ECE 111 - Homework #15

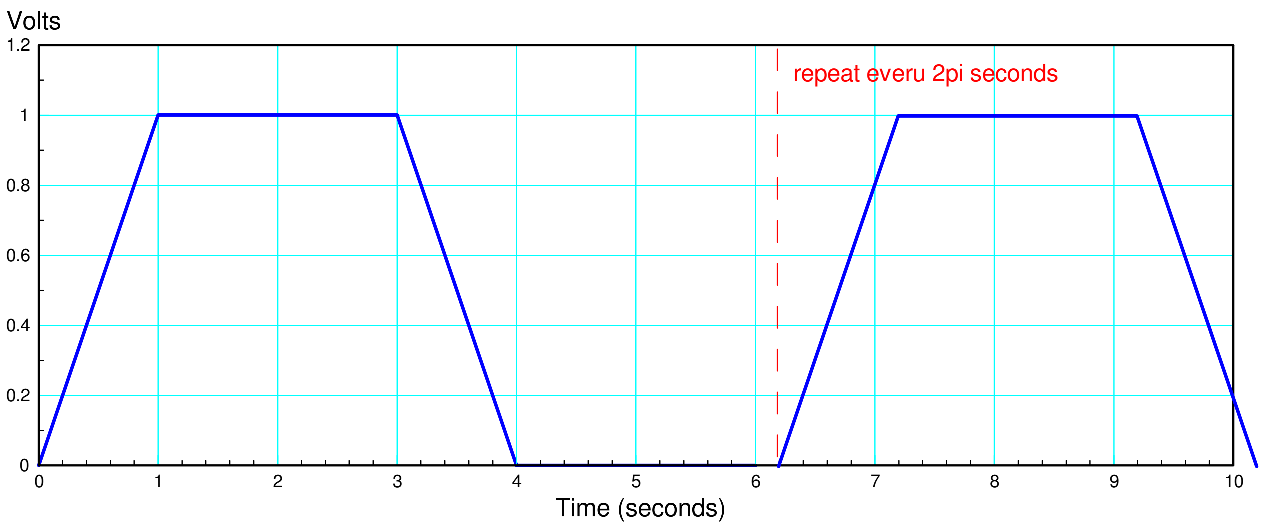
Week #15 - Signals & Frequency Content of a Signal

Problem 1-5) Let x(t) be a function which is periodic in 2π as shown below

$$x(t) = x(t + 2\pi)$$

or in Matlab:

```
t = [0:0.001:2*pi]' + 1e-6;
x = t .* (t<1)   +  1*(t>1).*(t<3) + (4-t).*(t>3).*(t<4);
plot(t,x)
```



x(t)   Note that x(t) repeats repeats every 2π seconds

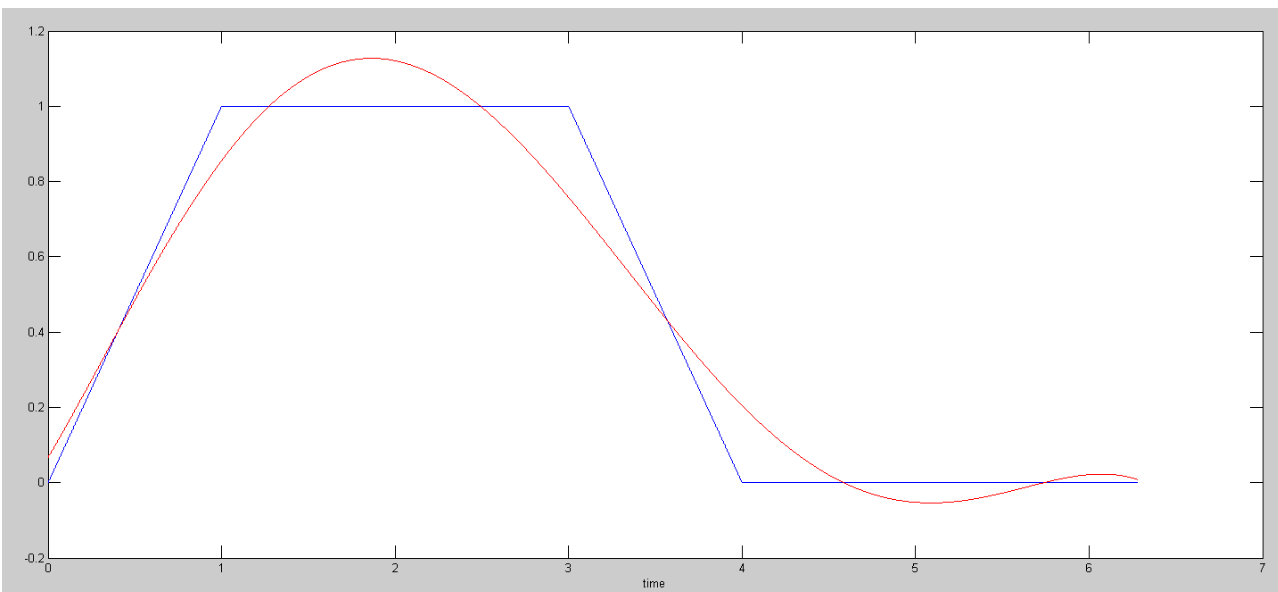
Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]' + 1e-6;
>> x = t .* (t<1)   +  1*(t>1).*(t<3) + (4-t).*(t>3).*(t<4);

>> B = [t.^0, t, t.^2, t.^3, t.^4, t.^5];
>> A = inv(B'*B)*B'*x

a0     0.0673
a1     0.7779
a2     0.2490
a3    -0.2990
a4     0.0641
a5    -0.0041

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('time');
```



Comments

- It's a curve fit
- Adding more terms will give a better approximation
- The result isn't useful: given x(t) I still don't know how to find y(t) in problem #3

$$Y = \left(\frac{1}{s^2+s+1}\right) X$$

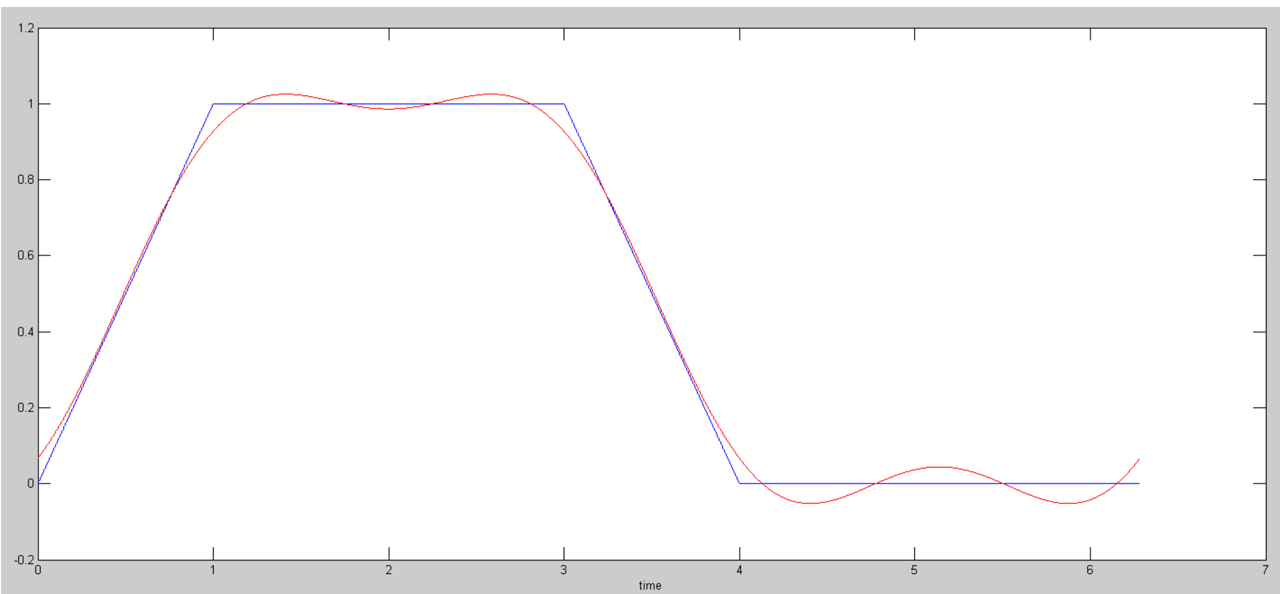
Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

Plot x(t) along with it's approximation.

```
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x

A =

a0  4.7746e-001
a1 -2.5341e-001
b1  5.5367e-001
a2 -2.4724e-002
b2 -2.8606e-002
a3 -1.3247e-001
b3  3.8544e-002

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('time');
>>
```



Comment

- It's another curve fit
- Adding more terms will give a better match
- This result *is* useful for problem #3, however. Now that x(t) is expressed in terms of sine waves, we can find y(t) using phasor analysis.

Superposition

3) Assume X and Y are related by

$$Y = \left(\frac{1}{s^2+s+1}\right)X$$

3a) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

Problem #2 used an orthogonal basis function. What this means is

- All terms can be determined separately
- Adding more terms do not change the previous results (terms are orthogonal)

Since x(t) is

$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

you can find the terms by hitting x(t) with sine waves

$$a_0 = mean(x)$$

$$a_n = 2 \cdot mean(x \cdot \cos(nt))$$

$$b_n = 2 \cdot mean(x \cdot \sin(nt))$$

The 2 term accounts for

$$mean(\cos^2(\omega t)) = \frac{1}{2}$$

This is an orthoganal bassis. If the mean of each basis element was 1.0000, it would be an orthonormal basis. You'll cover this when you take Math 129.

In Matlab, you can find the coefficients another way

```
>> a0 = mean(x)

a0 =  4.7740e-001

>> a1 = 2*mean(x .* cos(t))

a1 = -2.5336e-001

>> b1 = 2*mean(x .* sin(t))

b1 =  5.5359e-001

>> a2 = 2*mean(x .* cos(2*t))

a2 = -2.4704e-002

>> b2 = 2*mean(x .* sin(2*t))

b2 = -2.8603e-002

>> a3 = 2*mean(x .* cos(3*t))

a3 = -1.3243e-001
```

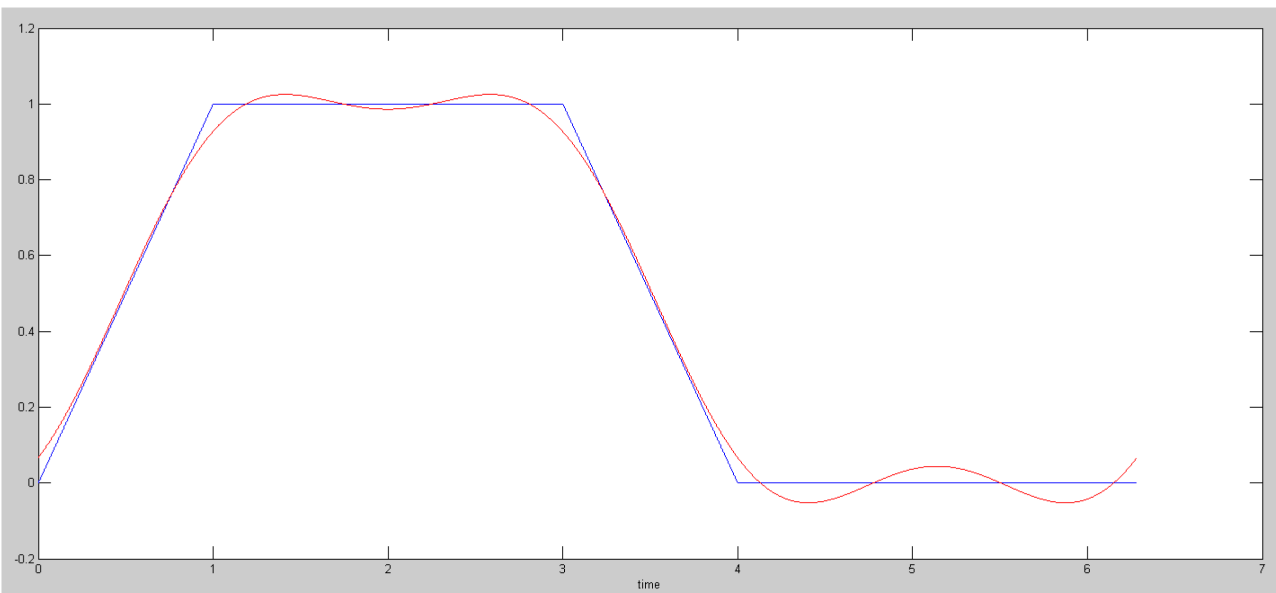
```
>> b3 = 2*mean(x .* sin(3*t))

b3 =   3.8539e-002
```

This should give you the same result as problem #2

term	a0	a1	b1	a2	b2	a3	b3
prob #2	0.4775	-0.2534	0.5537	-0.0247	-0.0286	-0.1325	0.0385
prob #3	0.4774	-0.2534	0.5536	-0.0247	-0.0286	-0.1324	0.0385

3b) Plot x(t) and its Fourier approximation taken out to 3 rad/sec

Same results as problem #2



4) Determine the output, y(t), at DC (w = 0)

```
>> s = j*0;
>> X0 = a0

X0 =    0.4774

>> Y0 = ( 1 / (s^2 + s + 1)) * X0

Y0 =    0.4774
```

$$y_0(t) = 0.4774$$

5) Determine the output, y(t), at 1 rad/sec

```
>> s = j*1;
>> X1 = a1 - j*b1

X1 =  -0.2534 - 0.5536i

>> Y1 = ( 1 / (s^2 + s + 1)) * X1

Y1 =  -0.5536 + 0.2534i
```

$$y_1(t) = -0.5536\cos(t) - 0.2534\sin(t)$$

6) Determine the output, y(t), at 2 rad/sec

```
>> s = j*2;
>> X2 = a2 - j*b2

X2 =  -0.0247 + 0.0286i

>> Y2 = ( 1 / (s^2 + s + 1)) * X2

Y2 =   0.0101 - 0.0028i
```

$$y_2(t) = 0.0101\cos(2t) + 0.0028\sin(2t)$$

7) Determine the output, y(t), at 3 rad/sec

```
>> s = j*3;
>> X3 = a3 - j*b3

X3 =  -0.1324 - 0.0385i

>> Y3 = ( 1 / (s^2 + s + 1)) * X3

Y3 =   0.0129 + 0.0097i
```

$$y_3(t) = 0.0129\cos(3t) - 0.0097\sin(3t)$$

8) Determine the total answer, y(t)

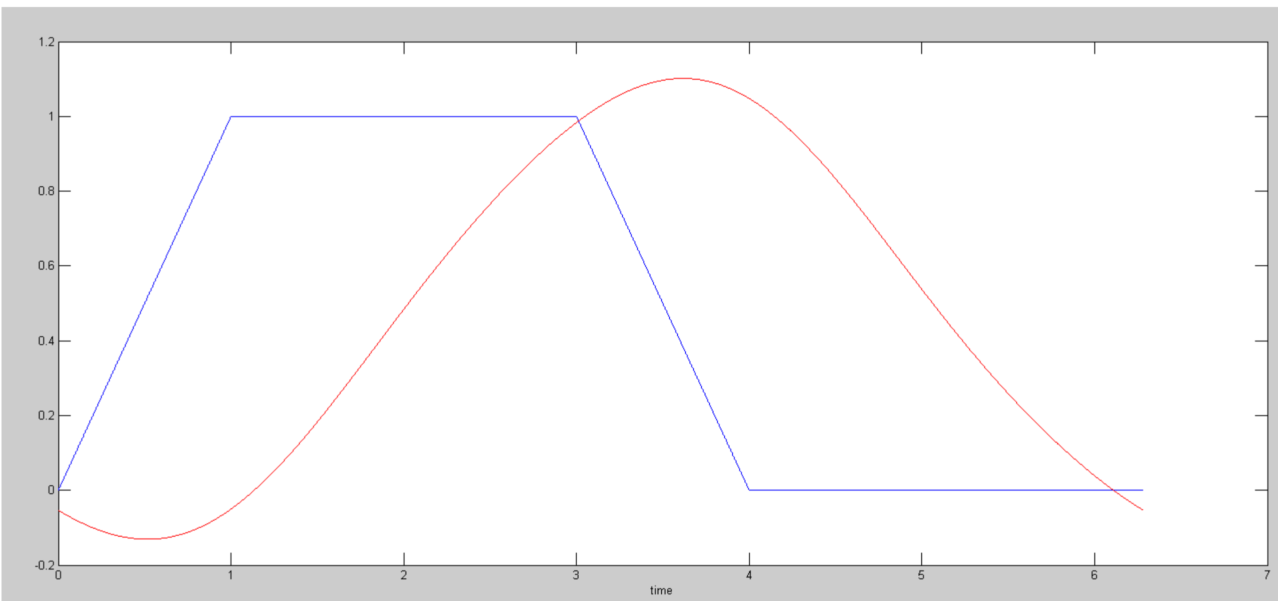
- Plot x(t) and y(t)

$$y(t) = y_0 + y_1 + y_2 + y_3$$

$$\begin{aligned} y(t) = & \quad 0.4774 \\ & -0.5536\cos(t) - 0.2534\sin(t) \\ & +0.0101\cos(2t) + 0.0028\sin(2t) \\ & +0.0129\cos(3t) - 0.0097\sin(3t) \end{aligned}$$

In Matlab

```
>> y0 = Y0;
>> y1 = real(Y1)*cos(t) - imag(Y1)*sin(t);
>> y2 = real(Y2)*cos(2*t) - imag(Y2)*sin(2*t);
>> y3 = real(Y3)*cos(3*t) - imag(Y3)*sin(3*t);
>> y = y0 + y1 + y2 + y3;
>> plot(t,x,'b',t,y,'r');
>> xlabel('time');
```



Comment:

- This is the power of Fourier transforms
- Phasor analysis only works with sine waves
- With Fourier transforms, you can find the output of a filter with any periodic input
- Fourier transforms is just curve fitting where the basis funciton consists of sine waves
- You'll do this a lot when you get to Signals and Systems (ECE 343)