

ECE 111 - Homework #12

Complex Numbers

Due Monday, April 14th. Please submit via email or on BlackBoard

Complex Numbers

1) Determine the rectangular or polar form of each complex number

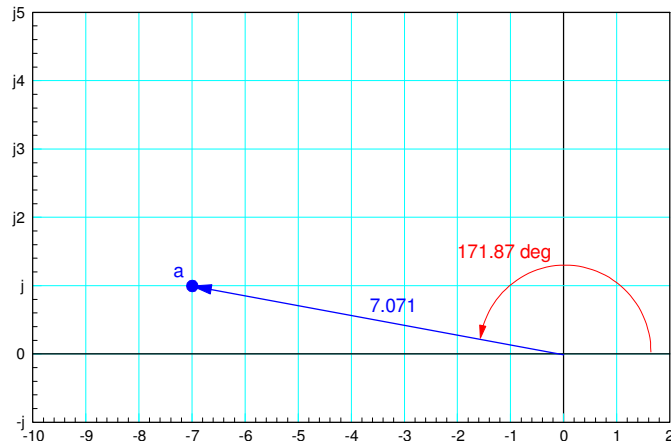
$$a = -7 + j$$

```
>> a = -7 + j  
a = -7.0000 + 1.0000i
```

```
>> abs(a)  
ans = 7.0711
```

```
>> angle(a)*180/pi  
ans = 171.8699
```

$$-7 + j = 7.0711 \angle 171.8699^\circ$$



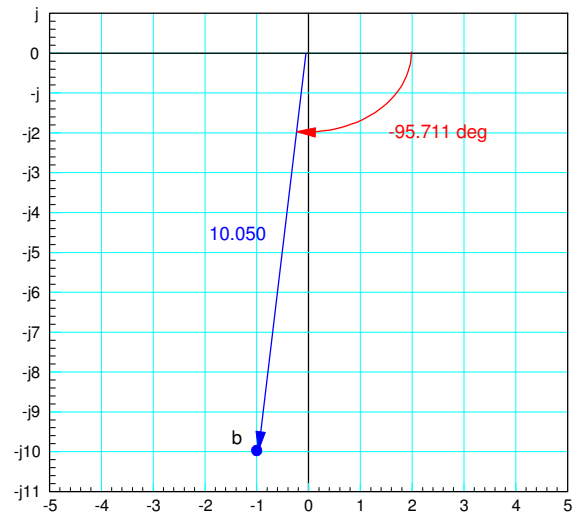
$$b = -1 - j10$$

```
>> b = -1 - 10i  
b = -1.0000 -10.0000i
```

```
>> abs(b)  
ans = 10.0499
```

```
>> angle(b)*180/pi  
ans = -95.7106
```

$$-1 - j10 = 10.0499 \angle -95.7016^\circ$$



$$c = 7 \angle -125^\circ$$

```
>> c = 7 * exp(-125*pi/180 * j)  
c = -4.0150 - 5.7341i
```

$$d = 8 \angle -37^\circ$$

```
>> d = 8 * exp(-37*pi/180 * j)  
d = 6.3891 - 4.8145i
```

2) Determine y as a complex number

$$y = (1 - j7) + (5 - j) + (-8 - j5)$$

```
>> y = (1-7i) + (5-1i) + (-8-5i)
```

```
y = -2.0000 -13.0000i
```

On an HP Prime calculator

```
1-i7  
enter  
5-i1  
+  
-8-i5  
+
```

$$y = (10\angle -84^0) + (5\angle 79^0) + (9\angle -99^0)$$

```
>> ya = 10*exp(-84*pi/180 * j)  
ya = 1.0453 - 9.9452i
```

```
>> yb = 5*exp(79*pi/180 * j)  
yb = 0.9540 + 4.9081i
```

```
>> yc = 9 * exp(-99*pi/180 * j)  
yc = -1.4079 - 8.8892i
```

```
>> y = ya + yb + yc  
y = 0.5914 -13.9263i
```

On an HP Prime calculator

```
10 angle -84  
enter  
5 angle 79  
+  
9 angle -99  
+  
angle
```

3) Determine y as a complex number

$$y = \left(\frac{(1-j7)(5-j)}{(-8-j5)} \right)$$

```
>> y = ( (1-7i) * (5-i) ) / (-8-5i)
```

y = 2.2022 + 3.1236i

On an HP-Prime

```
1-i7
enter
5-i1
*
-8-5i
/
```

$$y = \left(\left(\frac{7-j8}{-20-j5} \right) + \left(\frac{6-j}{8-j6} \right) \right) \left(\frac{-5-j7}{-7+j7} \right)$$

```
>> y = ( (7-8i) / (-20-5i) + (6-1i) / (8-6i) ) * ( (-5-7i) / (-7+7i) )
```

y = -0.6768 + 0.1556i

On an HP Prime

```
7-8i
enter
-20-5i
/
6-1i
enter
8-6i
/
+
-5-7i
enter
-7+7i
/
*
```

4) Determine y as a complex number

$$y = e^{(2+j)}$$

```
>> exp(2+1i)
```

```
ans = 3.9923 + 6.2177i
```

$$y = \ln(-7 + j7)$$

```
>> log(-7+7i)
```

```
ans = 2.2925 + 2.3562i
```

$$y = (2 - j3)^{(3-j2)}$$

```
>> (2-3i) ^ (3-2i)
```

```
ans = 4.7141 + 4.5698i
```

Comments:

- Functions of complex numbers produce a complex number.
- Matlab has no problem with these functions
- Some calculators (HP) can do complex operations. Others will give you an error message

Partial Fractions with Complex Numbers

5) Determine the partial fraction expansion

$$\left(\frac{3(x+1)(x+2)}{(x+1+j4)(x+1-j4)(x+5)} \right) = \left(\frac{a}{x+1+j4} \right) + \left(\frac{b}{x+1-j4} \right) + \left(\frac{c}{x+5} \right)$$

Option 1 in Matlab: Pick a point ϵ away and evaluate

```
>> x = -1-4i + 1e-9;
>> a = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+1-4i)*(x+5) ) * (x+1+4i)

a = 0.9375 - 0.5625i

>> x = -1 + 4i + 1e-9;
>> b = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+1-4i)*(x+5) ) * (x+1-4i)

b = 0.9375 + 0.5625i

>> x = -5 + 1e-9;
>> c = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+1-4i)*(x+5) ) * (x+5)

c = 1.1250
```

Option 2: Use the cover-up method

$$a = \left(\frac{3(x+1)(x+2)}{(x+1-j4)(x+5)} \right)_{x=-1-j4}$$

```
>> x = -1-4i;
>> a = 3*(x+1)*(x+2) / ( (x+1-4i)*(x+5) )

a = 0.9375 - 0.5625i
```

$$b = \left(\frac{3(x+1)(x+2)}{(x+1+j4)(x+5)} \right)_{x=-1+j4}$$

```
>> x = -1+4i;
>> b = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+5) )

b = 0.9375 + 0.5625i
```

$$c = \left(\frac{3(x+1)(x+2)}{(x+1+j4)(x+1-j4)} \right)_{x=-5}$$

```
>> x = -5;
>> c = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+1-4i) )

c = 1.1250
```

6) Determine the partial fraction expansion

$$\left(\frac{8(x+1+j2)(x+1-j2)}{(x+1)(x+3)(x+5+j2)(x+5-j2)} \right) = \left(\frac{a}{x+1} \right) + \left(\frac{b}{x+3} \right) + \left(\frac{c}{x+5+j2} \right) + \left(\frac{d}{x+5-j2} \right)$$

```
>> x = -1 + 1e-9;
>> a = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+3)*(x+5+2i)*(x+5-2i) ) * (x+1)

a =    0.8000

>> x = -3 + 1e-9;
>> b = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+3)*(x+5+2i)*(x+5-2i) ) * (x+3)

b =   -4.0000

>> x = -5-2i + 1e-9;
>> c = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+3)*(x+5+2i)*(x+5-2i) ) * (x+5+2i)

c =    1.6000 + 3.2000i

>> x = -5+2i + 1e-9;
>> d = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+3)*(x+5+2i)*(x+5-2i) ) * (x+5-2i)

d =    1.6000 - 3.2000i

>>
```