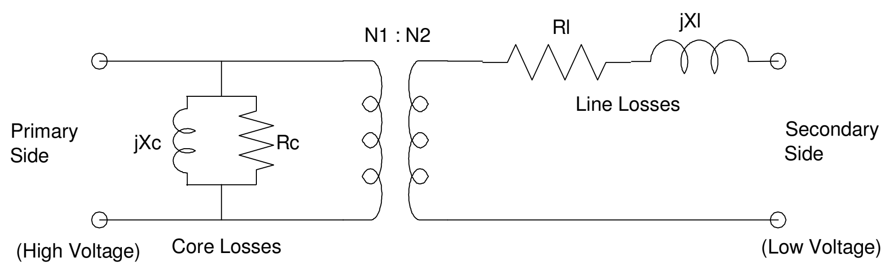
ECE 111: Homework 16

ECE 331 Energy Conversion
Due Monday, May 12th. Please submit via email or on BlackBoard

1) Determine the circuit model for a 13.2kV : 240V transformer is tested with the following test results:



Transformer Model

	V	Power	pf
Open-Circuit Test	V1 = 13.2kV	20 W	0.01
Short-Circuit Test	V2 = 24V	25 W	0.98

Core Model (open-circuit test)

```
>> V = 13.2e3;
>> P = 20;
>> pf = 0.01;
>> I = P / (V * pf)

I =     0.1515

>> Z = (V/I) * exp(j*acos(pf))

Z =   8.7120e+002 +8.7116e+004i
```

This is the series model. The paralel model comes from the inverse of Z

```
>> 1/Z

ans =   1.1478e-007 -1.1478e-005i

>> Rc = 1 / real(1/Z)
```

Rc =   8.7120e+006

```
>> Xc = -1 / imag(1/Z)
```

Xc =   8.7124e+004

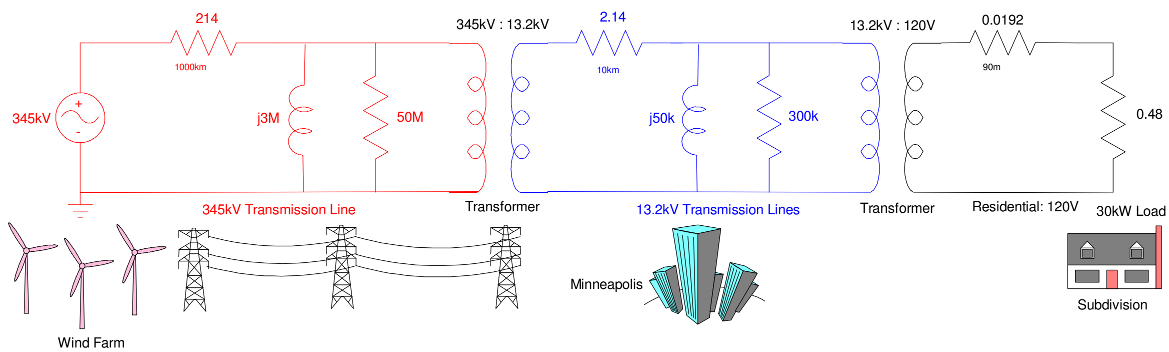
```
>>
```

Line Model (short-circuit test)

```
>> V = 24;
>> P = 25;
>> pf = 0.98;
>> I = P / (V * pf)

I =    1.0629

>> Z = (V/I) * exp(j*acos(pf))

Z =  22.1276 + 4.4932i

>> RL = real(Z)

RL =   22.1276

>> XL = imag(Z)

XL =    4.4932
```

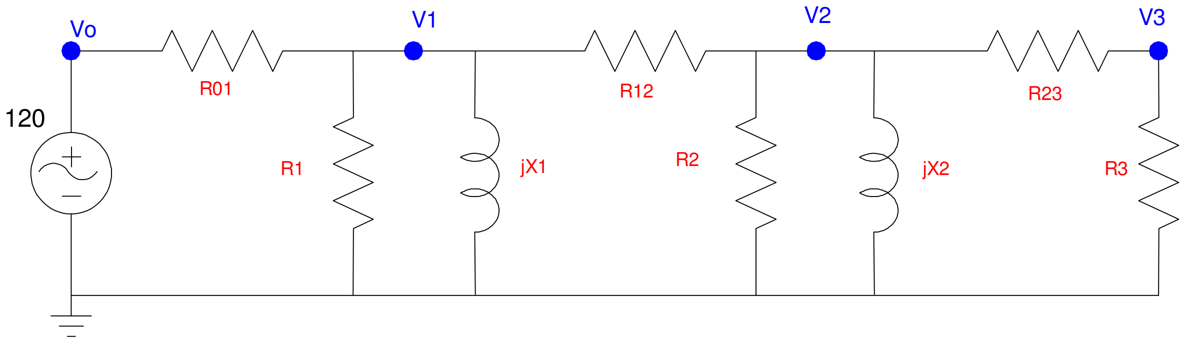
For the utility grid on the back of the page....

2) Convert the voltages and impedances to the 120V node (right side)




```
R01 = 214 * (120/345e3)^2
X1  = j*3e6 * (120/345e3)^2
R1  = 50e6 * (120/345e3)^2
R12 = 2.14 * (120/13.2e3)^2
R2  = 300e3 * (120/13.2e3)^2
X2  = j*50e3 * (120/13.2e3)^2
R23 = 0.0192
R3  = 0.48

R01 =   2.5890e-005
X1  =          0 + 0.3629i
R1  =     6.0491
R12 =   1.7686e-004
R2  =   24.7934
X2  =          0 + 4.1322i
R23 =     0.0192
R3  =     0.4800
```

3) Write the voltage node equations for this circuit (with transformers removed)



$$V_0 = 120$$

$$\left(\frac{V_1 - V_0}{R_{01}}\right) + \left(\frac{V_1}{R_1}\right) + \left(\frac{V_1}{jX_1}\right) + \left(\frac{V_1 - V_2}{R_{12}}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R_{12}}\right) + \left(\frac{V_2}{R_2}\right) + \left(\frac{V_2}{jX_2}\right) + \left(\frac{V_2 - V_3}{R_{12}}\right) = 0$$

$$\left(\frac{V_3 - V_2}{R_{23}}\right) + \left(\frac{V_3}{R_3}\right) = 0$$

4) Determine the voltages at each node

Group terms

$$V_0 = 120$$

$$-\left(\frac{1}{R_{01}}\right)V_0 + \left(\frac{1}{R_{01}} + \frac{1}{R_1} + \frac{1}{jX_1} + \frac{1}{R_{12}}\right)V_1 - \left(\frac{1}{R_{12}}\right)V_2 = 0$$

$$-\left(\frac{1}{R_{12}}\right)V_1 + \left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{jX_2} + \frac{1}{R_{23}}\right)V_2 - \left(\frac{1}{R_{23}}\right)V_3 = 0$$

$$-\left(\frac{1}{R_{23}}\right)V_2 + \left(\frac{1}{R_{23}} + \frac{1}{R_3}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{R_{01}}\right) & \left(\frac{1}{R_{01}} + \frac{1}{R_1} + \frac{1}{jX_1} + \frac{1}{R_{12}}\right) & \left(\frac{-1}{R_{12}}\right) & 0 \\ 0 & \left(\frac{-1}{R_{12}}\right) & \left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{jX_2} + \frac{1}{R_{23}}\right) & \left(\frac{-1}{R_{23}}\right) \\ 0 & 0 & \left(\frac{-1}{R_{23}}\right) & \left(\frac{1}{R_{23}} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
R01 = 214 * (120/345e3)^2;
X1 = j*3e6 * (120/345e3)^2;
R1 = 50e6 * (120/345e3)^2;
R12 = 2.14 * (120/13.2e3)^2;
R2 = 300e3 * (120/13.2e3)^2;
X2 = j*50e3 * (120/13.2e3)^2;
R23 = 0.0192;
R3 = 0.48;

b1 = [1,0,0,0];
b2 = [-1/R01,1/R01+1/R1+1/X1+1/R12,-1/R12,0];
b3 = [0,-1/R12,1/R12+1/R2+1/X2+1/R23,-1/R23];
b4 = [0,0,-1/R23,1/R23+1/R3];
B = [b1;b2;b3;b4];
A = [120;0;0;0];
V = inv(B)*A

V0   120.00
V1   119.99 + 0.01i
V2   119.95 + 0.01i
V3   115.34 + 0.01i
```

The voltage at the customer has drooped down to 115.34 Volts

5) Determine the efficiency of this system

- Ignoring the core losses
  - Assumes a large number of customers share these losses
- Including the core losses

Assumes a single customer

```
V0 = V(1);
V1 = V(2);
V2 = V(3);
V3 = V(4);

P01 = abs(V0 - V1)^2 / R01
P1 = abs(V1)^2 / R1
P12 = abs(V1 - V2)^2 / R12
P2 = abs(V2)^2 / R2
P23 = abs(V2 - V3)^2 / R23
P3 = abs(V3)^2 / R3

eff = P3 / (P1 + P12 + P2 + P23 + P3)

eff = P3 / (P12 + P23 + P3)


P01 =    5.1660
P1 =  2380.2
P12 =   10.7756
P2 =  580.3140
P23 =  1108.5
P3 =  27713
```

The customer is using 27.7kW of power

The efficiency if you incude everything is 87.17%

```
eff = P3 / (P1 + P12 + P2 + P23 + P3)
eff =    0.8717
```

The efficiency if you ignore the core losses (meaning their losses are distributed over a large number of customers) is 96.12%

```
eff = P3 / (P12 + P23 + P3)
eff =    0.9612
```

Transformers allow you to create a pretty efficient means of transporting electicity over large distances