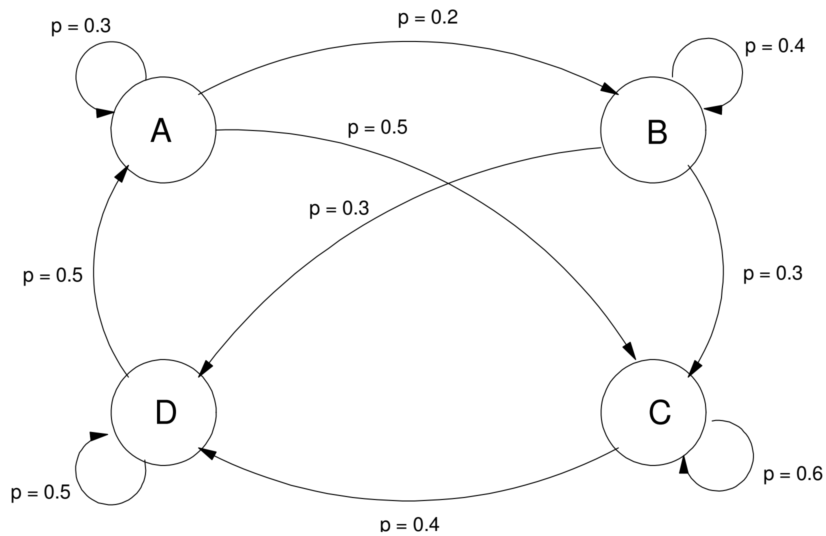
There is a 29.92% correlation between the low in January & February

```
>> July = DATA(:,8);
>> Cov = mean(Jan .* July) - mean(Jan)*mean(July)

Cov =

    5.1978

>> Correlation = Cov / ( std(Jan) * std(July) )

Correlation =

    0.1545
```

There is a 15.45% correlation between January & July

Markov Chains

Four people are playing a game of hot-potato. Each second, a player can keep the potato or pass it to another player. The probability of each decision are as follows



3) Assume player A starts with the potato. Determine the probability that each player has the potato after 10 tosses using matrix muplication.

$$\begin{bmatrix} A(k+1) \\ B(k+1) \\ C(k+1) \\ D(k+1) \end{bmatrix} = \begin{bmatrix} 0.3 & 0 & 0 & 0.5 \\ 0.2 & 0.4 & 0 & 0 \\ 0.5 & 0.3 & 0.6 & 0 \\ 0 & 0.3 & 0.4 & 0.5 \end{bmatrix} \begin{bmatrix} A(k) \\ B(k) \\ C(k) \\ D(k) \end{bmatrix}$$

```
>> M = [0.3,0,0,0.5 ; 0.2,0.4,0,0 ; 0.5,0.3,0.6,0 ; 0,0.3,0.4,0.5]

    0.3000         0         0    0.5000
    0.2000    0.4000         0         0
    0.5000    0.3000    0.6000         0
         0    0.3000    0.4000    0.5000

>> X0 = [1;0;0;0]

     1
     0
     0
     0

>> M^10 * X0
```

```
A(10)    0.2363
B(10)    0.0785
C(10)    0.3540
D(10)    0.3311
```

4) Assume player A starts with the potato. Determine the probability that player A has the potato after k tosses using z-transforms.

- What is the probability that A has the potato after an infinite number of tosses?

```
>> A = M

    0.3000         0         0    0.5000
    0.2000    0.4000         0         0
    0.5000    0.3000    0.6000         0
         0    0.3000    0.4000    0.5000

>> X0 = [1;0;0;0]

     1
     0
     0
     0

>> C = [1,0,0,0]

     1     0     0     0

>> D = 0;
>> G = ss(A,X0,C,D,1);

>> zpk(G)

         (z-0.5) (z-0.6) (z-0.4)
-----------------------------------------
(z-1) (z-0.3532) (z^2 - 0.4468z + 0.2322)

Sampling time (seconds): 1
```

Multiply by z to get the z-transform for A(z)

$$A(z) = \left(\frac{(z-0.4)(z-0.5)(z-0.6)z}{(z-1)(z-0.3532)(z^2-0.4468z+0.2322)} \right)$$

Do a partial fraction expansion

$$A(z) = \left(\left(\frac{0.2362}{z-1} \right) + \left(\frac{0.0132}{z-0.3532} \right) + \left(\frac{0.3963\angle-18.71^0}{z-0.4819\angle 62.376^0} \right) + \left(\frac{0.3969\angle+18.71^0}{z-0.4819\angle-62.376^0} \right) \right) z$$

Convert back to k

$$a(k) = 0.2362 + 0.0132(0.3532)^k + 0.7925(0.4819)^k \cos(62.376k^0 + 18.71^0)$$

5) Assume player A starts with the potato. Determine the probability that A has the potato after an infinite number of tosses using eigenvalues and eigenvectors.

Start with the state-transistion matrix:

```
>> A

    0.3000         0         0    0.5000
    0.2000    0.4000         0         0
    0.5000    0.3000    0.6000         0
         0    0.3000    0.4000    0.5000
```

Find the eigenvalues & eigenvectors. The one we care about is the eigenvector associated with the eigenvalue at z = +1 (show in red)

```
>> [M,V] = eig(A)

M =

  -0.4335            -0.6190                -0.6190                -0.1849
  -0.1445             0.1024 + 0.2476i       0.1024 - 0.2476i       0.7895
  -0.6503             0.4218 + 0.2809i       0.4218 - 0.2809i      -0.5849
  -0.6069             0.0948 - 0.5285i       0.0948 + 0.5285i      -0.0197


V =

   1.0000                  0                      0                      0
        0             0.2234 + 0.4269i            0                      0
        0                  0                 0.2234 - 0.4269i            0
        0                  0                      0                 0.3532

>> Xa = M(:,1)

   -0.4335
   -0.1445
   -0.6503
   -0.6069
```

Make this a valid probability (sum must be 1.0000)

```
>> Xa / sum(Xa)

    0.2362      Probability that A winds up with the ball
    0.0787
    0.3543
    0.3307
```

Note: This is the same result we got in problem #4