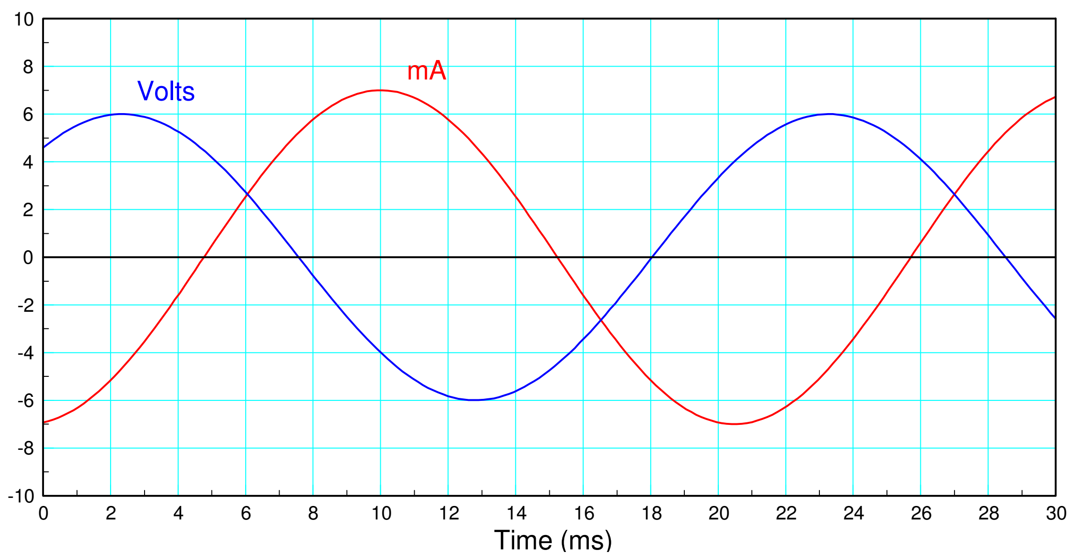
ECE 111 - Homework #13

ECE 311 Circuits II - Phasors

Phasor Voltages

1) Express V and I as phasors (i.e. as complex numbers)

- From this, determine the impedance, Z = V/I



Polar form is easier with waveforms

- The magnitude is the peak voltage
- The phase is the delay of the peak

Current (red):

- Peak = 7.0mA
- Delay to peak= 10ms
- Period = 21ms
- $\phi = -\left(\frac{10ms}{21ms}\right)360^0 = -171.4^0$ $\qquad I = 7.0\angle -171.4^0$ mA

Voltage (blue)

- Peak = 6mA
- Delay to peak = 2.2ms
- Period = 21ms (same as voltage)
- $\phi = -\left(\frac{2.2ms}{21ms}\right)360^0 = -37.7^0$ $\qquad V = 6.0\angle -37.7^0$ Volts

- Impedance

$$Z = \frac{V}{I}$$

$$Z = \left(\frac{6.0\angle{-37.7^0} V}{7.0\angle{-171.4^0} mA} \right)$$

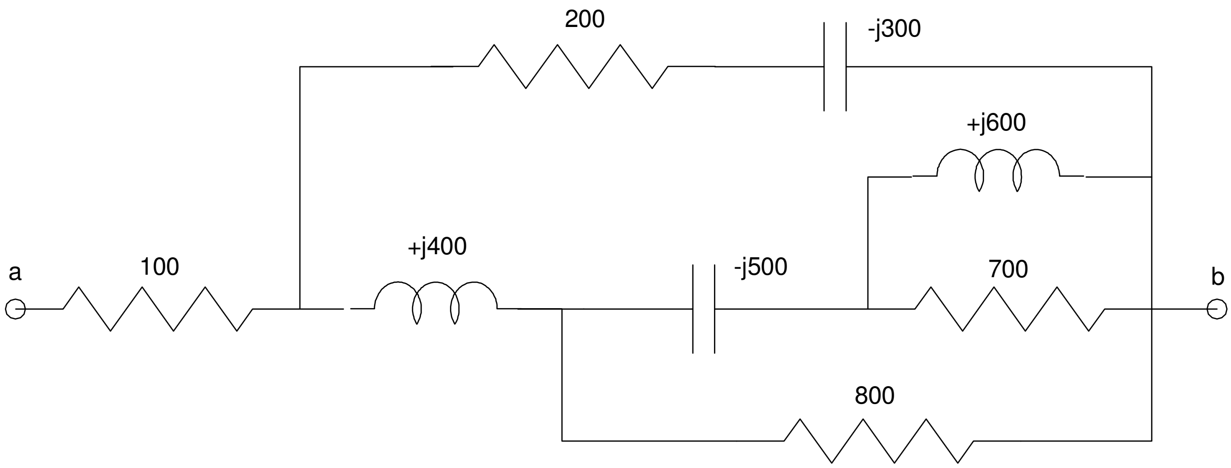
$$Z = 857.2\angle + 133.7^0$$

or in Matlab

```
>> V = 6*exp(-j*37.7*pi/180)

V =   4.7473 - 3.6692i

>> I = 7e-3 * exp(-j*171.4*pi/180)

I =  -0.0069 - 0.0010i

>> Z = V / I

Z = -5.9218e+002 +6.1969e+002i
```

Phasor Impedances

2) Determine the impedance, Zab



In Matlab

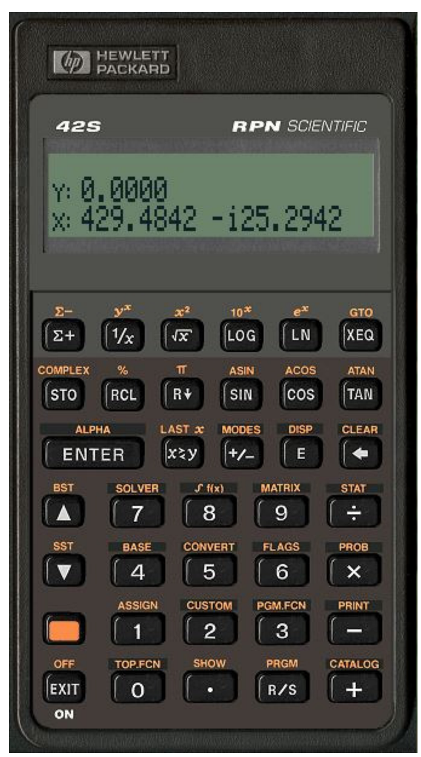
```
>> Z1 = 1 / ( 1/700 + 1/600i)
Z1 =  2.9647e+002 +3.4588e+002i

>> Z2 = Z1 - 500i
Z2 =  2.9647e+002 -1.5412e+002i

>> Z3 = 1 / (1/Z2 + 1/800)
Z3 =  2.2762e+002 -8.0453e+001i

>> Z4 = Z3 + 400i
Z4 =  2.2762e+002 +3.1955e+002i

>> Z5 = 1 / (1/Z4 + 1/(200-300i))
Z5 =  3.2948e+002 -2.5294e+001i

>> Zab = Z5 + 100
```
Zab = 4.2948e+002 -2.5294e+001i

HP42

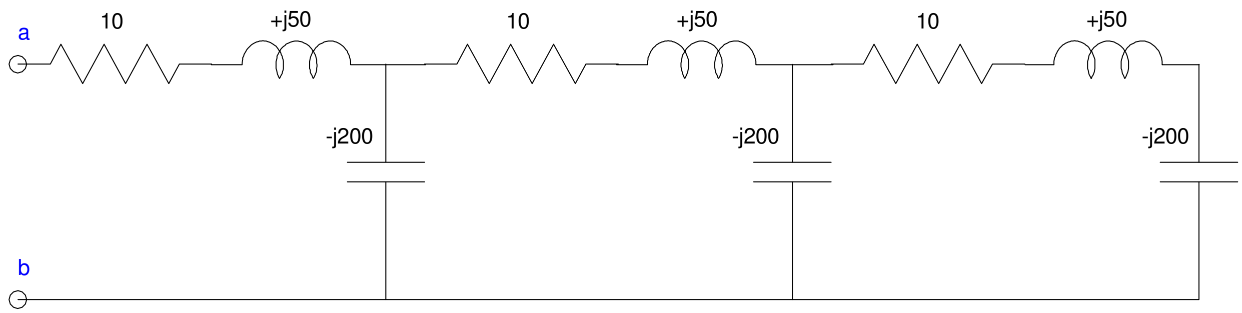
```
700
1/x
0
enter
600
complex
1/x
+
1/x

0
enter
-500
complex
+
```

```
1/x
900
1/x
+
1/x

0
enter
400
complex
+
```

```
1/x
200
enter
-300
compelx
1/x
+
1/x

100
+
```



3) Determine the impedance, Zab

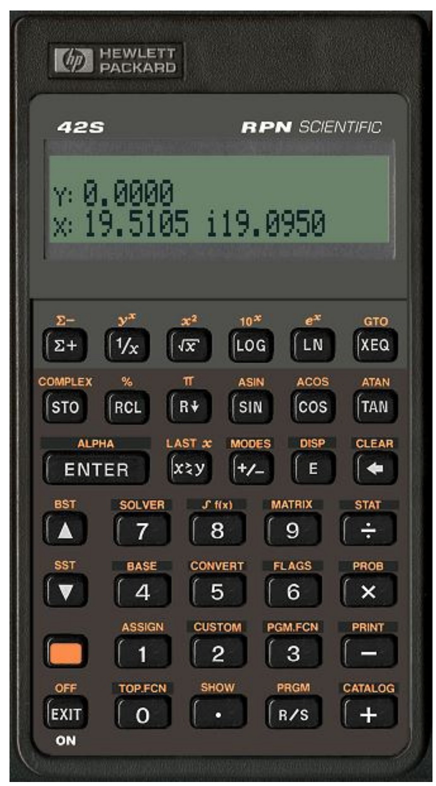


In Matlab

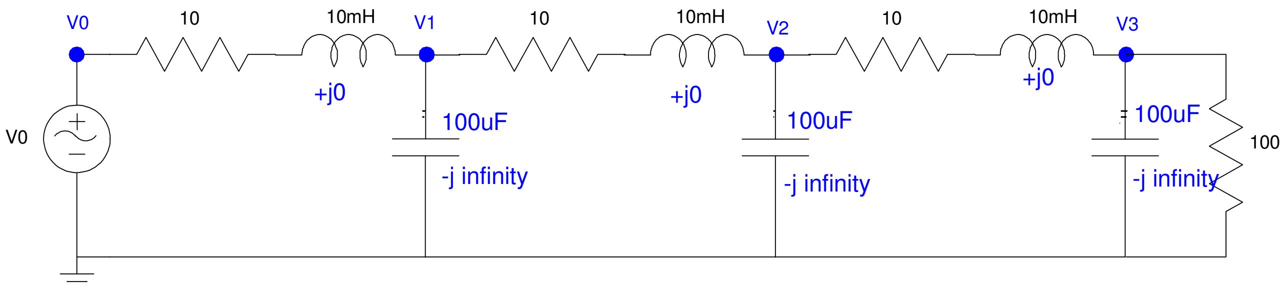
```
>> Z1 = 10 + 50i - 200i
Z1 =  1.0000e+001 -1.5000e+002i

>> Z2 = 1 / (1/Z1 + 1/(-200i))
Z2 =   3.2626 -85.8075i

>> Z3 = Z2 + 10 + 50i
Z3 =  13.2626 -35.8075i

>> Z4 = 1 / (1/Z3 + 1/(-200i))
Z4 =   9.5105 -30.9050i

>> Zab = Z4 + 10 + 50i
Zab =  19.5105 +19.0950i
```

HP42

```
10                10
enter             enter
50                50
complex           complex
                  +
0
enter             1/x
-200              0
complex           enter
+                 200
                  compelx
1/x               1/x
0                 +
enter             1/x
200
complex           0
1/x               enter
+                 50
1/x               complex
                  +
```



Voltage Nodes with Phasors



4) Assume V0 = 10.

a) Determine the impedances of each element at 0 rad/sec

$$L \rightarrow j\omega L = j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j\infty$$

b) Write the votlage node equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1 - V_2}{10}\right) = 0$$

$$\left(\frac{V_2 - V_1}{10}\right) + \left(\frac{V_2 - V_3}{10}\right) = 0$$

$$\left(\frac{V_3 - V_2}{10}\right) + \left(\frac{V_3}{100}\right) = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-0.1V_0 + 0.2V_1 - 0.1V_2 = 0$$

$$-0.1V_1 + 0.2V_2 - 0.1V_3 = 0$$

$$-0.1V_2 + 0.11V_3 = 0$$

Place in matrix form

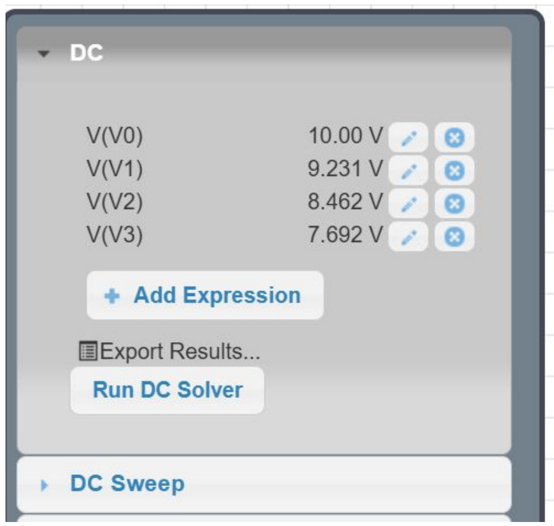
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.1 & 0.2 & 0.1 & 0 \\ 0 & -0.1 & 0.2 & 0.1 \\ 0 & 0 & -0.1 & 0.11 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

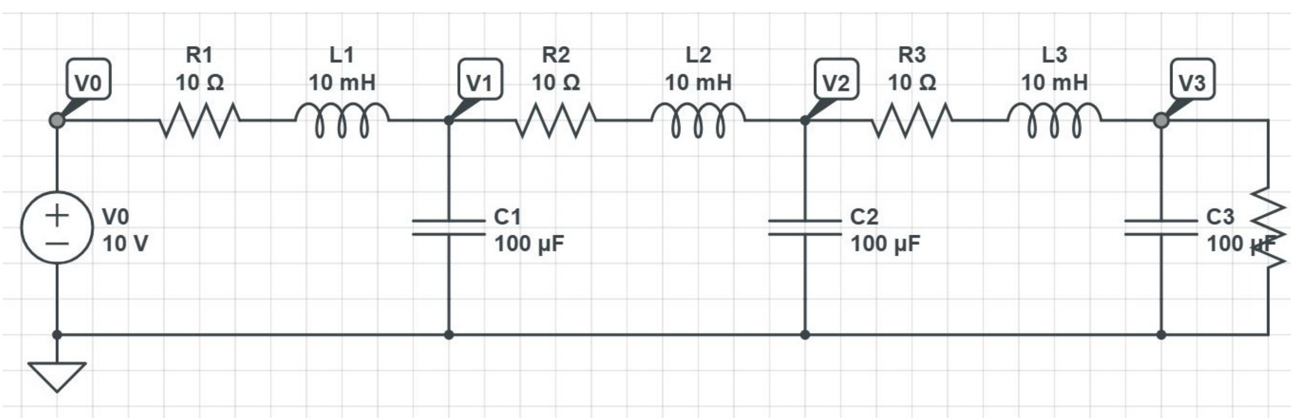
Solve using Matlab

```
>> B = [1,0,0,0;-0.1,0.2,-0.1,0;0,-0.1,0.2,-0.1;0,0,-0.1,0.11]

    1.0000         0         0         0
   -0.1000    0.2000   -0.1000         0
         0   -0.1000    0.2000   -0.1000
         0         0   -0.1000    0.1100

>> A = [10;0;0;0]

    10
     0
     0
     0

>> V = inv(B)*A

V0   10.0000
V1    9.2308
V2    8.4615
V3    7.6923
```

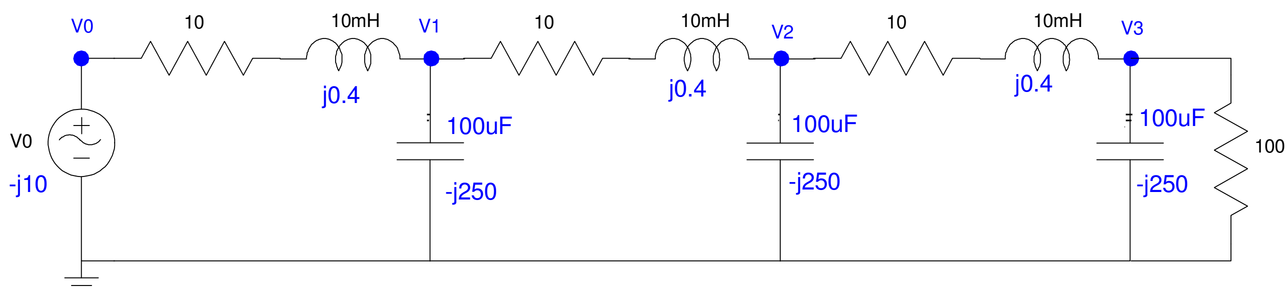
5) Check your results in CircuitLab

- The results match





6) Assume $V_0 = 10\sin(40t)$ *10V, 40 rad/sec sine wave (6.36Hz)*



a) Determine the impedances of each element at 40 rad/sec

$$V = 0 - j10$$

$$L \rightarrow j\omega L$$
$$= j \cdot 40 \cdot 0.01$$
$$= j0.4$$

$$C \rightarrow \frac{1}{j\omega C}$$
$$= \frac{1}{j \cdot 40 \cdot 100e{-6}}$$
$$= -j250$$

b) Write the votlage node equations

$$V_0 = -j10$$

$$\left(\frac{V_1 - V_0}{10+j0.4}\right) + \left(\frac{V_1}{-j250}\right) + \left(\frac{V_1 - V_2}{10+j0.4}\right) = 0$$

$$\left(\frac{V_2 - V_1}{10+j0.4}\right) + \left(\frac{V_2}{-j250}\right) + \left(\frac{V_2 - V_3}{10+j0.4}\right) = 0$$

$$\left(\frac{V_3 - V_2}{10+j0.4}\right) + \left(\frac{V_3}{-j250}\right) = 0$$

c) Solve for V1, V2, and V3 as complex numbers

Group terms and simplify

$$V_0 = -j10$$

$$-\left(\frac{1}{10+j0.4}\right)V_0 + \left(\left(\frac{1}{10+j0.4}\right) + \left(\frac{1}{-j250}\right) + \left(\frac{1}{10+j0.4}\right)\right)V_1 - \left(\frac{1}{10+j0.4}\right)V_2 = 0$$

$$-\left(\frac{1}{10+j0.4}\right)V_1 + \left(\left(\frac{1}{10+j0.4}\right) + \left(\frac{1}{-j250}\right) + \left(\frac{1}{10+j0.4}\right)\right)V_2 - \left(\frac{1}{10+j0.4}\right)V_3 = 0$$

$$-\left(\frac{1}{10+j0.4}\right)V_2 + \left(\left(\frac{1}{10+j0.4}\right) + \left(\frac{1}{-j250}\right)\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10+j0.4}\right) & \left(\left(\frac{2}{10+j0.4}\right)+\left(\frac{1}{-j250}\right)\right) & \left(\frac{-1}{10+j0.4}\right) & 0 \\ 0 & \left(\frac{-1}{10+j0.4}\right) & \left(\left(\frac{2}{10+j0.4}\right)+\left(\frac{1}{-j250}\right)\right) & \left(\frac{-1}{10+j0.4}\right) \\ 0 & 0 & \left(\frac{-1}{10+j0.4}\right) & \left(\left(\frac{1}{10+j0.4}\right)+\left(\frac{1}{-j250}\right)\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
a1 = [1,0,0,0];
a2 = [-1/(10+0.4j),2/(10+0.4j)+1/(-250j),-1/(10+0.4j),0];
a3 = [0, -1/(10+0.4j),2/(10+0.4j)+1/(-250j),-1/(10+0.4j)];
a4 = [0,0,-1/(10+0.4j),1/(10+0.4j)+1/(-250j)];
A = [a1;a2;a3;a4]

>> B = [-10i;0;0;0];

V = inv(A)*B

V0         0 -10.0000i
V1  -1.1737 - 9.8279i
V2  -1.9524 - 9.6871i
V3  -2.3405 - 9.6088i
```

meaning

```
V0 = 10 sin(40t)
V1  -1.1737 cos(40t) + 9.8279 sin(40t)
V2  -1.9524 cos(40t) + 9.68761 sin(40t)
V3  -2.3405 cos(40t) + 9.6088 sin(40t)
```

In polar form, the magnitude is

```
>> abs(V)

V0   10.0000
V1    9.8978
V2    9.8819
V3    9.8898
```

The phase shift in degrees is

```
>> angle(V)*180/pi

   -90.0000
   -96.8103
  -101.3952
  -103.6894
```

7) Check your results in CircuitLab using a transient simulation for 500ms (time step = 5us).

Use a sinusoidal source for V0

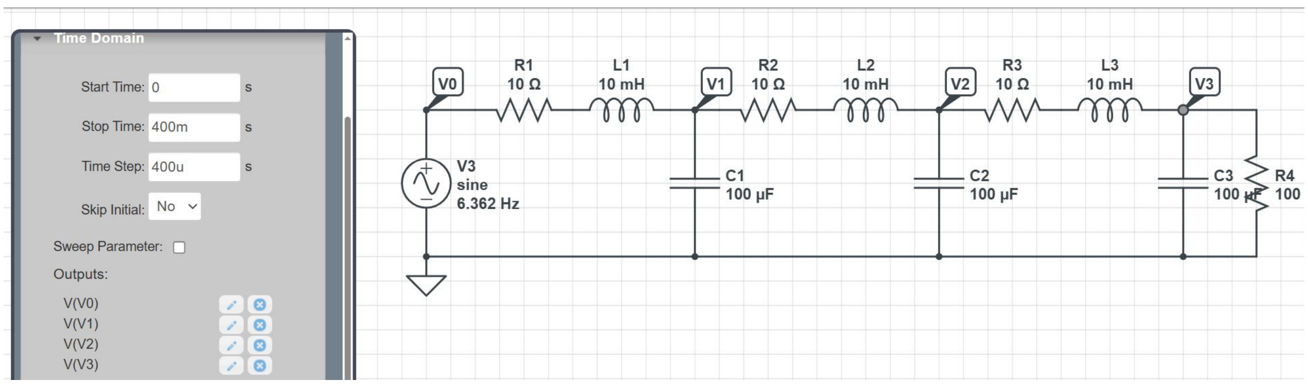
- Amplitude = 10 Volts
- Frequency = 6.3662Hz

$$\omega = 2\pi f$$

$$f = \frac{40}{2\pi} = 6.3622 \text{ Hz}$$

- Delay = 0 degrees (CircuitLab uses sine waves rather than cosine waves)

Run a time-domain simulation for a couple cycles



This shows the waveforms

- Amplitudes match calculations
- Phase shift (delays) match calculations
- AC circuit analysis is just like DC analysis, only you're using complex numbers