

ECE 111 - Homework #12

Complex Numbers

Complex Numbers

1) Determine the rectangular or polar form of each complex number

- note: Matlab works with complex numbers just like real numbers
- Default angles are in radians with Matlab
- Use Modes Rect / Polar to switch back and forth with an HP42

a) $3 + j5$

```
>> A = 3 + 5i;  
>> abs(A)  
ans = 5.8310
```

```
>> angle(A)  
ans = 1.0304
```

$$3 + j5 = 5.831 \angle 1.0304 \text{ radians}$$

b) $7 - j6$

```
>> B = 7 - 6i;  
>> abs(B)  
ans = 9.2195
```

```
>> angle(B)  
ans = -0.7086
```

$$7 - j6 = 9.2195 \angle -0.7086 \text{ radians}$$

c) $6 \angle 75^\circ$

```
>> C = 6 * exp(j*75*pi/180)
```

```
C = 1.5529 + 5.7956i
```

d) $8 \angle -120^\circ$

```
>> D = 8 * exp(j*(-120)*pi/180)
```

```
D = -4.0000 - 6.9282i
```

2) Determine y as a complex number

a) $y = (3 + j5) + (7 - j6) + (2 + j7)$

```
>> y = (3 + 5i) + (7 - 6i) + (2 + 7i)
```

y = 12.0000 + 6.0000i

With an HP42 calculator

```
modes
rect
3
enter
5
complex
7
enter
-6
complex
+
2
enter
7
complex
+
```

b) $y = (7\angle 35^\circ) + (8\angle 65^\circ) + (9\angle -22^\circ)$

```
>> b1 = 7 * exp(j*35*pi/180)
b1 = 5.7341 + 4.0150i
```

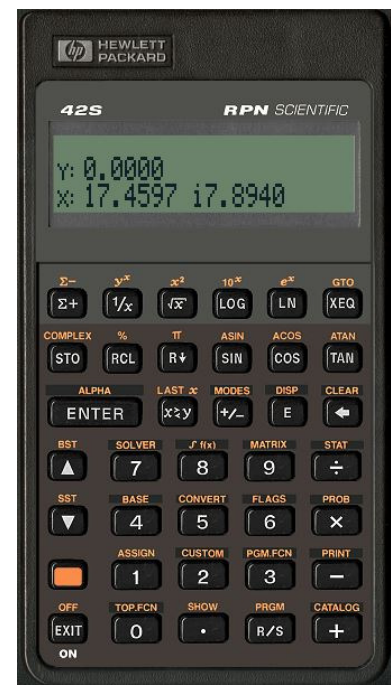
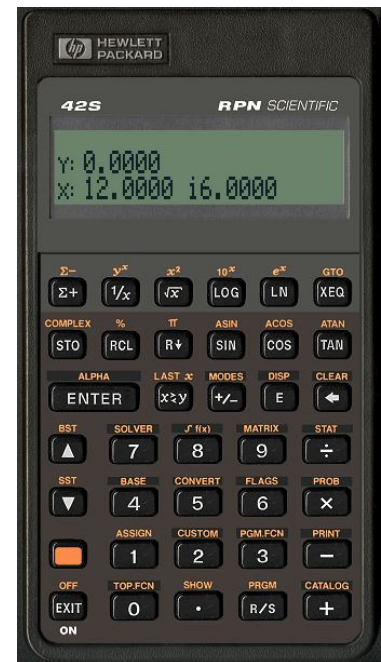
```
>> b2 = 8 * exp(j*65*pi/180)
b2 = 3.3809 + 7.2505i
```

```
>> b3 = 9 * exp(-j*22*pi/180)
b3 = 8.3447 - 3.3715i
```

```
>> B = b1 + b2 + b3
B = 17.4597 + 7.8940i
```

With an HP calculator

```
modes
polar
7
enter
35
complex
8
enter
65
complex
+
9
enter
-22
complex
+
modes
rect
```



3) Determine y as a complex number

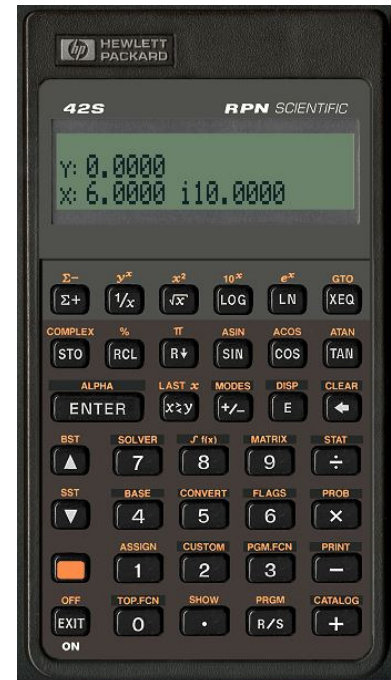
$$a) y = \left(\frac{(3+j5)(4-j6)}{(2-j3)} \right)$$

```
>> y = (3+5i)*(4-6i) / (2-3i)
```

y = 6.0000 +10.0000i

On an HP42 calculator

```
modes
rect
3
enter
5
complex
4
enter
-6
complex
*
2
enter
-3
complex
/
```



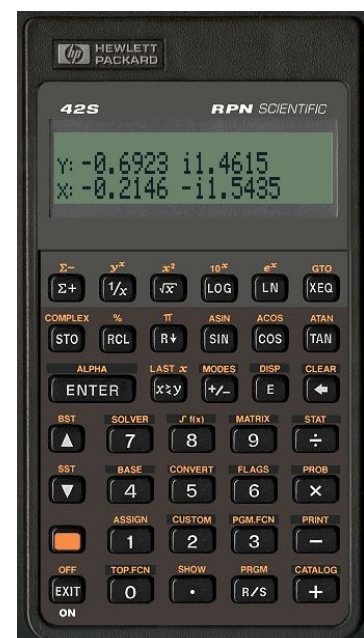
$$b) y = \left(\left(\frac{3+j5}{2-j3} \right) + \left(\frac{4-j6}{1+j3} \right) \right) \left(\frac{2+j4}{6-j1} \right)$$

```
>> y = ( (3+5i)/(2-3i) + (4-6i)/(1+3i) ) * ( (2+4i)/(6-1i) )
```

y = -0.2146 - 1.5435i

With an HP42

```
3          2
enter      enter
5          4
complex    complex
2          6
enter      enter
-3         -1
complex    complex
/          /
4          *
enter
-6
complex
1
enter 3
complex
/
+
```



4) Determine y as a complex number

$$y = e^{(1+j3)}$$

```
>> a = exp(1 + 3i)
```

```
a = -2.6911 + 0.3836i
```

$$y = \ln(1 + j3)$$

```
>> b = log(1 + 3i)
```

```
b = 1.1513 + 1.2490i
```

$$y = (1 + j3)^{(2+j4)}$$

```
>> c = (1+3i)^(2+4i)
```

```
c = 0.0461 + 0.0495i
```

note:

- Functions of complex numbers usually make sense and result in complex numbers
- Some calculators can do these operations (HP42 for example)
- Matlab can do these calculations

Partial Fractions with Complex Numbers

5) Determine the partial fraction expansion

$$\left(\frac{3(x+1)(x+2)}{(x+j4)(x-j4)(x+5)} \right) = \left(\frac{a}{x+j4} \right) + \left(\frac{b}{x-j4} \right) + \left(\frac{c}{x+5} \right)$$

Using the cover-up method

$$a = \left(\frac{3(x+1)(x+2)}{(x-j4)(x+5)} \right)_{x=-j4}$$

```
>> x = -4i;  
>> a = 3*(x+1)*(x+2) / ((x-4i)*(x+5))
```

a = 1.0610 - 0.2012i

$$b = \left(\frac{3(x+1)(x+2)}{(x+j4)(x+5)} \right)_{x=j4}$$

```
>> x = 4i;  
>> b = 3*(x+1)*(x+2) / ((x+4i)*(x+5))
```

b = 1.0610 + 0.2012i

$$c = \left(\frac{3(x+1)(x+2)}{(x+j4)(x-j4)} \right)_{x=-5}$$

```
>> x = -5;  
>> c = 3*(x+1)*(x+2) / ((x+4i)*(x-4i))
```

c = 0.8780

Note:

- Everything you do with real numbers also works with complex numbers
- It really helps to have a calculator which can work with complex numbers
- Matlab works with complex numbers
- So does the HP42 (free42)

6) Determine the partial fraction expansion

$$\left(\frac{8(x+j2)(x-j2)}{(x+1)(x+3)(x+5+j)(x+5-j)} \right) = \left(\frac{a}{x+1} \right) + \left(\frac{b}{x+3} \right) + \left(\frac{c}{x+5+j} \right) + \left(\frac{d}{x+5-j} \right)$$

Using the cover-up method

$$a = \left(\frac{8(x+j2)(x-j2)}{(x+3)(x+5+j)(x+5-j)} \right)_{x=-1}$$

```
>> x = -1;
>> a = 8*(x+2i)*(x-2i) / ((x+3)*(x+5+j)*(x+5-j))
```

a = 1.1765

$$b = \left(\frac{8(x+j2)(x-j2)}{(x+1)(x+5+j)(x+5-j)} \right)_{x=-3}$$

```
>> x = -3;
>> b = 8*(x+2i)*(x-2i) / ((x+1)*(x+5+j)*(x+5-j))
```

b = -10.4000

$$c = \left(\frac{8(x+j2)(x-j2)}{(x+1)(x+3)(x+5-j)} \right)_{x=-5-j}$$

```
>> x = -5-j;
>> c = 8*(x+2i)*(x-2i) / ((x+1)*(x+3)*(x+5-j))
```

c = 4.6118 +12.0471i

$$d = \left(\frac{8(x+j2)(x-j2)}{(x+1)(x+3)(x+5+j)} \right)_{x=-5+j}$$

```
>> x = -5+j;
>> d = 8*(x+2i)*(x-2i) / ((x+1)*(x+3)*(x+5+j))
```

d = 4.6118 -12.0471i