ECE 111 - Homework #14

ECE 343 Signals & Systems & Filters

Filter Analysis

1) A filter has the following transfer function

$$Y = \left(\frac{100(2s+1)}{(s+3)(s+4)(s+5)}\right)X$$

1a) What is the differential equation relating X and Y?

Cross multiply and multiply out

$$(s+3)(s+4)(s+5)Y = 100(2s+1)X$$

```
>> poly([-3,-4,-5])
ans =      1     12     47     60
```

$$(s^3 + 12s^2 + 47s + 60)Y = (200s + 100)X$$

Note that 'sY' means 'the derivative of y(t)'

$$\boxed{y''' + 12y'' + 47y' + 60y = 200x' + 100x}$$

1b) Find y(t) assuming $x(t) = 4$

Use phasor analysis

$$s = j0$$

$$X = 4$$

$$Y = \left(\frac{100(2s+1)}{(s+3)(s+4)(s+5)}\right)_{s=0} \cdot (4)$$

In Matlab

```
>> s = 0i;
>> X =4;
>> Y = 100*(2*s+1) / ( (s+3)*(s+4)*(s+5) ) * X

Y =     6.6667
```

$$y(t) = 6.6667$$

1c) Find y(t) assuming $x(t) = 5\sin(6t)$

Use phasor analysis

$$s = j6$$

$$X = 0 - j5$$

$$Y = \left(\frac{100(2s+1)}{(s+3)(s+4)(s+5)} \right)_{s=j6} \cdot (0 - j5)$$

in Matlab

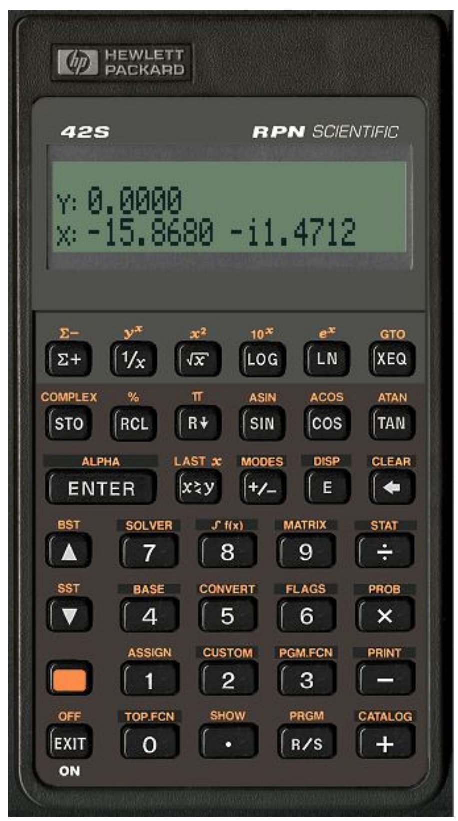
```
>> s = 6i;
>> X =0 - 5i;
>> Y = 100*(2*s+1) / ( (s+3)*(s+4)*(s+5) ) * X

Y = -15.8680 - 1.4712i
```

or

$$y(t) = -15.868\cos(6t) + 1.4712\sin(6t)$$

HP42

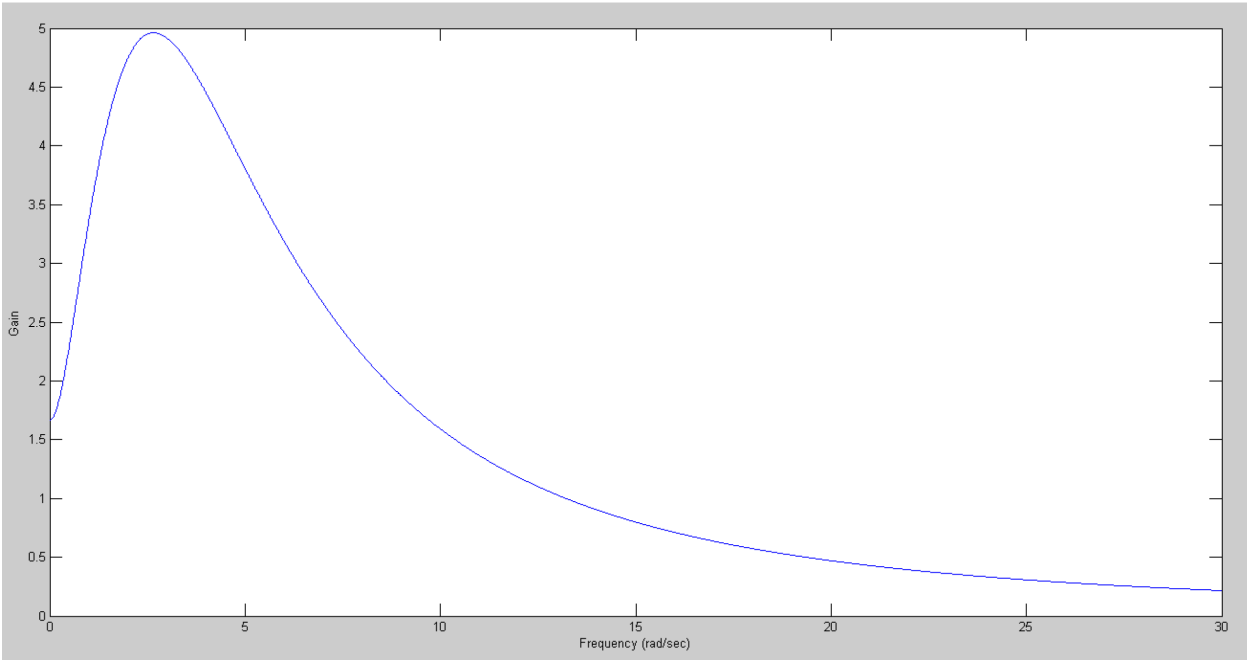
```
0
enter
6
STO
alpha
s
alpha
(this creates a complex variable 's')

2
RCL s
*
1
+
100
*
RCL s
3
+
/
RCL s
4
+
/
RCL s
5
+
/
0
enter
-5
complex
*
```



2) Plot the gain vs. frequency for this filter from 0 to 30 rad/sec.

$$Y = \left(\frac{100(2s+1)}{(s+3)(s+4)(s+5)} \right) X$$

```
>> w = [0:0.01:30]';
>> s = j*w;
>> G = 100*(2*s+1) ./ ( (s+3).*(s+4).*(s+5) );
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



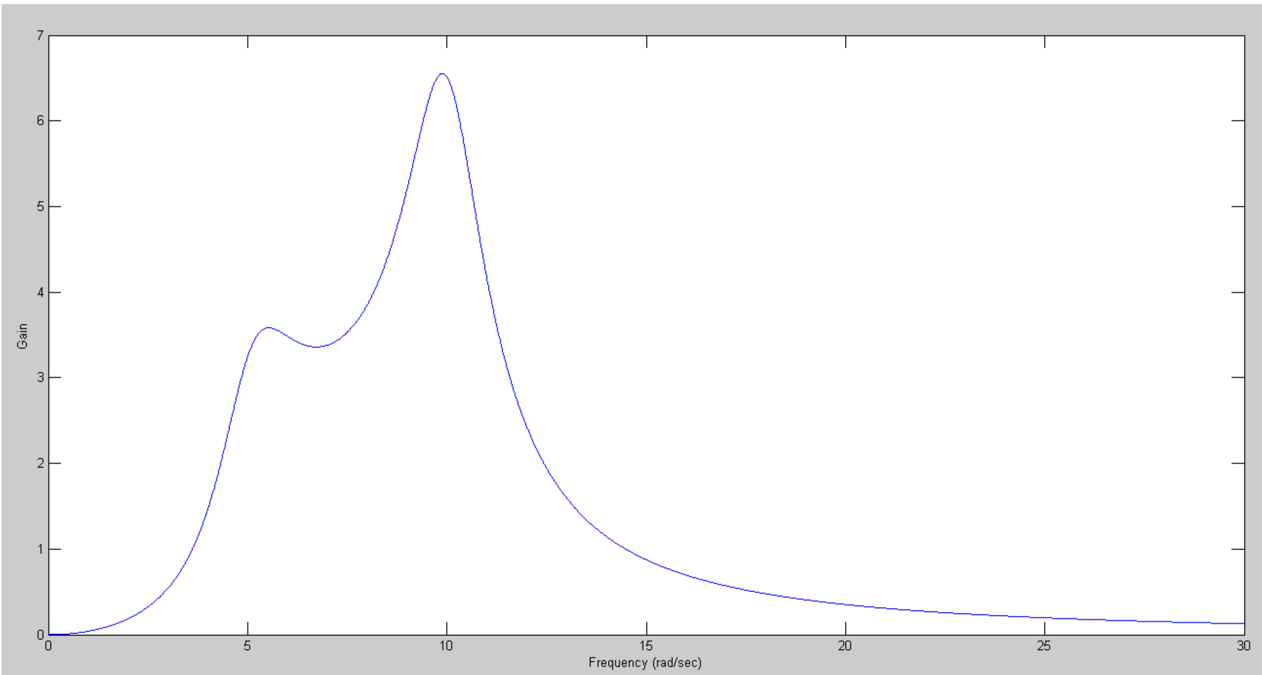
Comment

- This is low-pass filter
- At DC, the gain is 1.6 (problem 1b)
- At 6 rad/sec, the gain increases to 3.5 (problem 1c)
- The peak gain is about 5 at 2.5 rad/sec
- Then the gain drops as frequency goes up

3) Plot the gain vs. frequency for this filter from 0 to 30 rad/sec.

$$Y = \left(\frac{100s^2}{\left(s^2+2s+26\right)\left(s^2+2s+101\right)} \right) X$$

```
>> w = [0:0.01:30]';
>> s = j*w;
>> G = 100 * s.^2 ./ ( (s.^2 + 2*s + 26).*(s.^2 + 2*s + 101) );
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



Comment

- This is a band-pass filter
- The DC gain is zero
- The high-frequency gain goes to zero
- Frequencies between 2 and 15 rad/sec are passed
- Maximum gain is 6.5 at around 7 rad/sec

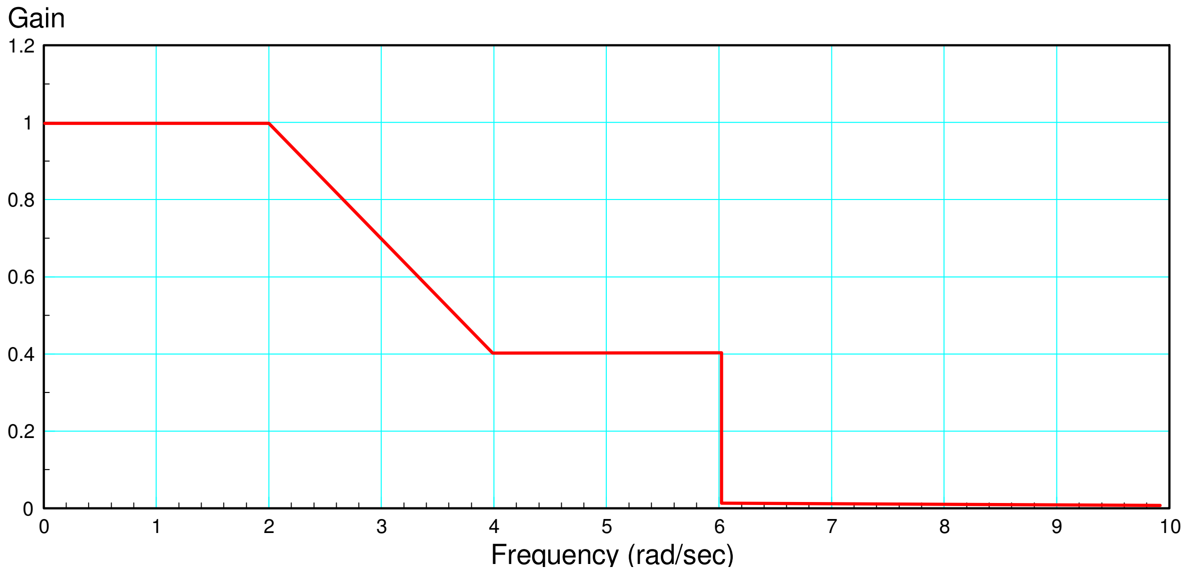
More comments:

- Filter analysis is pretty easy if you don't mind using complex numbers

Filter Design

Problem 4-6) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{a}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$



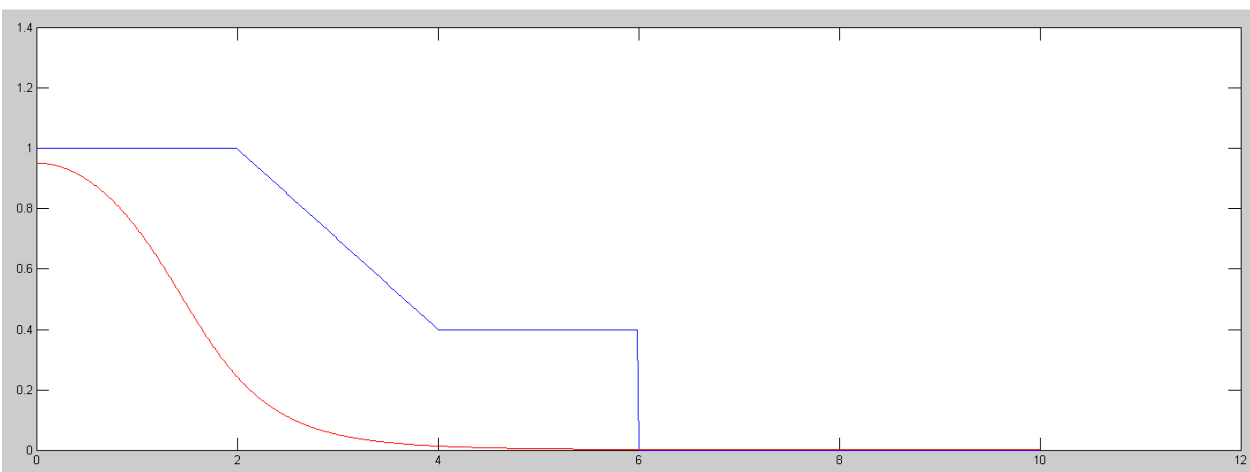
4) Write an m-file, cost.m, which

- Is passed an array, z, with each element representing (a, b, c, d, e, f, g)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

```
function [ J ] = costf( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);
 g = z(7);
 w = [0:0.01:10]' + 1e-6;
 s = j*w;
 g1 = 1 * (w < 2);
 g2 = (-0.3*w + 1.6).*(w>2).*(w<4);
 g3 = 0.4 * (w>4).*(w<6);
 Gideal = g1 + g2 + g3;
 G = a ./ ( (s.^2 + b*s + c) .* (s.^2 + d*s + e ).*(s.^2 + f*s + g) );
 G = abs(G);
 E = abs(Gideal) - abs(G);
 J = sum(E .^ 2);
 plot(w,Gideal,w,abs(G),'r');
 ylim([0,1.4]);
 pause(0.01);
end
```

Testing this code, from the command window

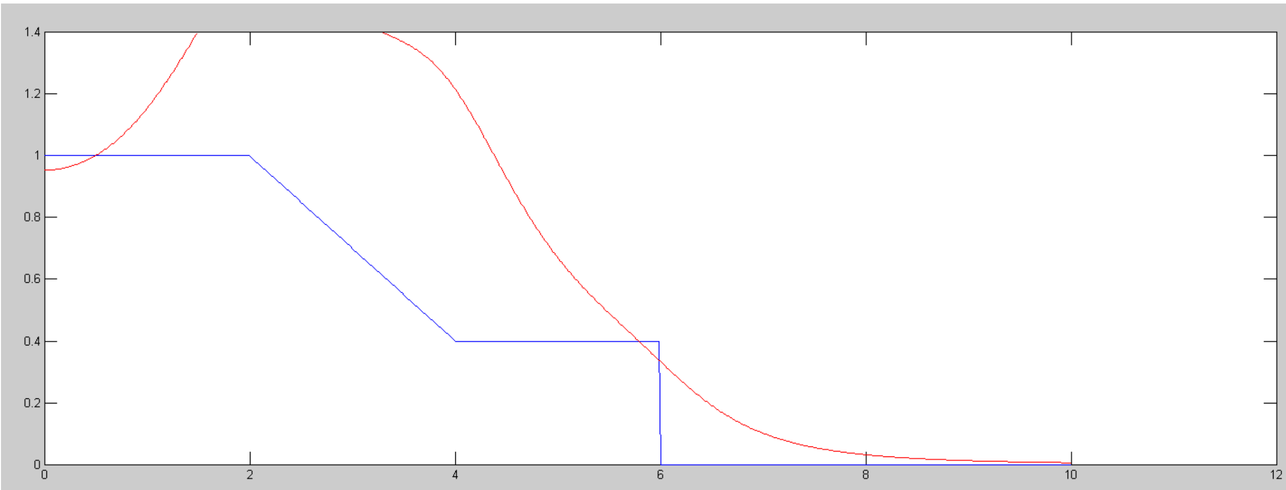
```
>> costf([100,2,3,4,5,6,7])

ans =  142.5987
```



5) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right) = \left(\frac{3000}{\left(s^2+2s+5\right)\left(s^2+2+17\right)\left(s^2+2s+37\right)} \right)$$

```
>> costf([3000,2,5,2,17,2,37])

ans =   172.4221
```



6) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$

a) Give the resulting (a, b, c, d, e, f, g)

```
>> [Z,e] = fminsearch('costf',[3000,2,5,2,17,2,37])

Exiting: Maximum number of function evaluations has been exceeded
         - increase MaxFunEvals option.
         Current function value: 1.907511

          a          b          c          d          e          f          g
Z =    1935.1     5.3539     8.2284     2.5951     7.7445     0.8267    30.9399

e =     1.9075
```

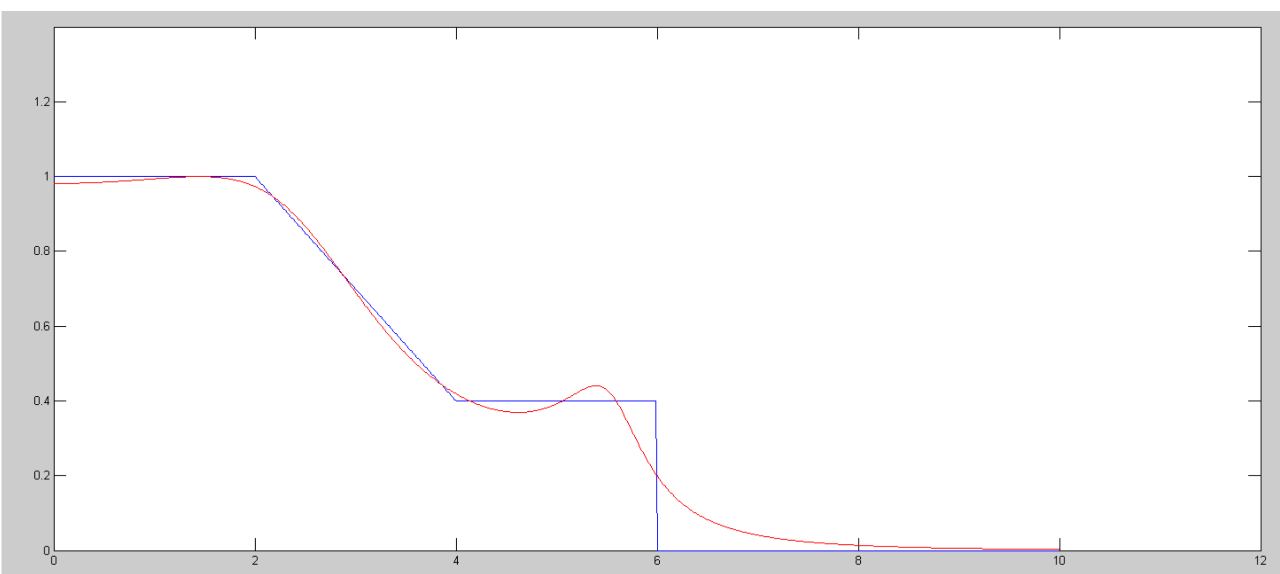
Try to improve the results by continuing where fminsearch left off...

```
>> [Z,e] = fminsearch('costf',Z)

        a          b          c          d          e          f          g
    1462.5     3.4278     4.8084     2.7771     9.9343     0.7309    31.0694

e =     1.7905
```

b) Give the resulting filter, and

$$G(s) = \left(\frac{1462.5}{\left(s^2+3.42s+4.808\right)\left(s^2+2.777s+9.9343\right)\left(s^2+0.7309s+31.069\right)} \right)$$

c) Plot the 'optimal' filter's gain vs. frequency



Comments

- You'll cover filter design when you take Electronics and Signals and Systems
- Without any background on filter design, you can design some pretty good filters using Matlab and fminsearch()
- fminsearch() is a really useful funciton