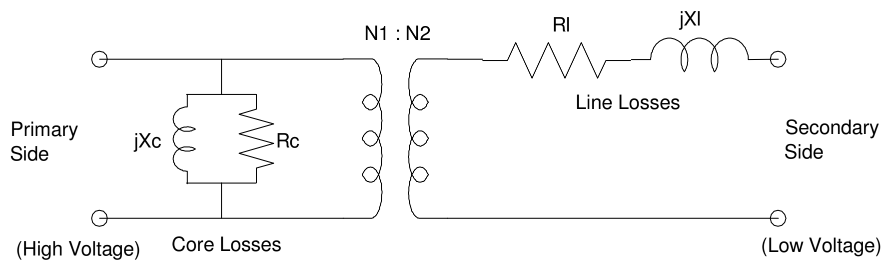
ECE 111: Homework 16

ECE 331 Energy Conversion

1) Determine the circuit model for a 13.2kV : 240V transformer is tested with the following test results:



Transformer Model

	V	Power	pf
Open-Circuit Test	V1 = 9.6kV	15 W	0.015
Short-Circuit Test	V2 = 20V	10 W	0.99

Open Circuit Test (Rc and jXc)

$$P = V \cdot I \cdot pf$$

$$15W = 9.6kV \cdot I \cdot 0.015$$

$$I = 0.1042A$$

$$Z = \left(\frac{V}{I}\right) \angle \arccos(pf)$$

$$Z = \left(\frac{9600V}{0.1042A}\right) \angle \arccos(0.015)$$

$$Z = 1382.4 + j92,150$$

This is the series model. To get the parallel model, take the inverse

$$\frac{1}{Z} = \frac{1}{R_c} + \frac{1}{jX_c}$$

In Matlab

```
>> V = 9600;
>> I = 15 / (9600*0.015)

I =    0.1042
```

```
>> Z = V/I * exp(j*acos(0.015))

Z =  1.3824e+003 +9.2150e+004i

>> 1/Z

ans =  1.6276e-007 -1.0849e-005i

>> Rc = 1 / real(1/Z)
```

`Rc =  6.1440e+006`

```
>> Xc = -1 / imag(1/Z)
```

`Xc =  9.2170e+004`

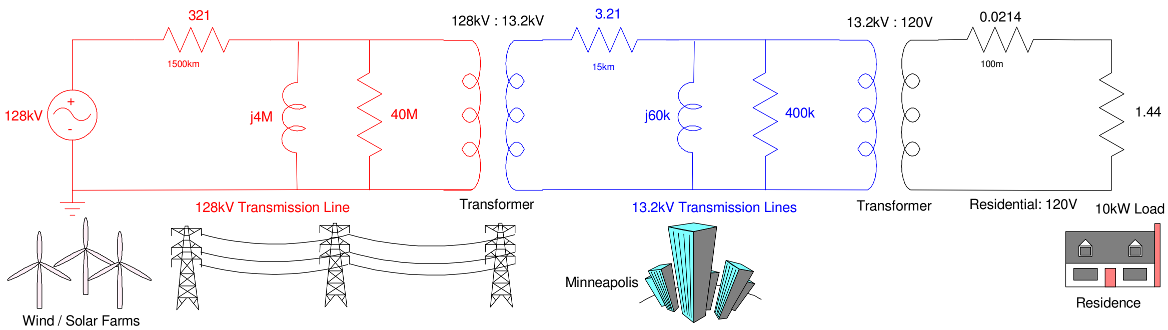
Short Circuit Test: This gives you RL and jXL

```
>> V = 20;
>> I = 10 / (20*0.99)

I =    0.5051

>> Z = V/I * exp(j*acos(0.99))
```

```
      RL       jXL
Z =  39.2040 + 5.5863i
```

For the utility grid on the back of the page....

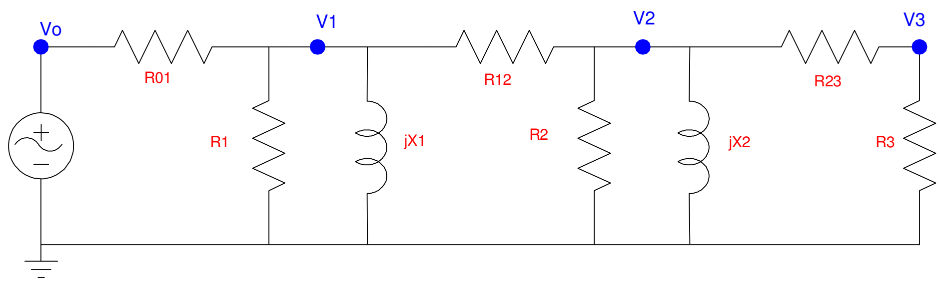
2) Convert the voltages and impeances to the 120V node (right side)



Impedances convert as the turn-ratio squared

```
>> V0 = 120;
>> R01 = 321 * (120/128000)^2

R01 =   2.8213e-004

>> R1 = 40e6 * (120/128000)^2

R1 =   35.1563

>> X1 = j*4e6 * (120/128000)^2

X1 =        0 + 3.5156i

>> R12 = 3.21*(120/13200)^2

R12 =   2.6529e-004

>> R2 = 400e3 * (120/13200)^2

R2 =   33.0579

>> X2 = j*60e3 * (120/13200)^2

X2 =        0 + 4.9587i

>> R23 = 0.0214;
>> R3 = 1.44;
>>
```

3) Write the voltage node equations for this circuit (with transformers removed)



$$V_0 = 120$$

$$\left(\frac{V_1 - V_0}{R_{01}}\right) + \left(\frac{V_1}{R_1}\right) + \left(\frac{V_1}{jX_1}\right) + \left(\frac{V_1 - V_2}{R_{12}}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R_{12}}\right) + \left(\frac{V_2}{R_2}\right) + \left(\frac{V_2}{jX_2}\right) + \left(\frac{V_2 - V_3}{R_{23}}\right) = 0$$

$$\left(\frac{V_3 - V_2}{R_{23}}\right) + \left(\frac{V_3}{R_3}\right)$$

4) Determine the voltages at each node

Group terms

$$V_0 = 120$$

$$-\left(\frac{1}{R_{01}}\right)V_0 + \left(\frac{1}{R_{01}} + \frac{1}{R_1} + \frac{1}{jX_1} + \frac{1}{R_{12}}\right)V_1 - \left(\frac{1}{R_{12}}\right)V_2 = 0$$

$$-\left(\frac{1}{R_{12}}\right)V_1 + \left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{jX_2} + \frac{1}{R_{23}}\right)V_2 - \left(\frac{1}{R_{23}}\right)V_3 = 0$$

$$-\left(\frac{1}{R_{23}}\right)V_2 + \left(\frac{1}{R_{23}} + \frac{1}{R_3}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{R_{01}}\right) & \left(\frac{1}{R_{01}} + \frac{1}{R_1} + \frac{1}{jX_1} + \frac{1}{R_{12}}\right) & \left(\frac{-1}{R_{12}}\right) & 0 \\ 0 & \left(\frac{-1}{R_{12}}\right) & \left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{jX_2} + \frac{1}{R_{23}}\right) & \left(\frac{-1}{R_{23}}\right) \\ 0 & 0 & \left(\frac{-1}{R_{23}}\right) & \left(\frac{1}{R_{23}} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
V0 = 120;
R01 = 321 * (120/128000)^2;
R1 = 40e6 * (120/128000)^2;
X1 = j*4e6 * (120/128000)^2;
R12 = 3.21*(120/13200)^2;
R2 = 400e3 * (120/13200)^2;
X2 = j*60e3 * (120/13200)^2;
R23 = 0.0214;
R3 = 1.44;

b1 = [1,0,0,0];
b2 = [-1/R01, 1/R01+1/R1+1/X1+1/R12, -1/R12, 0];
b3 = [0, -1/R12, 1/R12+1/R2+1/X2+1/R23, -1/R23];
b4 = [0,0,-1/R23, 1/R23+1/R3];
B = [b1;b2;b3;b4];
A = [120;0;0;0];
V = inv(B)*A
```

V0 120.00
V1 119.97 + 0.02i
V2 119.95 + 0.02i
V3 118.20 + 0.02i

Determine the efficiency of this system

- Ignoring the core losses
  - Assumes a large number of customers share these losses
- Including the core losses
  - Assumes a single customer

In Matlab

```
V0 = V(1);
V1 = V(2);
V2 = V(3);
V3 = V(4);

P01 = (abs(V0-V1))^2 / R01
P12 = (abs(V1-V2))^2 / R12
P23 = (abs(V2-V3))^2 / R23
P1 = (abs(V1))^2 / R1
P2 = (abs(V2))^2 / R2
P3 = (abs(V3))^2 / R3

eff1 = P3 / (P1 + P2 + P3 + P01 + P12 + P23)
eff2 = P3 / (P3 + P01 + P12 + P23)

P01 =    3.2002
P12 =    2.1041
P23 =  144.1755
P1 =  409.4284
P2 =  435.2524
P3 =  9.7015e+003
```

```
eff1 =    0.9071

eff2 =    0.9848
```

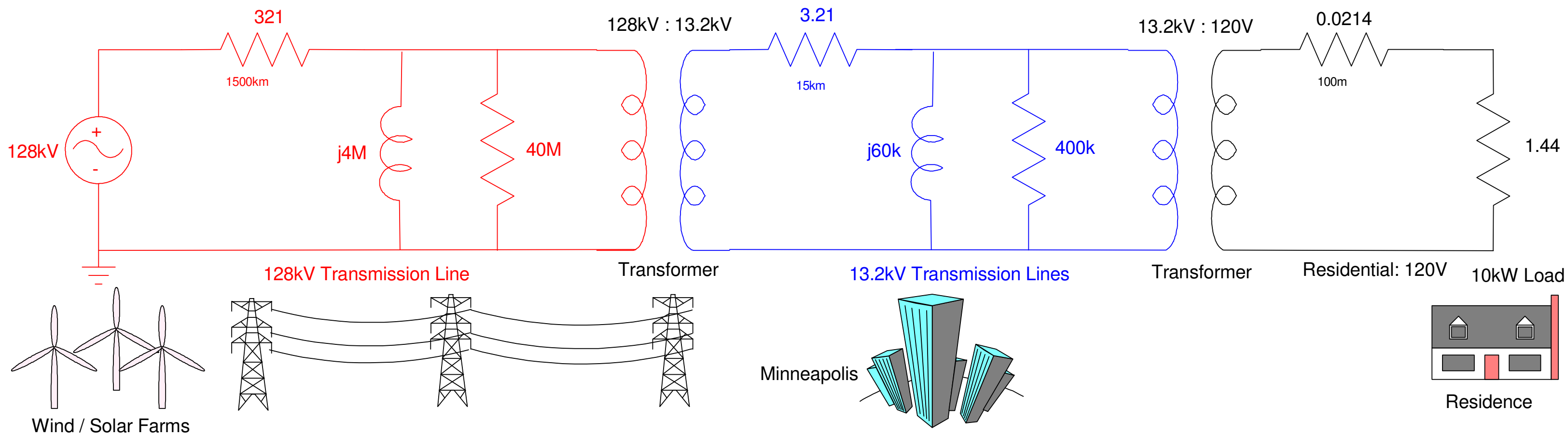
If you include everything, the power grid is 90.71% efficient

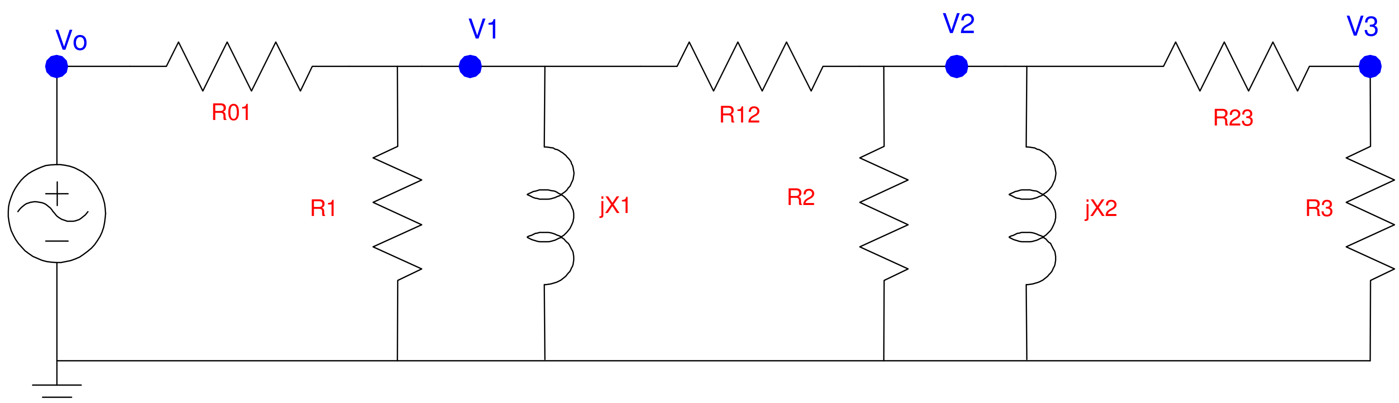
- If there is only one customer, 90.71% of the power generated gets to the customer

If you ignore the core losses, the power grid is 98.48% efficient

- If there are many customers (so the core losses are distributed across many accounts), the power grid is 98.48% efficient.

An AC power grid using Tesla's model is quite efficient. This only works for AC signals, however (which is why our power grid is 60Hz rather than Edison's DC power grid).


321
1500km
128kV
j4M
40M
128kV : 13.2kV
3.21
15km
j60k
400k
13.2kV : 120V
0.0214
100m
1.44
128kV Transmission Line
Transformer
13.2kV Transmission Lines
Transformer
Residential: 120V
10kW Load
Wind / Solar Farms
Minneapolis
Residence


Vo
V1
V2
V3
R01
R12
R23
R1
jX1
R2
jX2
R3