Inductors and Capacitors in the LaPlace Domain

Inductors

From before, the VI characteristics for an inductor are

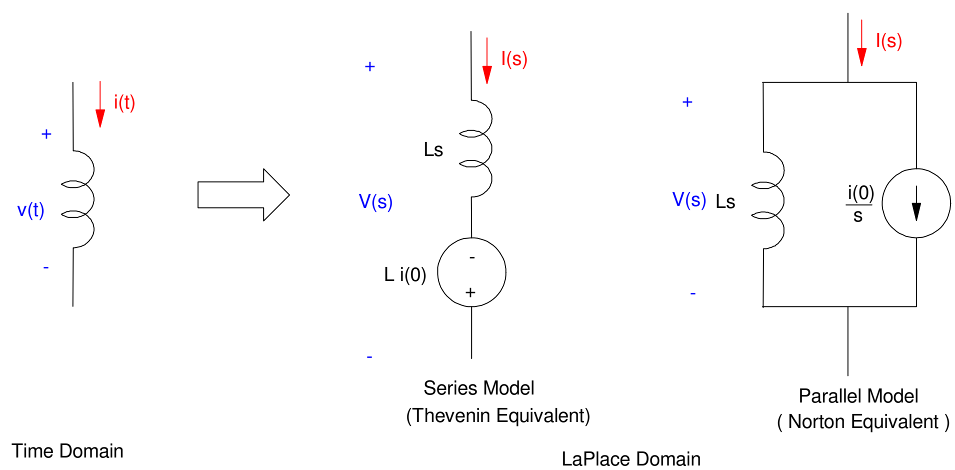
$$v(t) = L\frac{di(t)}{dt}$$

The LaPlace transform is

$$V = L \cdot (sI - i(0))$$

Voltages in series add, meaning this is the series connection of two elements: an impedance (Ls) and a voltage source (- L i(0)):

$$V = Ls \cdot I - Li(0)$$



This results in the Thevenin equivalent for an inductor. The Norton equivalent is then

$$I_n = \frac{V_{th}}{R_{th}}$$

$$I_n = \frac{L \cdot i(0)}{Ls}$$

$$I_n = \frac{i(0)}{s}$$

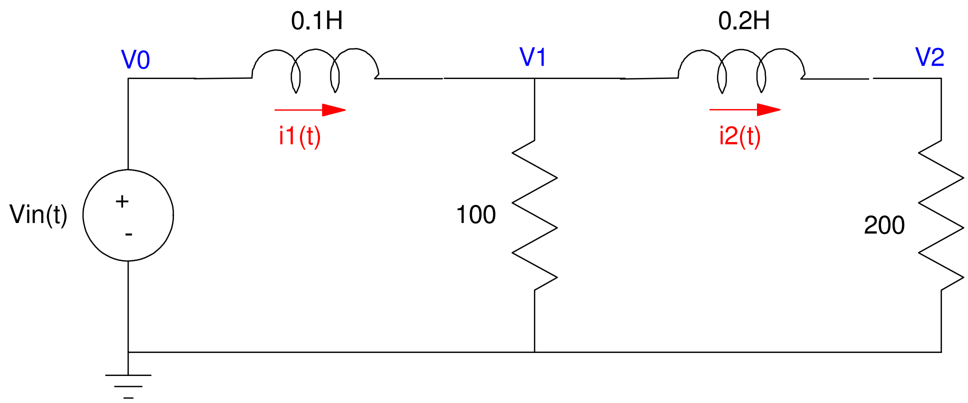
Note that

- The series model is more useful when writing current loop equations
- The parallel model is more useful when writing voltage node equations.

Both models are valid, however.

Example 1: Determine the voltages for the following circuit. Assume

$$v_{in}(t) = \begin{cases} 5V & t < 0 \\ 10V & t > 0 \end{cases}$$



Solution: First, find the currents at t = 0. Using phasor analysis, the inductors become

$$L \to j\omega L = 0$$

For t<0, this results in

$$v_1 = v_2 = 5V$$

$$i_2 = \frac{5V}{200\Omega} = 25mA$$

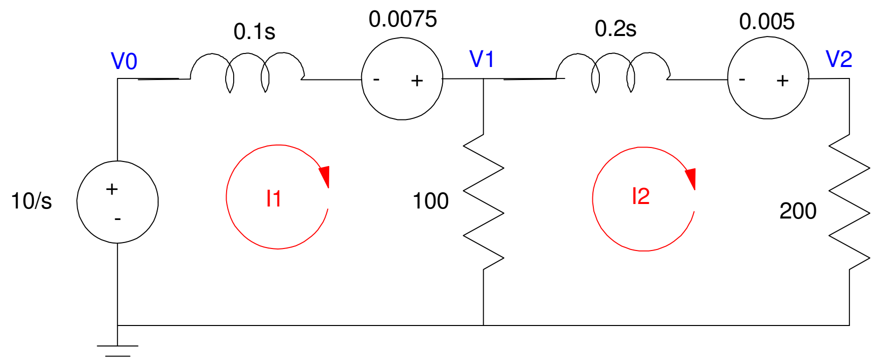
$$i_1 = \frac{5V}{100\Omega} + i_2 = 75mA$$

Now, take the LaPlace transform. Assuming we're going to be using current loops, use the series model for the inductors.

$$v_{in} \to \frac{10}{s}$$

$$0.1H \to 0.1s - 0.1 \cdot i_1(0)$$
$$\to 0.1s - 0.0075$$

$$0.2H \to 0.2s - 0.2 \cdot i_2(0)$$
$$\to 0.2s - 0.005$$

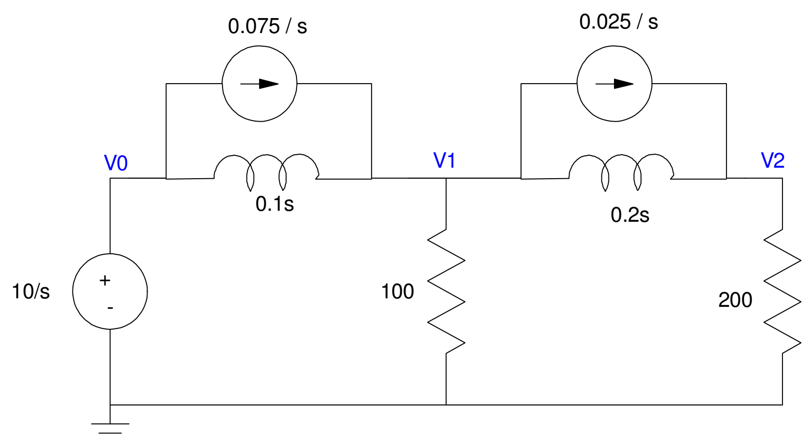


Writing the current loop equations:

$$-\frac{10}{s} + 0.1s \cdot I_1 - 0.0075 + 100(I_1 - I_2) = 0$$

$$100 \cdot (I_2 - I_1) + 0.2s \cdot I_2 - 0.005 + 200 \cdot I_2 = 0$$

The parallel model (for voltage nodes) would be



with the voltage node equations being

$$V_0 = \frac{10}{s}$$

$$\left(\frac{V_1 - V_0}{0.1s}\right) - \left(\frac{0.075}{s}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{0.025}{s}\right) + \left(\frac{V_1 - V_2}{0.2s}\right) = 0$$

$$\left(\frac{V_2 - V_1}{0.2s}\right) + \left(\frac{V_2}{200}\right) - \left(\frac{0.025}{s}\right) = 0$$

Capacitors

From before, the VI characteristics for a capacitor is

$$i(t) = C\frac{dv}{dt}$$

Taking the LaPlace transform

$$I(s) = C \cdot (sV - v(0))$$

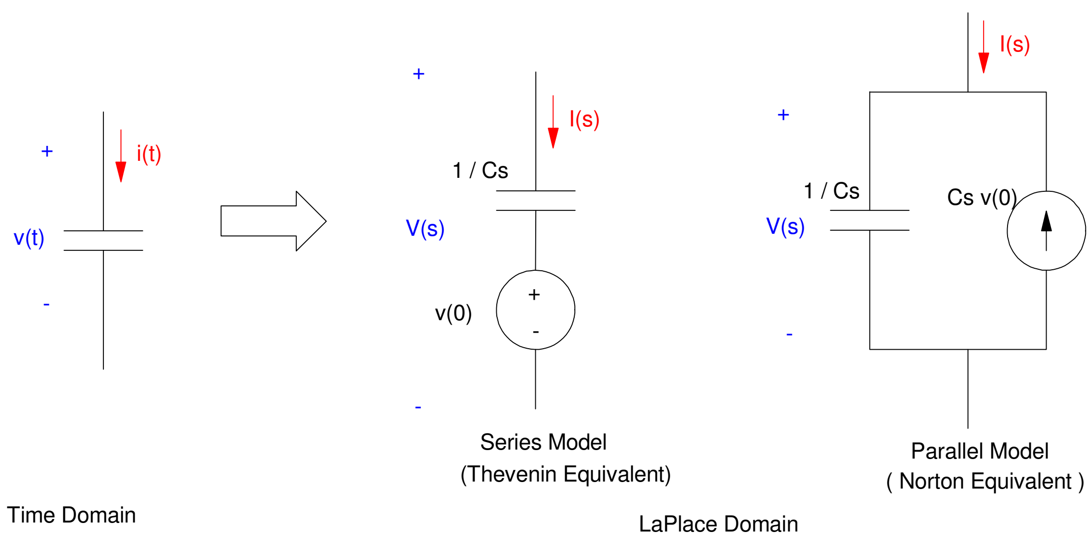
$$I(s) = CsV - Cv(0)$$

Solving for V

$$V = \left(\frac{1}{Cs}\right) I + v(0)$$

This gives the series (Thevenin) model for a capacitor. The parallel model has

$$I_{short} = \frac{V_{th}}{R_{th}} = \frac{v(0)}{1/Cs} = Csv(0)$$

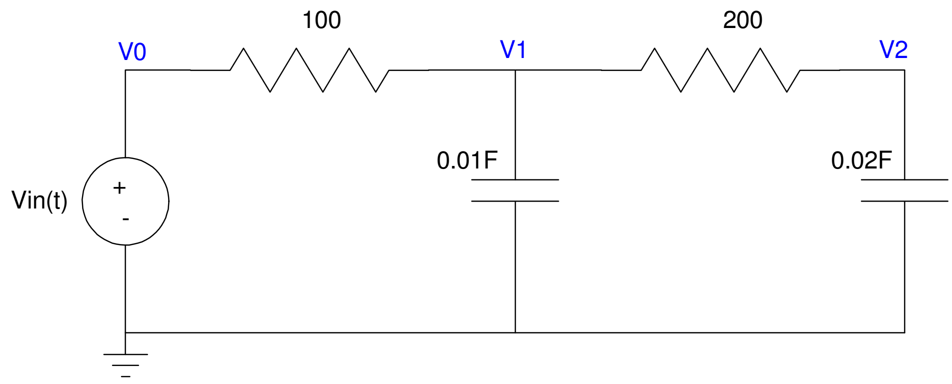


Note that

- The series model is more useful when writing current loop equations
- The parallel model is more useful when writing voltage node equations.

Example: Convert the following circuit to the LaPlace domain and write the voltage node and current loop equations.

$$v_{in}(t) = \begin{cases} 5V & t < 0 \\ 10V & t > 0 \end{cases}$$



Solution: First find the initial conditions. For t<0, the voltage is a constant (5V). Using phasor analysis,

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

$$V_1 = V_2 = 5V$$

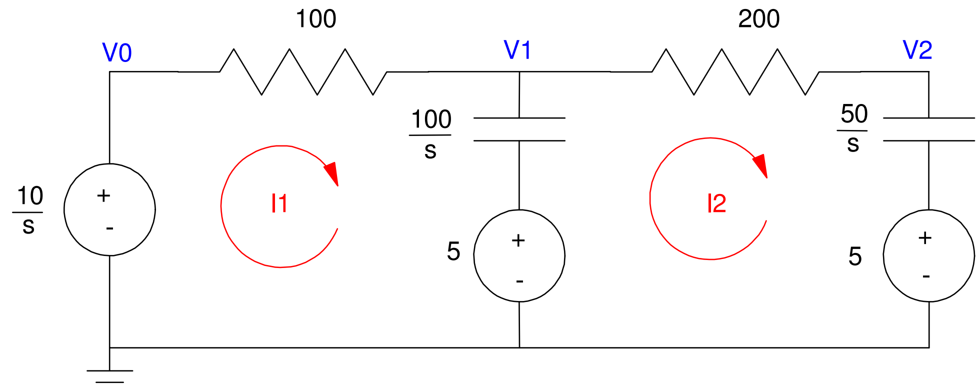
For the series (Thevenin) model, you have a voltage source with

$$V_{th} = v(0) = 5$$

For the parallel (Norton) model, you have a current source with

$$I_N = \frac{V_{th}}{R_{th}} = \frac{v(0)}{1/Cs} = C \cdot v(0) \cdot s$$

Series Model for t > 0:

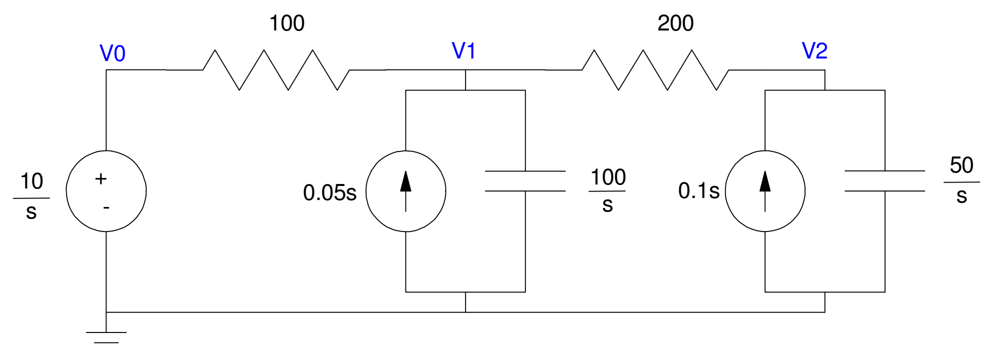


The current loop equations are then:

$$-\tfrac{10}{s} + 100I_1 + \left(\tfrac{100}{s}\right)(I_1 - I_2) + 5 = 0$$

$$-5 + \left(\tfrac{100}{s}\right)(I_2 - I_1) + 200I_2 + \left(\tfrac{50}{s}\right)I_2 + 5 = 0$$

Parallel Model for t > 0:



the voltage node equations become

$$V_0 = \tfrac{10}{s}$$

$$\left(\frac{V_1 - V_0}{100}\right) - 0.05s + \left(\frac{V_1}{100/s}\right) + \left(\frac{V_1 - V_2}{200}\right) = 0$$

$$\left(\frac{V_2 - V_1}{200}\right) - 0.1s + \left(\frac{V_2}{50/s}\right) = 0$$