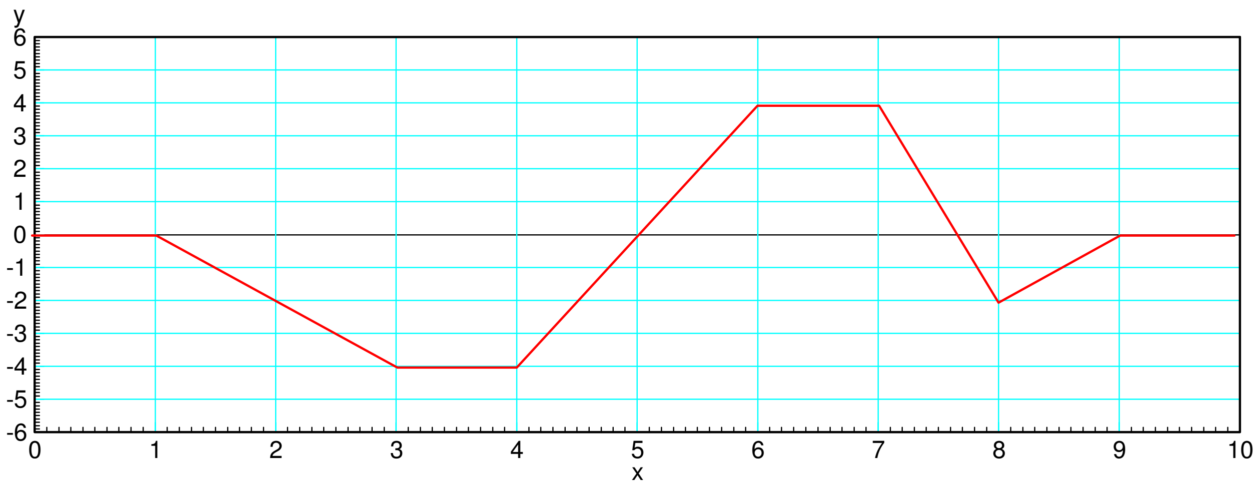
ECE 111 - Homework #10

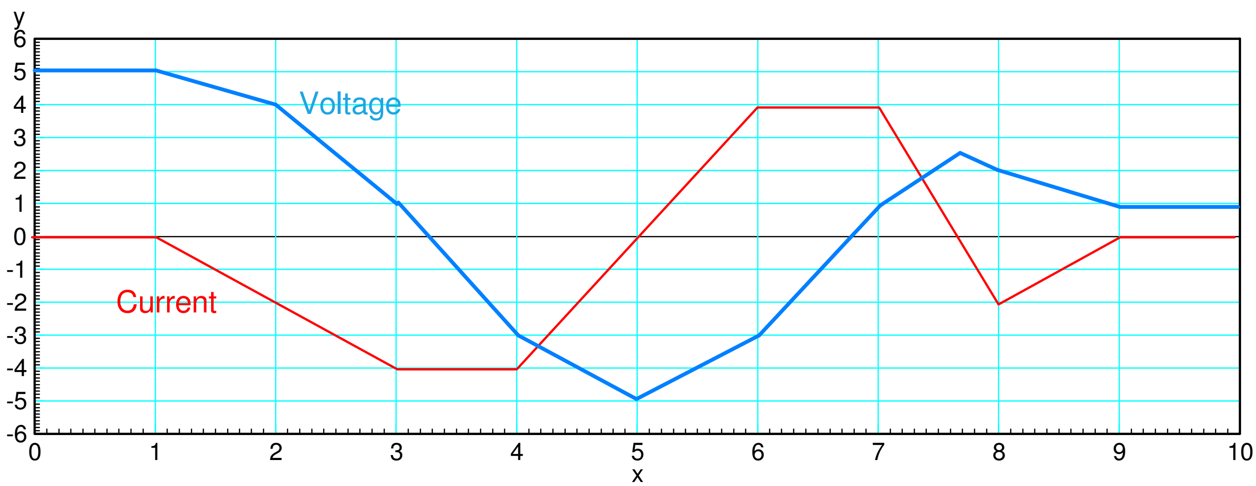
ECE 311 Circuits II - Heat Equation

Due Monday, October 30th

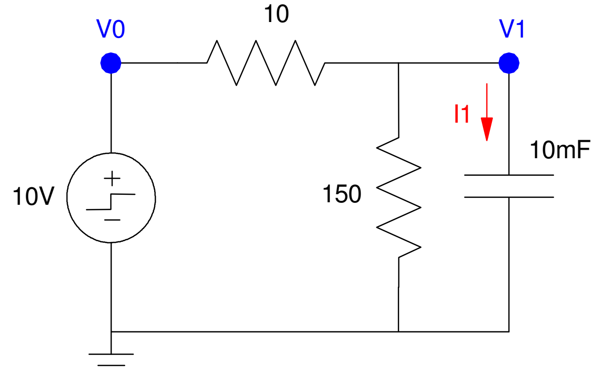
1) Assume the current flowing through a one Farad capacitor is shown below. Sketch the voltage. Assume V(0) = 5V. The voltage is the integral of the current (capacitors are integrators)

$$V = \frac{1}{C}\int I \cdot dt$$





1-Stage RC filter:



2) Write the differential equation that describe this circuit. Note:

$$I_1 = C\frac{dV_1}{dt} = \sum(\text{current to node}V_1)$$

$$I_1 = 0.01 \cdot \frac{dV_1}{dt} = \left(\frac{V_0 - V_1}{10}\right) - \left(\frac{V_1}{150}\right)$$

$$\boxed{\frac{dV_1}{dt} = 10V_0 - 10.667V_1}$$

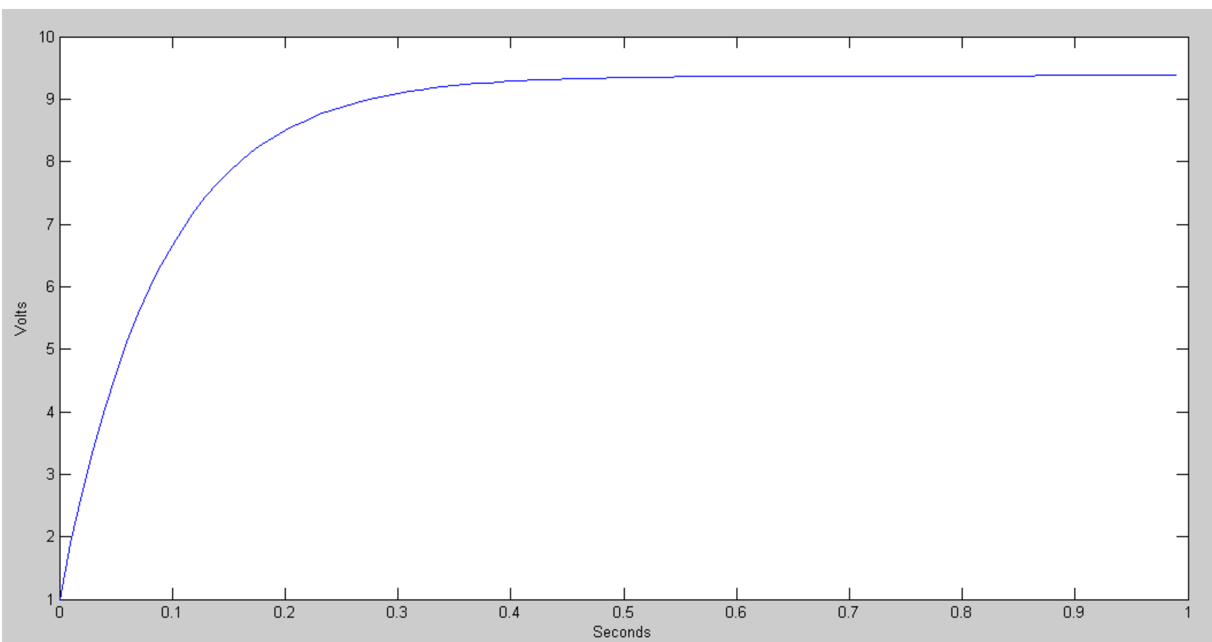
3) Find and plot V1(t) for five seconds using Matlab.

Solve in Matlab

$$\frac{dV_1}{dt} = 10V_0 - 10.667V_1$$

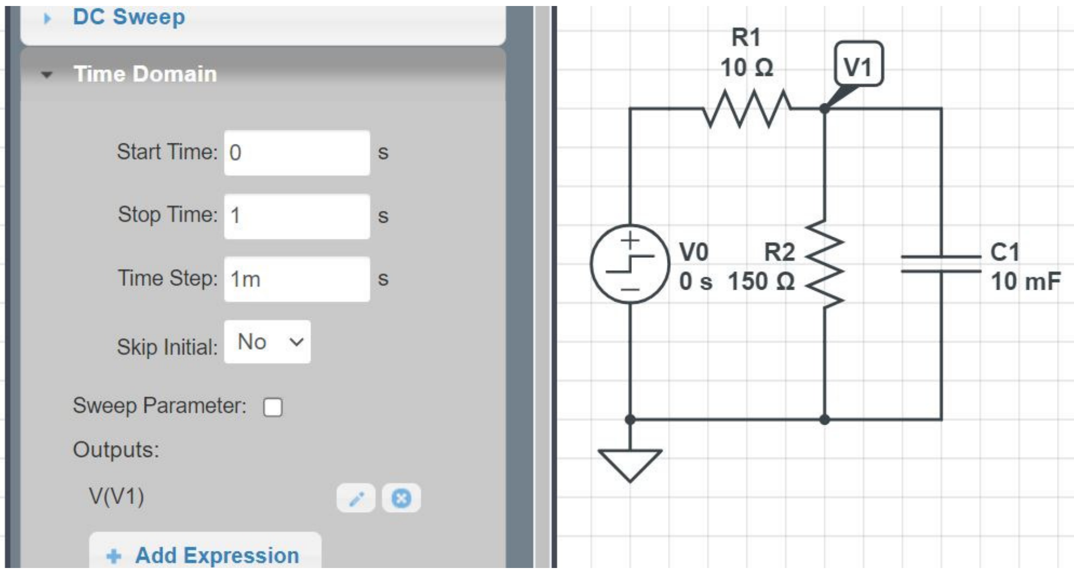
Matlab Code

```
V1 = 0;
t = 0;
dt = 0.01;
y = [];
V0 = 10;

while(t < 1)
    dV1 = 10*V0 - 10.667*V1;

    V1 = V1 + dV1*dt;
    t = t + dt;

    y = [y ; V1];
end

t = [0:length(y)-1]' * dt;
plot(t,y);
xlabel('Seconds');
ylabel('Volts');
xlim([0,1])
```



4) Find and plot V1(t) for one second using CircuitLab

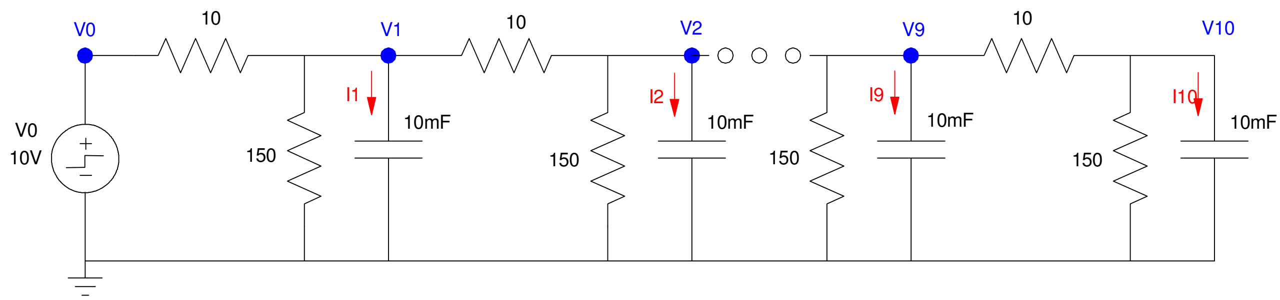
- Same plot we got in Matlab
- CircuitLab is using numerical integration to find the votlages



10-Stage RC Filter

5) Write the dynamics for this system as a set of ten coupled differential equations:

$$I_1 = C\frac{dV_1}{dt} = \sum(\text{current to node}V_1)$$



Nodes #1 through #9 will all be the same due to symmetry. Writing the node equation at V1:

$$I_1 = 0.01 \cdot \frac{dV_1}{dt} = \left(\frac{V_0 - V_1}{10}\right) - \left(\frac{V_1}{150}\right) + \left(\frac{V_2 - V_1}{10}\right)$$

$$\frac{dV_1}{dt} = 10V_0 - 20.667V_1 + 10V_2$$

$$\frac{dV_2}{dt} = 10V_1 - 20.667V_2 + 10V_3$$

$$\frac{dV_3}{dt} = 10V_2 - 20.667V_3 + 10V_4$$

$$\vdots$$

$$\frac{dV_9}{dt} = 10V_8 - 20.667V_9 + 10V_{10}$$

Node #10 is the oddball due to only having one 10 Ohm resistor connected to it

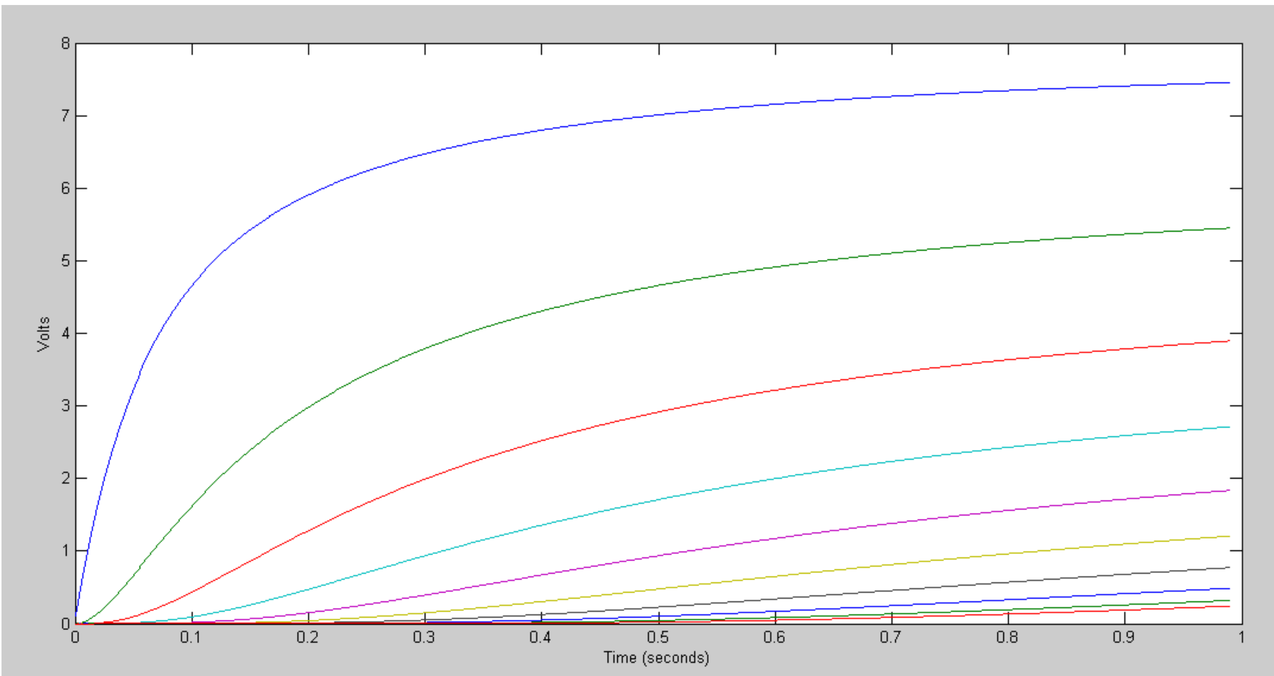
$$I_{10} = 0.01 \cdot \frac{dV_{10}}{dt} = \left(\frac{V_9 - V_{10}}{10}\right) - \left(\frac{V_{10}}{150}\right)$$

$$\frac{dV_{10}}{dt} = 10V_9 - 10.667V_{10}$$

6) Using Matlab, solve these ten differential equations for $0 < t < 1$ s assuming

- The initial voltages are zero, and
- V0 = 10V.

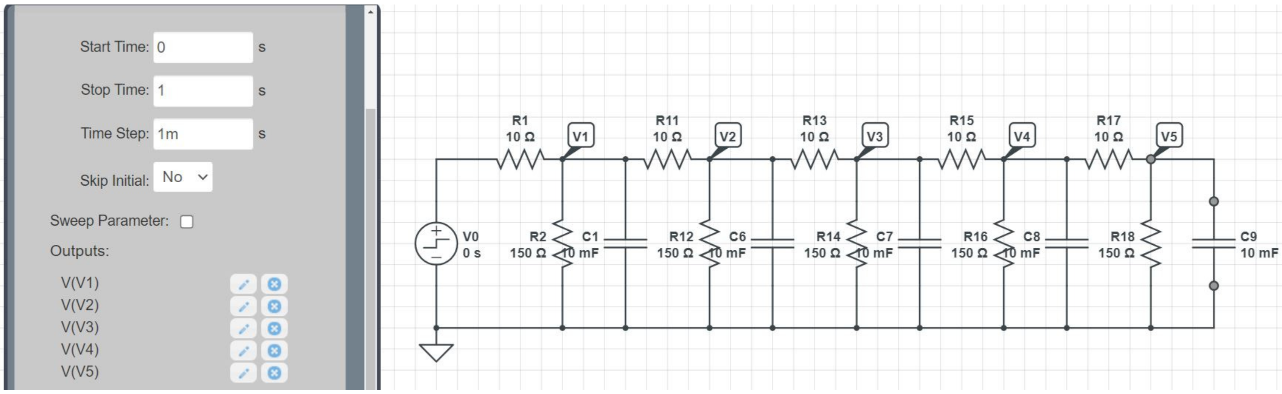
Same result as CircuitLab

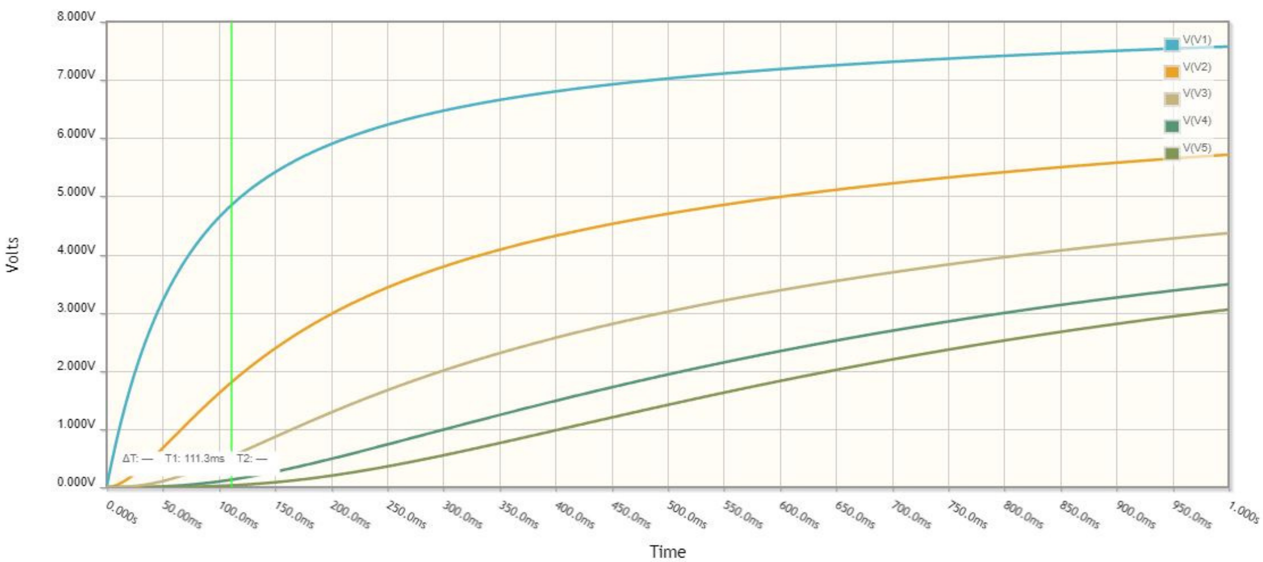


Code:

```
% 10-stage RC Filter

V = zeros(10,1);

dV = zeros(10,1);
V0 = 10;
dt = 0.001;
t = 0;
i = 0;

y = [];

while(t < 0.99)

   dV(1)  = 10*V0   - 20.667*V(1) + 10*V(2);
   dV(2)  = 10*V(1) - 20.667*V(2) + 10*V(3);
   dV(3)  = 10*V(2) - 20.667*V(3) + 10*V(4);
   dV(4)  = 10*V(3) - 20.667*V(4) + 10*V(5);
   dV(5)  = 10*V(4) - 20.667*V(5) + 10*V(6);
   dV(6)  = 10*V(5) - 20.667*V(6) + 10*V(7);
   dV(7)  = 10*V(6) - 20.667*V(7) + 10*V(8);
   dV(8)  = 10*V(7) - 20.667*V(8) + 10*V(9);
   dV(9)  = 10*V(8) - 20.667*V(9) + 10*V(10);
   dV(10) = 10*V(9) - 10.667*V(10);

   V = V +  dV*dt;
   t = t + dt;

   y = [y ; V'];

   plot([0:10], [V0;V], '.-', t, 0, 'b+');
   xlim([0,10]);
   ylim([0,10]);
   pause(0.01);

end

pause(10);
hold off

t = [1:length(y)]' * dt;
plot(t,y);
xlabel('Time (seconds)');
ylabel('Volts');
```

7) Using CircuitLab, find the response of this circuit to a 10V step input. *note: It's OK if you only build this circuit to 3 nodes...*





Natural Response: Eigenvectors and Eigenvalues

8) Assume V0 = 0V. Determine the initial conditions of V1..V10 so that

- The maximum voltage is 10V and
- 5a) The voltages go to zero as slow as possible
- 5b) The voltages go to zero as fast as possible.

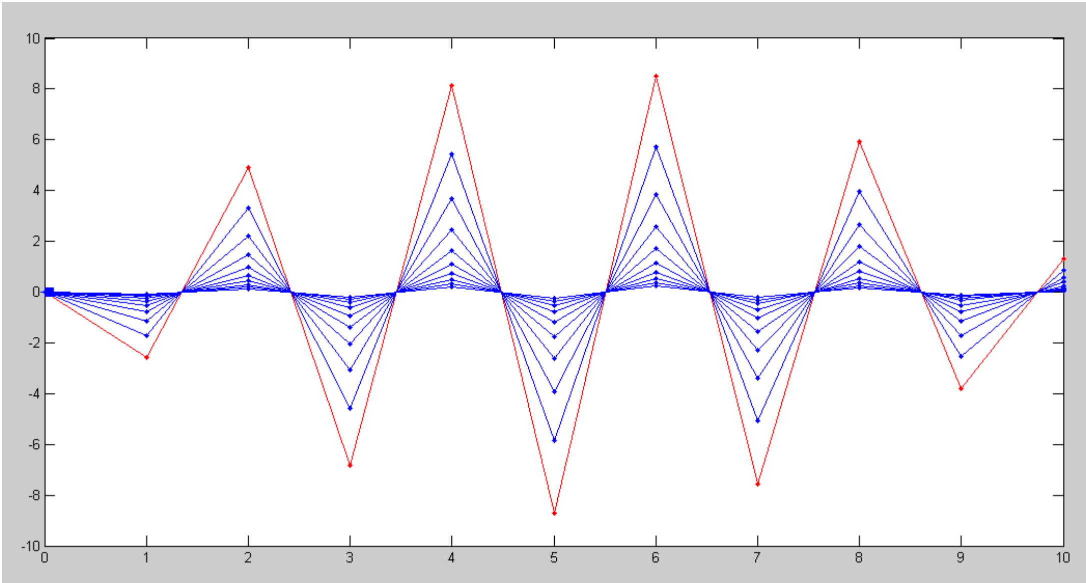
Simulate the response for these initial conditions in Matlab.

This is an eigenvlaue / eigenvector problem. Placing the dynamics in matrix form

$$dV = A \cdot V + B \cdot V_0$$

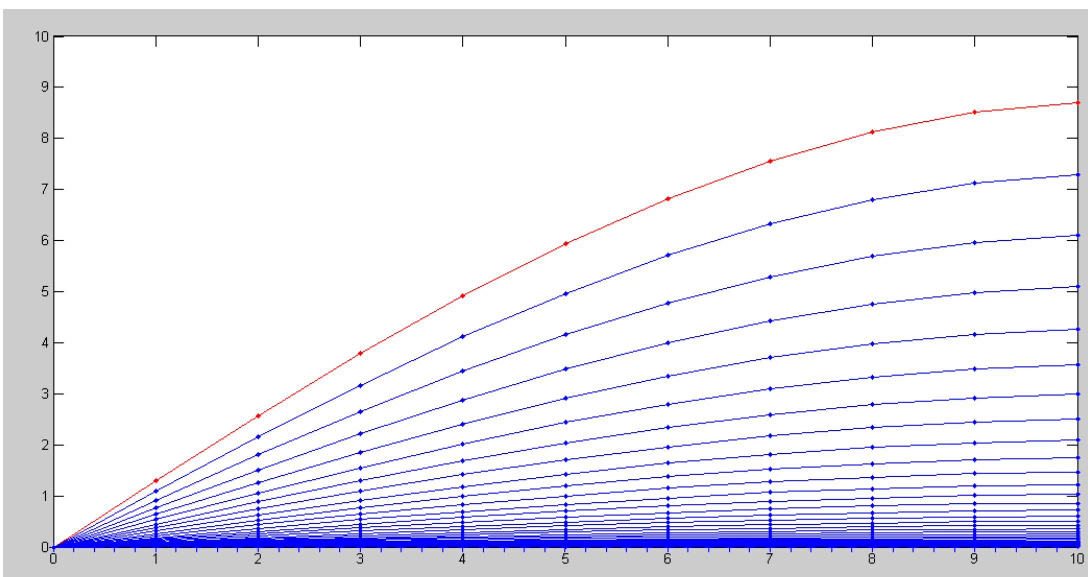
```
>> A = zeros(10,10);
>> for i=1:9
A(i,i) = -20.667;
A(i,i+1) = 10;
A(i+1,i) = 10;
end
>> A(10,10) = -10.667

A =

  -20.6670   10.0000         0         0         0         0         0         0         0         0
   10.0000  -20.6670   10.0000         0         0         0         0         0         0         0
         0   10.0000  -20.6670   10.0000         0         0         0         0         0         0
         0         0   10.0000  -20.6670   10.0000         0         0         0         0         0
         0         0         0   10.0000  -20.6670   10.0000         0         0         0         0
         0         0         0         0   10.0000  -20.6670   10.0000         0         0         0
         0         0         0         0         0   10.0000  -20.6670   10.0000         0         0
         0         0         0         0         0         0   10.0000  -20.6670   10.0000         0
         0         0         0         0         0         0         0   10.0000  -20.6670   10.0000
         0         0         0         0         0         0         0         0   10.0000  -10.6670

>> [M,V] = eig(A)

M = Eigenvectors
    fast                                                                                      slow
   -0.1286   -0.2459    0.3412    0.4063    0.4352    0.4255    0.3780    0.2969   -0.1894    0.0650
    0.2459    0.4063   -0.4255   -0.2969   -0.0650    0.1894    0.3780    0.4352   -0.3412    0.1286
   -0.3412   -0.4255    0.1894   -0.1894   -0.4255   -0.3412   -0.0000    0.3412   -0.4255    0.1894
    0.4063    0.2969    0.1894    0.4352    0.1286   -0.3412   -0.3780    0.0650   -0.4255    0.2459
   -0.4352   -0.0650   -0.4255   -0.1286    0.4063    0.1894   -0.3780   -0.2459   -0.3412    0.2969
    0.4255   -0.1894    0.3412   -0.3412   -0.1894    0.4255    0.0000   -0.4255   -0.1894    0.3412
   -0.3780    0.3780    0.0000    0.3780   -0.3780   -0.0000    0.3780   -0.3780   -0.0000    0.3780
    0.2969   -0.4352   -0.3412    0.0650    0.2459   -0.4255    0.3780   -0.1286    0.1894    0.4063
   -0.1894    0.3412    0.4255   -0.4255    0.3412   -0.1894    0.0000    0.1894    0.3412    0.4255
    0.0650   -0.1286   -0.1894    0.2459   -0.2969    0.3412   -0.3780    0.4063    0.4255    0.4352


>> eig(A)'

Eigenvalues

  -39.7785  -37.1918  -33.1368  -27.9738  -22.1616  -16.2166  -10.6670   -6.0060   -2.6476   -0.8904

>>
```

Fast Mode:



Fast Mode: The shape remains the same (the eigenvector).
The amplitude quickly decays to zero (the eigenvalue)

Slow Mode

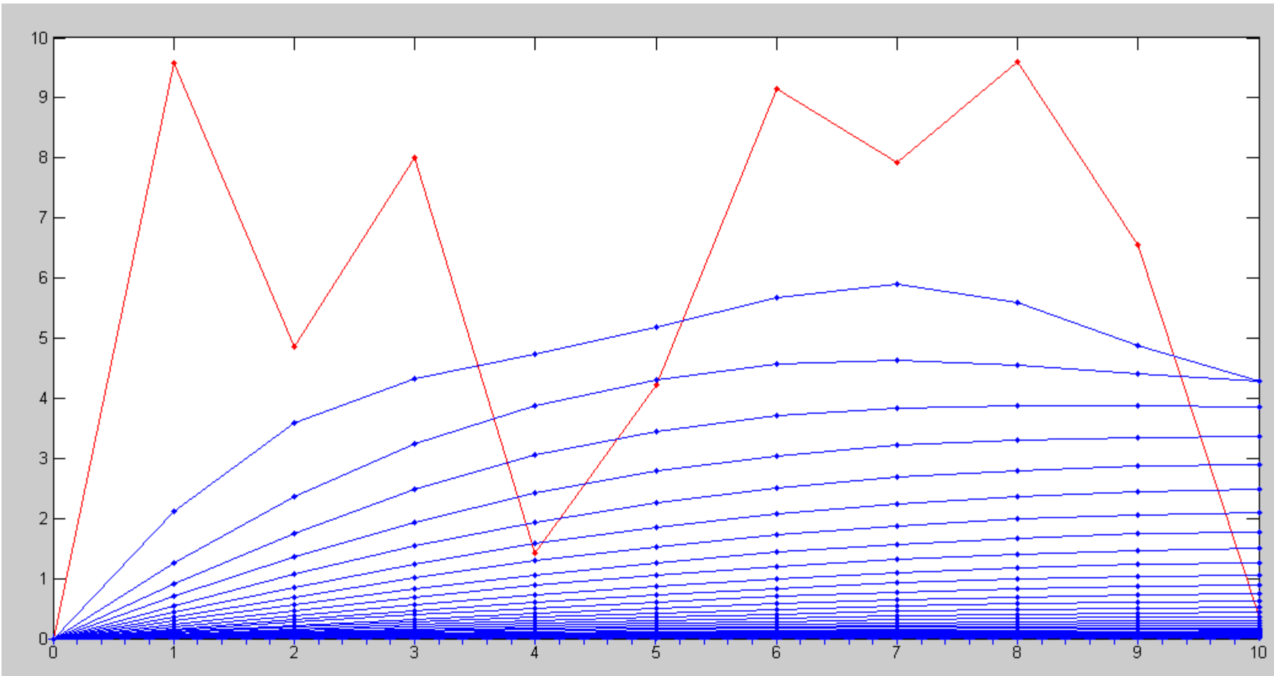


Slow Mode: The shape remains the same (the eigenvector).
The amplitude slowly decays to zero (the eigenvalue)

9) Assume Vin = 0V. Pick random voltages for V1 .. V10 in the range of (0V, 10V):

```
V = 10 * rand(10,1)
```

Plot the votlages at t = 2. Which eigenvector does it look like?



Response to a random initial condition
The fast modes decay quickly, leaving the slow mode

Matlab Code

```
% Fast Mode
 V = 20*M(:,1);

% Slow Mode
 V = 20*M(:,10);

% Random Initial Condition
 V = 10*rand(10,1);

dV = zeros(10,1);
V0 = 0;
dt = 0.002;
t = 0;
i = 0;

while(t < 9.9)
   < rest of code >
```