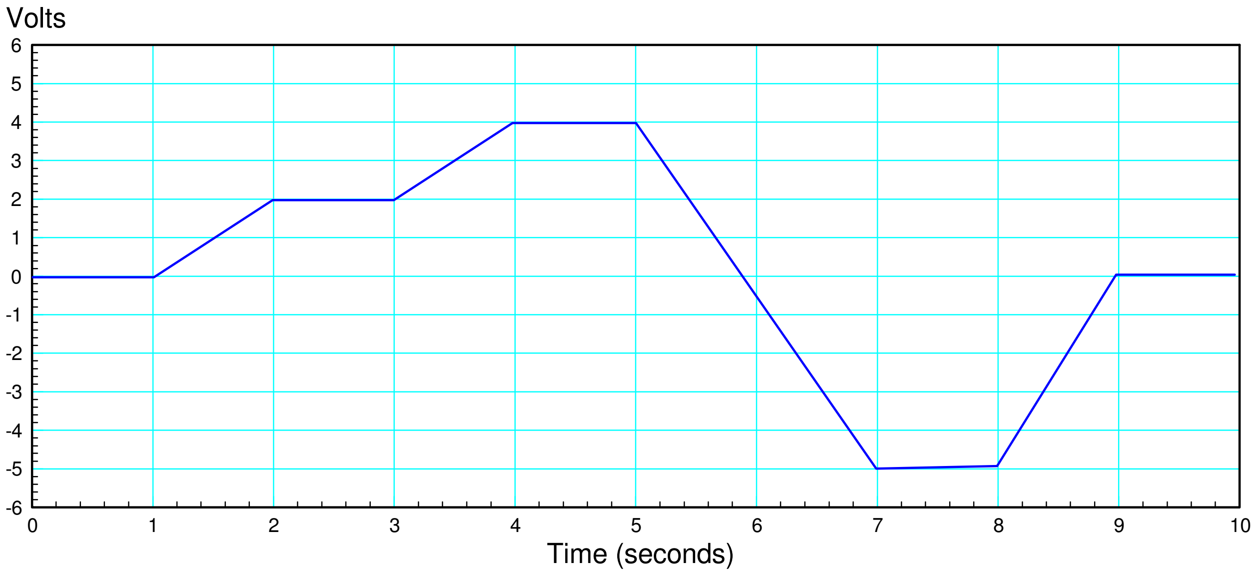
ECE 111 - Homework #11

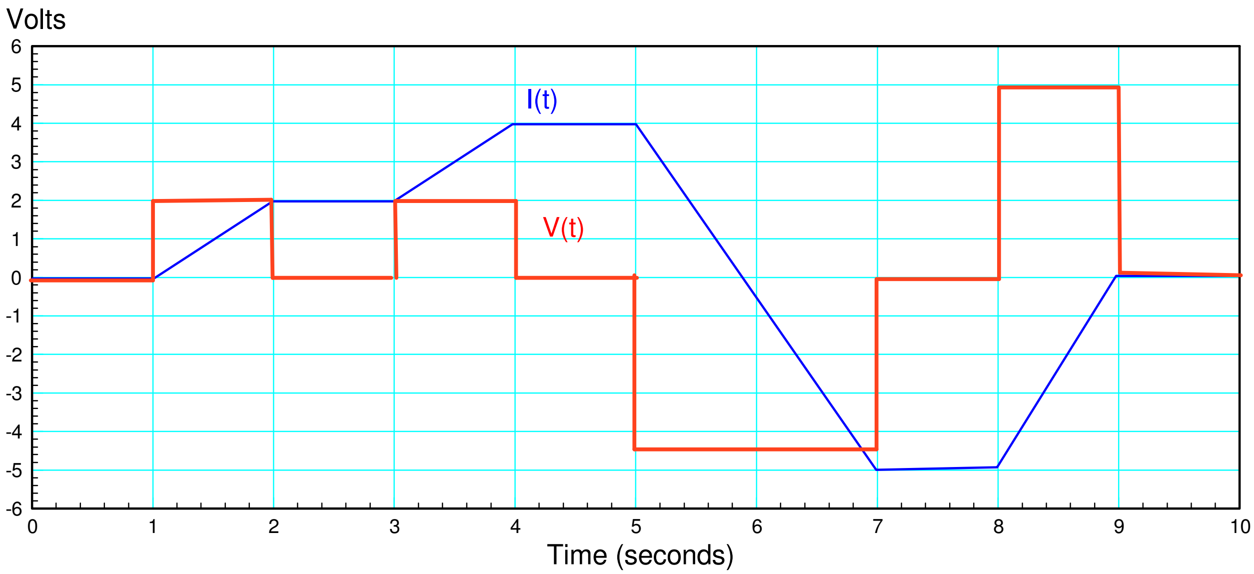
ECE 351 Electromagnetics & The Wave Equation

1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

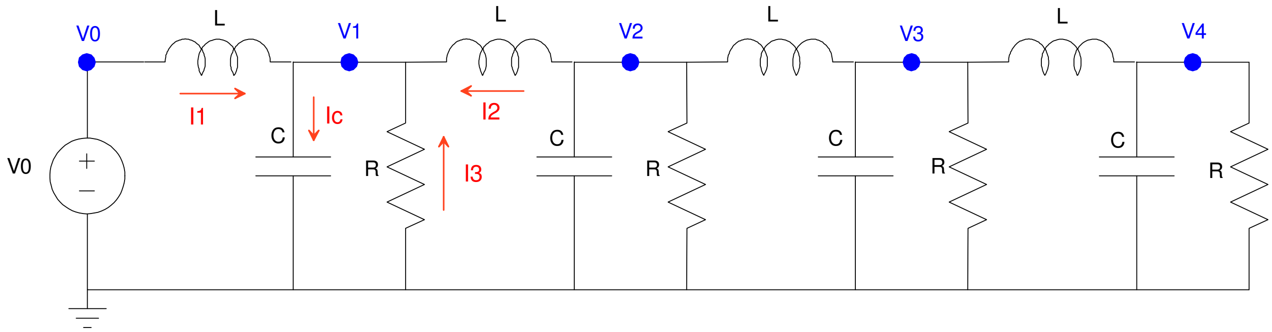
$$V = L\frac{dI}{dt}$$



Plot the derivative (slope) of the voltage



4-Node RLC Circuit



R = 220Ω, C = 0.2F, L = 0.15H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

This is a little tricky. Conservation of current tells you that

$$I_c = I_1 + I_2 + I_3$$

From the capacitor equation

$$I_c = CV_1' = I_1 + I_2 + I_3$$

Take the derivative again

$$CV_1'' = I_1' + I_2' + I_3'$$

For inductors

$$V = L\frac{dI}{dt}$$

$$LI_1' = V_0 - V_1$$

$$LI_2' = V_2 - V_1$$

Substituting

$$CV_1'' = \left(\frac{V_0 - V_1}{L}\right) + \left(\frac{V_2 - V_1}{L}\right) + \left(\frac{0 - V_1}{R}\right)'$$

Simplifying

$$V_1'' = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)V_1'$$

Plugging in numbers

$$V_1'' = 33.333V_0 - 66.667V_1 + 33.333V_2 - 0.023V_1'$$

Node #2 and #3 are similar due to symmetry

$$V_2'' = 33.333V_1 - 66.667V_2 + 33.333V_3 - 0.023V_2'$$

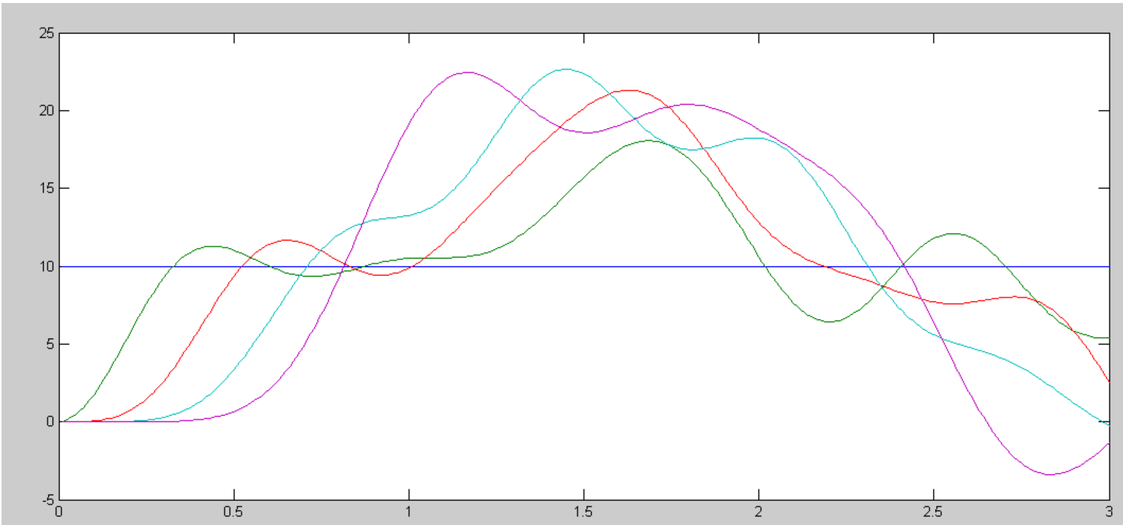
$$V_3'' = 33.333V_2 - 66.667V_3 + 33.333V_4 - 0.023V_3'$$

Node #4 is missing a term (since there is no node #5)

$$V_4'' = \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)V_4'$$

$$V_4'' = 33.333V_3 - 33.333V_4 - 0.023V_4'$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*



Voltages vs. Time

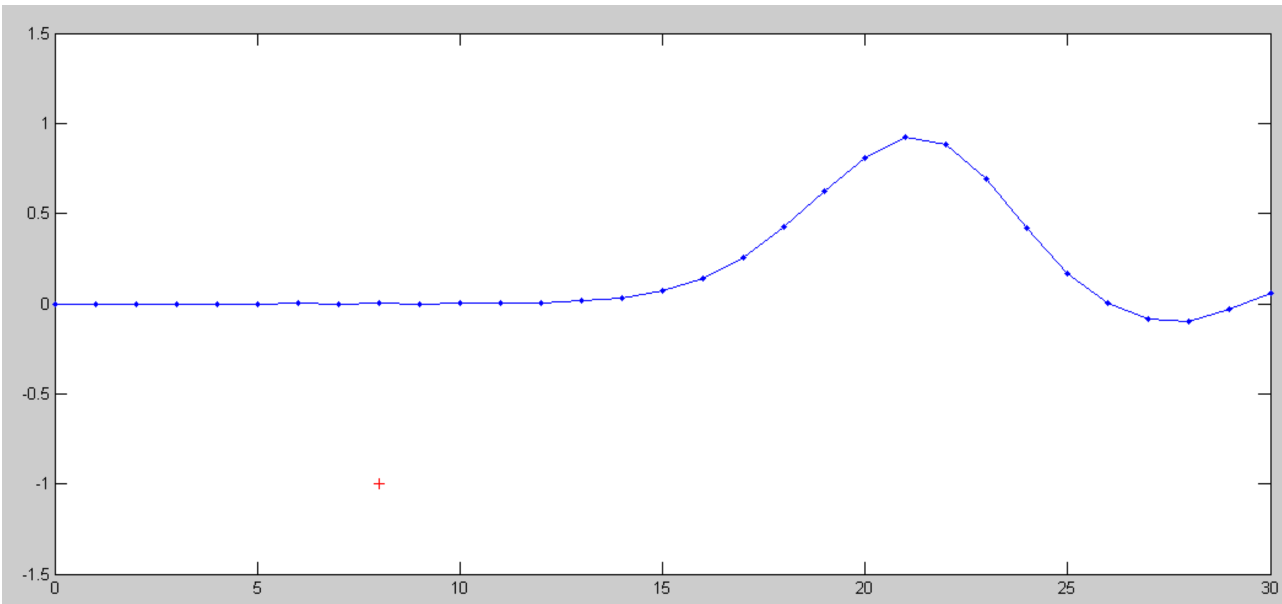
Matlab Code

```
V0 = 10;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;
dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;
V = [];
t = 0;
dt = 0.01;
while(t < 3)
 ddV1 = 33.333*V0 - 66.667*V1 + 33.333*V2 - 0.023*dV1;
 ddV2 = 33.333*V1 - 66.667*V2 + 33.333*V3 - 0.023*dV2;
 ddV3 = 33.333*V2 - 66.667*V3 + 33.333*V4 - 0.023*dV3;
 ddV4 = 33.333*V3 - 33.333*V4 - 0.023*dV4;
 dV1 = dV1 + ddV1*dt;
 dV2 = dV2 + ddV2*dt;
 dV3 = dV3 + ddV3*dt;
 dV4 = dV4 + ddV4*dt;
 V1 = V1 + dV1*dt;
 V2 = V2 + dV2*dt;
 V3 = V3 + dV3*dt;
 V4 = V4 + dV4*dt;
 t = t + dt;
 plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
 ylim([-5,25]);
 pause(0.01);
  V = [V ; [V0,V1,V2,V3,V4]];
 end
pause(2)
t = [1:length(V)]' * dt;
plot(t,V)
```

30-Node RLC Circuit (hint: modify the program Wave.m)

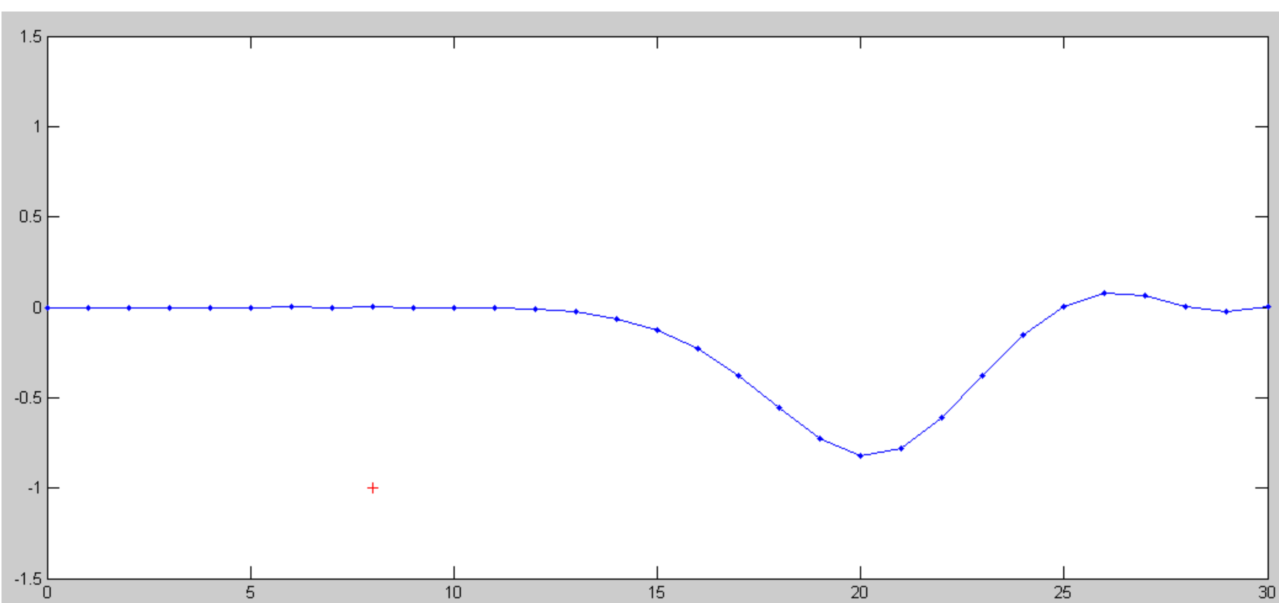
4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 8 seconds (just after the reflection) for $1 / R_{30}C = 0.01$

- A plus reflection results



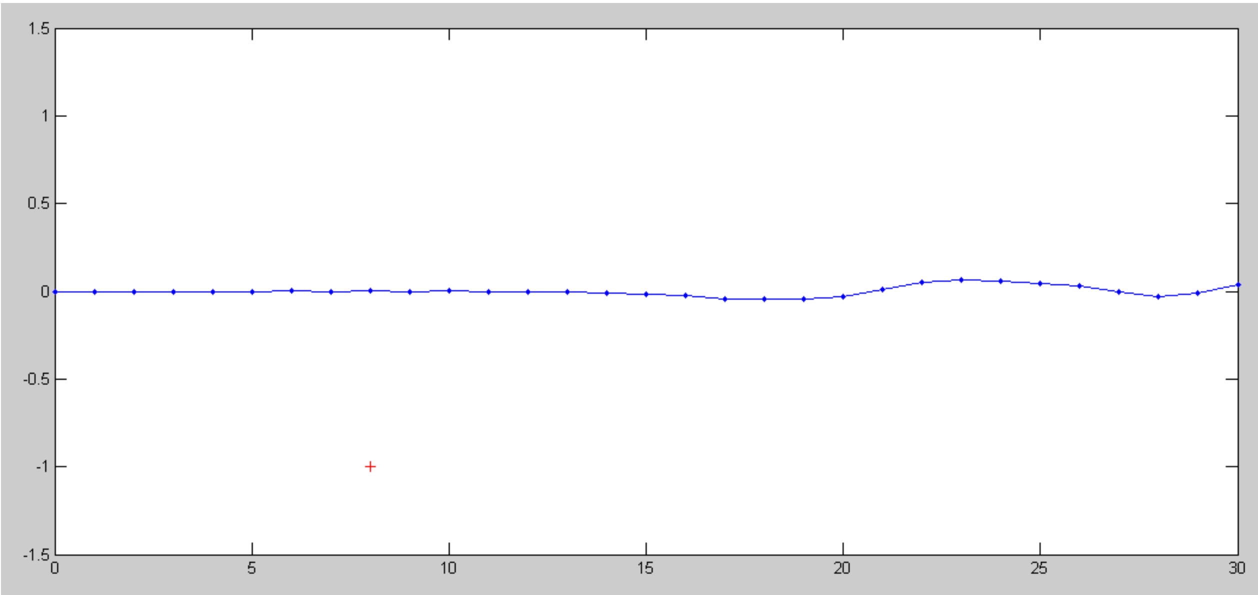
5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$

- A negative reflection results



6) Determine experimentally R_{30} so that the reflection is almost zero

- $1/RC = 5.8$ eliminates almost all reflections



Note:

- The heat equation is described by coupled 1st-order differential equations (HW10)
- The wave equation is described by coupled 2nd-order differential equations (HW11)
- Both are very common and behave very differently.
- When you take ECE 351 Electromagnetics, you'll cover how to calculate the resistance that eliminates all reflections
- In ECE 111, you can find the answer using Matlab along with trial and error with a numeric simulation